TEACHING MATHEMATICS DEVELOPMENTALLY: EXPERIENCES FROM NORWAY

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Mona That's where the motivation comes from. When they have to struggle a bit, and then can manage. They're like "Ooh, it's fun", right? And that's where the motivation lies to have a go at anything later on.

'Mona' is a mathematics teacher at a Norwegian elementary school. She is one of a group of local teachers who shared a particular concern: that their students did not find the regular mathematics lessons challenging enough and thus were bored.

Mona and the four other teachers whose voices are heard in this article have, along with hundreds of other elementary teachers, met this teaching problem by replacing a widespread Norwegian mathematics textbook series with a textbook series developed in the 1990s under the supervision of the Russian mathematician and educator Iren Arginskaya. The textbooks were inspired by the educational system of Leonid V. Zankov (1901–1977), a student and colleague of Lev S. Vygotsky.

Here we present a short account, based on the teachers' unique experiences, of the challenges they have faced and how the use of Russian textbooks has changed their views of mathematics teaching and learning.

The state of mathematics teaching in Norway

Norwegian teachers are free to choose their teaching material and methods as long as they follow the general curriculum. Previous research on mathematics in Norwegian classrooms shows that this subject has often been taught in traditional ways, focusing on routine skills, memorization of isolated facts and algorithms, relying strongly on textbooks (Alseth, Brekke & Breiteig, 2003). Furthermore, the mathematics textbook has traditionally been more dominant in teaching in the Nordic countries than elsewhere (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002). The current situation is not satisfactory, as reflected in the results of a test in secondary school mathematics that the Norwegian Mathematics Council arranges for first year university students. In 2017, an average engineering student had 46% correct answers, while the average for teacher-training and economics students was 41% and 31%, respectively [1].

Zankov's system of developmental education

Zankov's system for elementary education was developed through 20 years of extensive school research in the former Soviet Union. Although it has been introduced to an international audience before (Boguslavsky, 2015; Guseva & Solomonovich, 2017; Zankov, 1977), this article provides a first glimpse of its application in mathematics education in a Western context [2].

The main goal of Zankov's system is "striving to reveal the potential of [each] pupil and to create favorable conditions for its development" (Zankov, 1977, p. 62)—meaning not only their mental development, but also development of their "abilities, natural gifts, independence, and initiative" (p. 255). The system rests on five interconnected principles (Zankov, 1977):

- Teaching at a high level of difficulty
- The leading role of theoretical knowledge
- Proceeding at a rapid pace
- Promoting the students' awareness of the learning process
- Systematic development of each student in the classroom

The Norwegian project of implementing Zankov's system in mathematics education is called Developmental Education in Mathematics (DEM) and exists for grades 1–4.

Introduction of DEM in Norway

The DEM project began in 2009, when one elementary school teacher, unhappy with her practice at the time, approached Natalia for advice. Together, they looked abroad for new impulses. They chose Arginskaya's books since they saw particular qualities in them, some of which are briefly explained here. Arginskaya's textbooks cover the Norwegian mathematics curriculum, and so can be smoothly integrated into teaching.

Natalia, together with other scholars at our university, translated and adapted the textbooks to a Norwegian context, wrote teachers' guides, and offered seminars to interested teachers [3]. More than 70 elementary schools across Norway are currently using DEM. A point to keep in mind is that the teachers use the system with limited support except occasional evening seminars.

Our study

The question we pose is whether and how using these textbooks has enabled the teachers in our study to come to new awareness of how to provide their students with a deeper

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learning experience in mathematics. The teachers in this study are only a small sample of many teachers who use DEM. Four of them, Mona, Siri, Henry and Anne (pseudonyms), were among those who joined DEM in its early stages. Vivi, on the other hand, was an 'inexperienced' teacher who had only used DEM for about three months at the time of the interviews.

Methodologically, we build on Coles (2018) who discusses and classifies different ways teacher learning can be observed from talk. We see the teachers' reflections presented here as *self-reporting new awareness,* where "listeners are offered a description of a change and an associated new awareness" (p. 24), thus providing evidence for teachers' professional learning.

