negative quantities. To grow more powerful in mathematical thinking requires a focus on the essentials that operate in a more general manner.

The historical allusions to Norse mythology make fascinating reading and I would not be without such a vision. Yet, at higher levels, mathematics must operate in a wide range of situations where particular embodiments give meanings that may not be applicable in other contexts. Consider, for example, the interpretation of a function as distance given in terms of time, and all the meanings and allusions involved as its derivative is called velocity, with velocity having a derivative called acceleration, and its derivative often called "jerk" because a sudden change in acceleration is felt as a jerk. But this meaning does not fit well with simple harmonic motion, such as the case where the distance is sin*t*, the velocity is cos*t*, the acceleration is -sin*t* and the jerk is -cos*t*. In what sense is the smooth function -cos*t* a "jerk"? While some embodiments may be supportive in many ways, they may also have particular characteristics that are problematic and impede generalization.

While mathematics should have meaning, such meanings should be *flexible* to take account of other situations where the mathematics may be applicable. Embodied representations are powerful in giving fundamental meanings but they usually lack the power of symbolism to formulate ideas precisely and to find exact solutions to complex problems. They may also involve problematic met-befores that impede understanding in new contexts.

Mathematics must harness gesture and embodiment in enactive and visual representations, but it must also develop a flexibility in the use of symbolism and, where appropriate, the later development of mathematical formalism and proof. Mathematics needs to be related to real world situations but it also needs a symbolic fluency and power of its own that sets it free to move on to previously unimaginable ideas.

We therefore need to look at our own theories in more humble ways, to seek simple ways of expressing practical insights into teaching and learning. There are huge problems to address. There is much for us still to do to understand the nature of mathematical thinking with its crystalline structure and longer-term supportive and problematic met-befores. We should also apply our analysis not only to the learning of students but also to the supportive and problematic aspects of our own theories.

References

Gerofsky, S. (2011) Ancestral genres of mathematical graphs. *For the Learning of Mathematics* **31**(1), 14-19.

McGowen, M. & Tall, D. O. (2010) Metaphor or Met-before? The effects of previous experience on practice and theory of learning mathematics. *Journal of Mathematical Behavior* **29**(3), 169-179.

Meaney, T. (2011) Only two more sleeps until the school holidays: one child's home experiences of measurement. *For the Learning of Mathematics* **31**(1), 31-36.

Oldernburg, R. (2011) Some thoughts on mathematics in the workplace and in school. *For the Learning of Mathematics* **31**(1), 28-30.

Perry, W. G. (1970) *Forms of Intellectual and Ethical Development in the College Years: A Scheme.* New York, NY: Holt, Rinehart & Winston.

Renert, M. (2011) Mathematics for life: sustainable mathematics education. *For the Learning of Mathematics* **31**(1), 20-26.

Samson, D. & Shäfer, M. (2011) Enactivism, figural apprehension and knowledge objectification: an exploration of figural pattern generalisation. *For the Learning of Mathematics* **31**(1), 37-43.

von Glasersfeld, E. (1990) An exposition of constructivism - why some like it radical. In Davis, R. B. Maher, C. A. & Noddings, N. (Eds.) *Constructivist Views on Teaching and Learning Mathematics. Journal for Research in Mathematics Education* Monograph 4, pp. 19-30.

Modeling for life

WALTER WHITELEY

Almost all of us grew up mathematically with an assumption of the certainty of mathematical reasoning. Given certain input, we have certain output. However, problems like climate change, the degrading changes in the sea, and health treatment decisions have an unavoidable level of uncertainty in the input, as well as some complex connections. What is causal in a multi-variable set of complex connections? (See [1] for reflections on such a fundamental shift). It is a basic characteristic of modeling in situations such as climate change, or health outcomes, that the input and the conclusions are stochastic (subject in essential ways to uncertainty) but still have an appropriate form of reliability. This characteristic leads into probabilistic reasoning, a new topic not yet well supported in the curriculum or in the preparation of teachers.

As Renert (2011) notes, transforming action on such topics (and on others), also requires essential learning about modeling. Students need to practice a form of problem-based learning and simulation with multiple variables. In this approach, students and teachers search for mathematics sufficient to run a simulation of changes and then make some sense of why or why not to consider information from models as reliable. As Renert notes, this will require the use of modeling software, often in a black box form. This focus on modeling represents a shift from Platonist, deterministic, deductive, proof-based reasoning, towards modeling-based and stochastic-based reasoning. This shift positions applied mathematics as a key defining experience of current mathematics, in place of an approach to mathematics education which takes pure mathematics as the ideal to be emulated by classrooms full of students, few of whom would become users of formal proofs.