We have organized this article according to Zankov's five theoretical principles, while drawing on quotes from the interviews in order to bring out the teachers' experiences with Zankov's system.

Teaching at a high level of difficulty

> You may be very much interested in your students and guide them well, but if you don't give them an opportunity to suffer a little on their own with some problems, as a rule, nothing happens afterwards. But if you get them excited about doing something, they will fight through. (Arnold Ross, cited in Jackson, 2001, p. 698)

Zankov and his team were the first to investigate Vygotsky's cultural-historical theory in large-scale school experiments (Zankov, 1977). At the heart of the system lies Vygotsky's concept of the Zone of Proximal Development (ZPD), an area of action where the classroom collaboration may bring about beneficial developmental outcomes.

This principle is characterized "primarily by the fact that it reveals the child's mental powers and gives it room and direction". It involves both "the complexity of the material and [...] the exertion of effort by the pupils" (Zankov, 1977, p. 55). This resonated with all our participating teachers. Vivi, who was quite critical about other aspects of DEM, viewed the principle of a high level of difficulty as central to the system and was very supportive about it:

> *Vivi* What I really like is that the students are not supposed to learn one way to do it and that's it. They actually have to ponder, to think, to dare to make mistakes. And they have to challenge themselves and be in that scary zone [*laughs*] before we can find a solution together.

Above, Vivi touches on several important aspects related to this principle. She notes the importance of the students' own activity (*they* have to ponder, think and make mistakes) and the emotional investment that is required from them (*to dare* to make mistakes, that *scary* zone) in order to engage with challenging problems. And saying that *we* can find a solution together, she frames mathematics teaching as a social activity, as a collaboration between a teacher and her students, which is at the heart of the concept of the ZPD.

Consider, for example, the task presented in Figure 1, from the second-grade textbook. Often, such tasks that introduce new concepts or procedures are presented step by step by the teacher in whole-class discussions, keeping the task sequence and not giving away the main conclusion. The task is designed in a way that makes it clear for the students that their previous solution methods are inadequate (task b). This raises an obstacle and a need for new knowledge to overcome it (task c). Note that the book does not provide a ready-made answer in task c. This makes the students responsible for their choice and provides an opportunity to co-create new knowledge.

66 a) Copy the equations in your workbook.

$9 + 7 = 16$	$c = c$	$15 - 8 < 10$
$x - 9 = 5$	$14 - k = 8$	$16 - y = 8$
$7 + 6 < 16$	$a + 3 = 12$	$12 - 7 = 5$
$b + 1 = b$	$17 - 6 > 8$	$11 + 5 = 16$

b) Try to solve the equations. Did you have trouble with some of them? Why?

c) Is there an equation in a) that has no solution? Which one?
Is there an equation in a) that has infinitely many solutions? Which one?

d) Read and try to remember:

A number is called a **root of an equation** if the equality becomes true when the unknown is replaced with this number.

Solving an equation means finding all roots of the equation or showing that the equation has no solution.

e) Find the roots of the equations.

$m + 7 = 13$ $9 + p = 11$
$15 - n = 6$ $e - 7 = 8$

f) Write several equations with root 3.

g) Write an equation that has no solution.

h) Write an equation that has infinitely many solutions.

Figure 1. Task 66, a problem-oriented task requiring new knowledge. (Arginskaya et al., 2015, p. 32).

DEM is demanding for the students, but this does not imply that it is stressful. In fact, the teachers considered challenge a key factor for motivating their students, as in Mona's opening quote of this article. Siri pointed out an additional benefit:

> *Siri* I can use it in Norwegian class as well; I can use texts that I think are a bit above their level. But if we work on it together, I think we are able to understand them anyway, texts from higher grades. And they accept it, because they're used to it from mathematics. "OK, we're going to work on something difficult" and they accept it.

Siri points to the generality of this principle: that students' ability and inclination to take on challenges is beneficial for their academic activity overall. An important consequence of the high level of difficulty is that it is necessary that the teacher assumes a central role in supporting the students in all stages of their problem solving, including task understanding:

Anne There were many times, even in fourth grade, when the students were struggling just reading by themselves what they were supposed to do. *[...]* So you have to talk with them about it, "what does it say here" and "what's important to draw out from this", maybe simplify a few words or use other words that mean the same, and then they're like "Aha, OK!"

This means that teachers who prefer having their students work individually on a set of tasks during class will probably not find DEM attractive. While many of Arginskaya's tasks are suitable for individual work, the whole-class discussion is the centerpiece of 'Zankov's lesson' [4]. Given the high level of difficulty, our participants stressed the importance of developing not only the intellectual aspects of mathematical activity, but the affective as well—in particular reducing anxiety and increasing motivation. They advised establishing a safe classroom environment where students can offer novel or different ideas without fear of ridicule or rejection. According to our participants, this means treating wrong answers as learning opportunities, praising students for revising their thinking, cultivating a generally supportive atmosphere and developing in students a positive disposition.

The leading role of theoretical knowledge

The leading role of theoretical knowledge means that "skills are formed on the basis of general development and of the deepest possible understanding of various concepts, relationships and dependences" (Zankov 1977, p. 57). The principle requires that the topics studied should be systematically and logically interconnected, so that the students gradually will be able to see the whole picture of the subject and understand deeper the connections between different topics and mathematical ideas during the learning process. To achieve this, students are asked to compare, analyze, synthesize, justify, distinguish the essential from the non-essential, evaluate procedures and outcomes, and to generalize and explain concepts, symbols and definitions. However, this does not mean emphasizing the formalized aspects of mathematics at every possible turn, which would be alienating to the students and run counter to the philosophy of DEM. The mathematical activity should be both meaningful for the students and lead to a rich system of conceptual knowledge.

In the textbooks, this principle is operationalized in various ways. One way is to offer students opportunities to analyze relationships within the task, such as in Figure 2. Arginskaya (in Zankov, 1977, pp. 182–203) studied students' formation of the concept of a 'problem'. Her textbooks orient students toward the different parts of the general structure of word problems, such as the information, the question, superfluous or missing information, and known and unknown quantities. One goal is for students to be able to schematize the problems in various ways, for example like the schema suggested in task (b). Another goal is for students to manipulate the general structure of the problem by

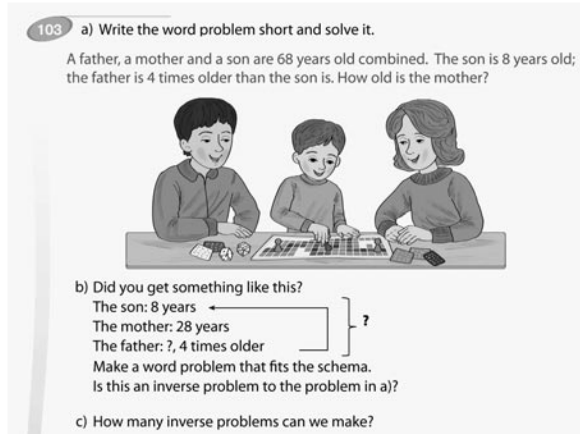

Figure 2. Task 103, illustrating a focus on the general structure of word problems. (Arginskaya, et al., 2016, p. 57).

creating 'inverse' problems where knowns and unknowns switch roles (tasks b and c).

Many of Arginskaya's tasks promote relational thinking and are connected to each other in various ways. They present topics from multiple perspectives and complement and enrich each other. This requires that the knowledge created by the student is systematized on a theoretical basis. In fact, when asked about the core of the DEM system, the feature that sets it apart from her previous practice, Mona simply said, "Learning to think!" She contrasted Zankov's ideals of independent reasoning with the more imitative approach of the textbooks she had used before.