Modeling will require the essential use of technology, and data for which we do not have even a good approximating formula. This work can be supported by information presented and reasoned about visually, rather than being immediately transferred to algebra. A move towards qualitative reasoning

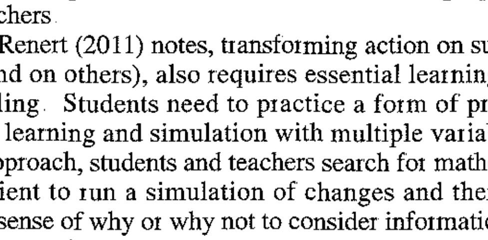

Figure 1. Simplified representation of a temperature distribution (vertical height represents number of days at a given temperature)

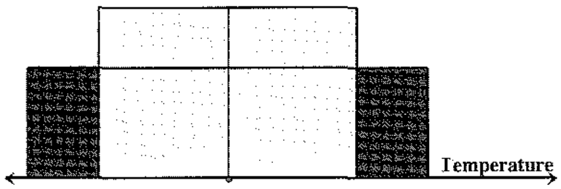

Figure 2. *Simplified representation superimposing increased variability in in temperatures*

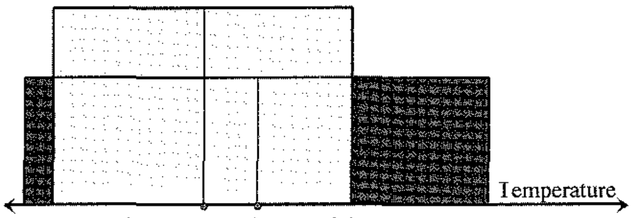

Figure 3 *Simplified representation superimposing increased variability in temperature plus increased average temperature to generate more events at both extremes, but many more extreme heat events*

(including qualitative visual reasoning) would be a big shift in the current curriculum. Many mathematics undergraduates (and future teachers) still think of formulae as the primary representation of functions, as I know from recent classes Representation via formulae worked for Euler, but it will not work for us with these problems

Here is a very simple example of "visual modeling" of the frequency of extreme events in the climate (the statistics of weather) The representations shown in Figures 1-3 are designed to support reasoning about how global warming could generate many more extreme hot events *but also* some more extreme cold events, with modest increases in mean temperature [2] Figure 1 is a very simplified visual representation of the variety of temperatures over a year. The second image (Figure 2) superimposes what the weather distribution would look like if the model gives a wider variation of weather, with the same average temperature.

The third diagram (Figure 3) superimposes a new distribution, with the wider variation plus an increased average temperature. What may seem as contrary evidence (more extreme cold events) becomes part of a larger visual comparison that can be further adapted to investigate patterns with more complex distributions. Similar changes in distribution can be explored for other extremes such as in rainfall (*i.e.*, drought and flood). Using visual representations as the initial model makes the discussion "reasonable" (one about which we can reason).

Reliance on modeling, with its corresponding uncertainty, is a factor in debates about the regulation of DDT, acid rain, tobacco smoke and secondary smoke, depletion of the ozone layer, and now CO_2 loads and climate change. In the recent book *Merchants of Doubt,* the authors, Naomi Oreskes and Erik M. Conway (2010), describe several decades over which key people have worked to prevent any form of regulation on all of these issues, in part by questioning the science (and mathematics) of predictions based on modeling and by contesting any decisions made under these stochastic characteristics (uncertainty).

The development of the capacity to assess information emerging from modeling, with a described range of uncertainty, and to still make decisions within this context is a task that cannot happen without effective mathematics education Developing this capacity is also essential in a culture that often interprets "likely" as a synonym for "do I want it to happen"! I look forward to richer examples and more pointed discussion within the mathematics education community.

Notes

[1] AAKKOZZLL: chance, statistics and a new paradigm: www.aakkozzll.com Note the pronunciation of the site name is 'acausal'.
[2] These images are simplified and adapted from Weaver (2008, p 8).

References

Oreskes, N. & Conway, E M. (2010) *Merchants of Doubt: How a Handful of Scientists Obscured the Truth on Issues from Tobacco Smoke to Global Warming.* New York, NY: Bloomsbury Press
Renert, M (2011) Mathematics for life: sustainable mathematics education *For the Learning of Mathematics* **31**(1) 20-26.
Weaver, A. (2008) *Keeping Our Cool: Canada in a Warming World* Toronto, ON: Penguin.

Now it concerns us! A reaction to Sustainable Mathematics Education

UWE GELLERT

The third millennium has come with a remarkably increased awareness of ecological dynamics triggered by the way we live on earth. Sustainability appears as a new key concept for thinking and acting Do we need a Sustainable Mathematics Education? If so, what could Sustainable Mathematics Education look like? I will briefly comment on the not-so-altruistic nature of the sudden concern for sustainability before arguing that the political and sociological dimensions of the relationship between mathematics, technology and society are fundamental to an "ethic of mathematics for life" (Renert, 2011, p. 25).

The catastrophic side effects of industrial production and increasing consumption have always been part and parcel of economic growth during the last century. The catastrophes have been more visible when abrupt, *e.g.* the disasters of Bhopal or Chernobyl of Fukushima, although the pollution caused continuously by major and minor raw material plants in Latin America, Africa and Asia may have caused even more damage to individuals and the environment This destruction of life and lifeforms has not directly affected all those who have profited most from the outsourcing of the