The formation of mathematical language is an important feature of the developmental approach, and Arginskaya's textbooks promote precise use of mathematical terms. Our participants noted the difference from Norwegian textbooks, particularly in the first grade books. One goal in the first year is to establish a broad repertoire of mathematical vocabulary that is constantly repeated and expanded in the subsequent years—a 'concept bank', as Henry put it. Our participants noted that the shared and precise vocabulary had made the mathematical communication of their students far more clear and effective than that of their previous 'traditional' students.

However, the vocabulary can be off-putting to teachers who are not used to it and do not expect their students to understand. This was Vivi's main criticism against DEM:

Vivi Well, I spoke with the teacher in first grade, and she is new as a teacher and with DEM as well. And we agree that it is really text heavy and really difficult to get the students to work independently and to understand the tasks. I feel like I use a lot of time translating the tasks: "What are we actually supposed to do here?" *[...]* I see a lot of blank stares if I just read them what's in the book. I've worked with language for many years, and my concern is that it should be a means for communication, not a barrier. But from my experience here, it quickly becomes a barrier.

Recounting her students' frustrations, Vivi further said she had resorted to other means to make mathematics more accessible for them, such as designing practical activities (*e.g.,* baking) and using another textbook on several occasions. She did not mention task reading as a particular mathematical ability that needs to be supported and developed. While Anne (above) expected to spend time in her lessons explaining and discussing the tasks with her students, Vivi would like the tasks to be easier to access so that her students could read them by themselves and be immediately engaged in finding solutions. She expressed serious doubts about how the textbooks affected her weaker students because the tasks were difficult to access. She did not approve of the books and asked us to revise them.

The rigorous language initially proved a challenge for our 'experienced' teachers as well. Both Mona and Siri said that they became very self-conscious about their word use, catching themselves making mistakes and worrying that they were not 'doing it right'. After four years of DEM, however, they had become far more confident:

Siri They are used to, all the way from the first grade, the many mathematical words and expressions. And at first I was *[…]* afraid to make mistakes, "Oh, I have to use the correct mathematical concepts all the time." But in time I've become more relaxed. *[…]* For instance, sometimes I say *quotient*, sometimes I say division expression, using them interchangeably. And I think that this creates more meaning for the students, more than if you just use the correct mathematical concepts all the way.

Siri also recounted a lesson in second grade in which she and her students were discussing a geometrical figure, which different students described as a quadrilateral, a rectangle, and a closed curve, respectively. According to her, a rich, connected mathematical vocabulary not only deepens the students' understanding but also enables more students to be active in the discussion, since there is always more to say about the mathematical object. Developing the students' precision and flexibility in concept use is part of the job of the teacher, and it requires that s/he is precise and flexible her/himself. For Siri, this came gradually through experience, using the books and teaching her students.

Proceeding at a rapid pace

This principle which can be called *festina lente*—'hurrying slowly'—is closely related to the first principle. It primarily means constant movement forward and an absence of multiple repetitions. This does not imply 'undue haste'; rather, it means "continuous enrichment of the pupils' minds, where a broad curriculum creates favorable conditions for the ever deeper understanding of the information obtained, since this information is incorporated into a widely developed system" (Zankov, 1977, pp. 57–58). Lessons are typically characterized by high intensity and a variety of forms of classwork. Rhythm and tempo hold the students' attention and maintain a certain degree of emotional intensity, and are achieved in large measure through the variety of actions and procedures in which the students are involved, and through the thought-out progression of these actions and procedures (Karp, 2004). Applied to mathematics, this means largely eschewing repetitive drills of isolated procedures and instead continuously repeating previous concepts in new problem situations.

Repetition is distributed across the entire school year (spaced repetition), so that key concepts are constantly revisited and connected to related concepts. This means that there are no chapters dealing with topics in isolation, *e.g.,* a chapter on measurement first and then one on addition and subtraction and so on. Rather, all mathematical topics of a school year are taught in parallel and connected to each other. There is evidence that spaced repetition has advantages for long-term learning compared to massed repetition (Kang, 2016).

Mona Since they have such varied tasks, they don't get stuck in geometry for two months, and then fractions for two months. The fact that we do different topics all the way opens up for them finding their own strategies. They don't get stuck with the *algorithm*.

Spaced repetition allows the teacher to move on, even though s/he is not completely satisfied with the students' understanding after a particular lesson, since there will be many opportunities for repeating the same mathematical content later on. Our participants described this particular change of practice as new, 'scary' and something of a leap of faith. It was only in retrospect that they saw that student understanding came anyway through the regular repetitions. However, the systematic structure of the books was not at all clear for the teachers at first. Through using the books in their classes over several years, they eventually became familiar with how the tasks relate to each other, and how Arginskaya gradually develops mathematical concepts. Mona said, "After four years, we've understood that *every single* task has a point. There are no 'fillers' where we do something just to do something." According to the teachers, after about four years, they had eventually learned not to worry about how a single lesson went and to have long-term development in mind instead.

Promoting the students' awareness of the learning process

Students not only benefit from reflecting on the subject matter of mathematics, but also on their own learning activity: what they are doing, how and why. This can be achieved in various ways, such as in this task from the third grade textbook (Arginskaya *et al.,* 2016):

> Write a short version of the word problem and solve it: *Joakim picked 18 boxes of strawberries in two days. How many boxes of strawberries will he pick in six days if he keeps on like this?*

> Find another way to solve the problem. Compare the different strategies. Which strategy do you think is best? (p. 64)

The teachers' guide offers different solution strategies and suggestions for a whole-class discussion about their general advantages and drawbacks. The prompt to "write a short version" can indicate a similar approach as shown in Figure 2. The explicit mentions of different solutions and general heuristics orient the students' attention toward their own and others' ways of thinking and doing mathematics, turning awareness "inward, toward study activity" (Zankov, 1977, p. 60). According to our participants, making the students aware of their own learning activity has made them more deliberate in their problem solving, enhancing their independence as learners:

Anne In traditional math, you learn *one* way to do it, and then you just do that. And if you get the wrong number, well, you just continue. I've seen a *lot* of students do that. They just race through, and they don't notice their mistakes. While our students now, a lot of them, seem more attentive to "what am I seeing here?" They don't just race through, they stop and they think. Some strategies, like "how many digits must this answer have", they've learned those and they discover that, "oh, this has to be wrong". [...] Well, most do. You can't get to all of them.

The other participants agreed with Anne. Making students aware about general problem solving strategies was a recurring theme in the interviews, and something which the teachers held as special for DEM, suggesting a change in their view about this aspect of mathematics teaching.

Systematic development of each student in the classroom

Given the high level of difficulty, a tempting conclusion is that Arginskaya's textbooks are best suited for mid- to high-performing students, and that those at a lower level will be better off with a more traditional approach. Our 'experienced' participants contested this notion strongly, stating that DEM had allowed them to raise their entire student groups to a higher level than they had been able to before. Anne described a student of hers who experienced great difficulties in all school subjects and qualified for special education, but who was able to enjoy mathematics because of Arginskaya's tasks:

Anne He didn't learn to write the numbers until almost third grade, and I'm talking about zero to ten. But because of all the discussions and the fact that there were a lot of pictures and such, this student was really active and felt a lot of mastery. I would start with something simple, and then he would be active. *[...]* And I think that there were a lot of tasks that he actually *got*, and that I wouldn't have thought to give him otherwise. If I had used a traditional system, I think it would have been tiresome for him, because there would have been a lot of sitting down and calculating, which he couldn't do.

Above, Anne ascribes the relative success of her student to the variation and richness in the tasks, a view generally shared among the 'experienced' participants. Many of Arginskaya's problems are inquiry-based, with a low threshold to engage the problem, several possible solutions and opportunities for extending the problem. Such problems allow a diversity of students to learn at different levels, and there is evidence that a creative, problem-based approach is, in fact, particularly effective for so-called cognitively weaker students (Norqvist, Lithner, Jonsson & Liljekvist, 2015). According to the teachers, both their 'strong' and 'weak' students had benefited from the switch to DEM.

Concluding reflections

Going back to our problem—has DEM enabled the teachers to come to new awareness of how to engage and challenge their students and provide them with a deeper learning experience in mathematics? The stories told by our 'experienced' participants answer this question in the affirmative. From the way they compared their current and previous practices, we can sense a major shift in how they approached their mathematics teaching before and after DEM. They had changed their views of learning mathematics, mathematical creativity, developing student thinking with a long-term perspective in mind, and of challenging and engaging both high- and low-achieving students.

As for Vivi, Arginskaya's textbooks did not meet her expectations for what elementary school mathematics should look like, and, according to her, her students had met difficulties with the tasks. However, the more experienced teachers told us they too had difficulties at first, and it would be interesting to hear about Vivi's experiences in a few years from now.

One reason for the teachers' development might lie in the way they used the teachers' guides:

Siri At first, I read the guide like this [*pretends to read a book closely*], but then I noticed, the more and more secure I got, that I could put the book away. Just read it quickly to get the idea behind a task and then I could teach.

Ball and Cohen (1996) suggested that detailed teachers' guides are important for the contribution of curricular material to teacher learning. Teachers' ability to benefit from the scaffolding function of the teachers' guide that Siri described might be key to their learning from using DEM.

In changing her/his practice, the teacher needs a conviction that s/he is doing what is best for the students, as Siri told us:

Siri I think you need teachers who believe in it. I think it is teacher dependent. If you don't believe in it, I don't think it will work for the students.

However, in the case that a teacher should find the system to suit the needs of her/his students, Zankov (1999) offered extensive practical advice. Many of his suggestions concern supporting students' independence and initiative, such as pausing after questions, not interfering with student thinking and allowing them the freedom to make their own solutions and mistakes. Zankov also advocated active teaching methods such as play or drama for developing the creative potential of the students, as well as cultivating students' sympathy for one another and allowing them the freedom to express their emotions.

Last, but not least, Zankov suggested that the teacher should have the ability to become an 'equal partner' to the students, a sentiment that is reflected in a particular understanding of the Russian term *obučenie* (education). According to Roth (2017), the dialectical notion of *obučenie* denotes the organized and integral activity of teaching-learning. Not only does this imply interdependency and inseparability of the acts of teachers and learners, but also that teachers can learn and learners can teach (*e.g.,* by providing unexpected or thought-provoking responses), going beyond their institutionalized roles. Such a teaching philosophy requires the teacher to be flexible, open-minded and attuned to students' solutions and ways of thinking.

Some ideas presented in this article might seem familiar to many readers, even though most have never heard of Zankov before. Still, some aspects of his system contribute to the current discourse of mathematics education scholarship. One is captured in the slogan *festina lente:* The idea that teaching all mathematical topics of a school year in parallel and connected to each other throughout allows for a higher pace of student learning and development. The principle is simple and intuitive and finds support in research studies (Kang, 2016), yet many mathematics teachers and textbooks authors create artificial divides between concepts by treating topics separately—contrary to a vision of teaching mathematics rich in relationships.

Mason (2010) said about the field of mathematics education that "each generation [is] revisiting and reconstructing classic insights and awareness" and that despite the proliferation of academic publications, "rarely do we get evidence that the framework has enabled teachers to modify their practice and so influence student learning" (p. 6). Zankov's developmental approach might have the potential of being such a framework.

This article represents only a small first exploration into the implementation of DEM in Norway. There remain many unanswered questions, for instance, whether or not Norwegian DEM students have any advantages over 'traditional' students in terms of skills and knowledge, learning pace or attitudes toward mathematics.

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Notes

[1] https://matematikkradet.no/ (in Norwegian).

[2] While Zankov's system originally covers all elementary school subjects, the Norwegian implementation is in mathematics only.

[3] Website for the DEM project: https://en.matematikklandet.no/ (in English)

[4] For more about Zankov's lesson: http://en.matematikklandet.no/questions-answers/

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