make mathematics more 'real'?', *For the Learning of Mathematics* **13**(2), 12-17
Gee, J. (1996, second edition) *Social linguistics and literacies: ideology in discourses*, London, Falmer
Heath, S B (1983) *Ways with words: language, life and work in communities and classrooms*, Cambridge, Cambridge University Press
Gregory, E. and Williams, A. (2000) *City literacies: learning to read across generations and cultures*, London, Routledge.
Nunes, T., Schliemann, A. and Carraher, D (1993) *Street mathematics and school mathematics*, Cambridge, Cambridge University Press
Skovsmose, O. (1994) *Towards a philosophy of critical mathematics education*, Dordrecht, Kluwer Academic Publishers
Skovsmose, O (2000) 'Aporism and critical mathematics education', *For the Learning of Mathematics* **20**(1), 2-8
Street, B (1984) *Literacy in theory and practice*, Cambridge, UK, Cambridge University Press
Street, B. (1995) *Social literacies: critical approaches to literacy in development, ethnography and education*, Harlow, Essex, Longman

Mathematics and mathematical practices: where to draw the line?

BILL BARTON

A comment on 'Solid geometry in the works of an iron artisan', Castro, 23(3): It is always interesting to read about the implicit mathematical knowledge of ironworkers, weavers, tailors and other practitioners as they undertake their various crafts and professions Quantity, relations, and space are integral parts of many aspects of human life as we both make sense of the world around us and also manipulate it for our own ends. The formalities of conventional mathematics are only one mode of recording and developing these aspects of our lives - mathematicians are by no means the only ones who work with number, logic, and shape. But therein lies a conundrum. At what point do we call these activities mathematics? When is an iron artisan doing mathematics, and when is he or she crafting iron? Can they be doing both at the same time? How do the two activities relate to each other?

Fernando Castro (2003) describes in detail a particular incident in which an ironworker demonstrates, during the construction of part of a toy truck, the knowledge that a right parallelepiped has equal diagonals. Or does he? It certainly seems unlikely that these terms are used in the same way by the ironworker and Castro (in fact they may be, but it is not clear from the article, and it is easily possible to imagine that the craft worker understands the concepts of 'right' and 'parallelepiped' but does not use those terms). I also imagine that this knowledge is, for the ironworker, neither formulated as the statement of a theorem, nor as a logical 'if and only if' relation.

In which case, just how is this knowledge formulated? My reading of the article leads me to believe that there are some fundamental differences about the way Castro and the artisan relate to this knowledge. Castro's development of the activity as a classroom exercise that focuses on the knowledge in isolation is definitely mathematical (and as such may not always be recognised in all practical situations). The artisan's knowledge is embedded in practical knowledge in a particular context, (and also may not always be recognised in other contexts, including a formal mathematical one).

There have been many studies done on these issues: studies in ethnomathematics, studies in mathematics education, studies in situated cognition, studies in anthropology, studies in the history of mathematics and studies in indigenous knowledge. The writers will be familiar to many. Ascher, D'Ambrosio, Gerdes, and Knijnik are early (and continuing) writers in ethnomathematics - the many later ones can be found in the proceedings of the two International Congresses on Ethnomathematics (ICEMs) (Contreras, Morales and Ramírez, 1999; Monteiro, 2002), or the movement's newsletter. Zaslavsky is the most well-known writer in adapting cultural materials for the mathematics classroom, but members of NCTM and others have been active in this area (Jones, 1998; Krause, 1983; Trentacosta, 1997). Lave (1988) is credited with the theory of situated cognition, also cited are the street-vendor studies of Carraher, Carraher and Schliemann (1985), and the carpentry investigations of Millroy (1992). The anthropologist most noted in the mathematical arena is Pinxten, and there are myriads of studies of the number systems of different cultural groups. Joseph has, in recent years, highlighted non-Western perspectives on the history of mathematics, but other writers have also contributed (e.g. Berggren, 1990). The area of indigenous knowledge is relatively new compared with the others: a starting point for those interested is Semali and Kincheloe (1999).

The question raised by Castro's article, however, is the relationship between these areas of study. This is an urgent question because there is the potential for serious misunderstandings within this body of work. For example, writing in one area has been criticised as if it was from another. Rowlands and Carson's (2002) critique of ethnomathematics as if it is an educational movement is a case in point, the rebuttal (Adam, Alangui and Barton, 2003) differentiating between the open educational questions, and the ethnomathematical issue of relativity in mathematical thought. Similarly, motivational materials using culturally specific material have been critiqued as if they were part of the mathematics curriculum rather than as attempts to address equity-related educational issues. Another example is the welcoming of Joseph's historical works (e g. 1992), by ethnomathematicians, although Joseph himself has some ambivalence about the basic philosophies behind ethnomathematics (personal communication).

First, it should be noted that the boundaries between these areas of study overlap - the differences are often ones of emphasis and focus rather than distinct features. Furthermore, many writers deliberately address more than one of these areas in the same article, for example Knijnik (1999) addresses ethnomathematics, indigenous knowledge and education simultaneously.

Castro, despite the later, education-oriented section of the article, is addressing ethnomathematical issues. As this is my orientation also, what follows makes a first attempt to distinguish that field from the others.

We would do well, as Castro does, to refer back to Ubiratan D'Ambrosio's original writing about ethnomathematics. The clarity of his vision back in the mid-eighties remains, even if the reality of two decades of writing and research has not quite fulfilled the all-encompassing nature of that vision D'Ambrosio regards ethnomathematics as a research programme in the history and philosophy, not just of mathematics but of all knowledge (Ascher and D'Ambrosio, 1994) This is a re-appropriation of the original meaning for 'mathematics' as "that which is worth learning" (Miller, 2003), or, to paraphrase D'Ambrosio, the knowledge we gain as we seek to comprehend our world.

Ethnomathematics has its focus firmly fixed on mathematical knowledge – its aim is the illumination and extension of this knowledge, its methods are to expand the ambit of what can be legitimately regarded as mathematics, by including mathematical practices and systems wherever they occur, and, in particular, where they occur in culturally specific contexts, such as:

- *a more anthropological orientation* – some study of the nature of mathematical activity in relation to these contexts
- *a more historical orientation* – the way this mathematical activity develops and merges with other knowledge
- *situated cognition and educational orientations* – implications for learning and education.

How does ethnomathematics extend mathematical knowledge? There are some examples of direct contributions from culturally specific knowledge to the general body of conventional mathematics. Ascher (2002) recounts such an example, and some historical studies can be interpreted this way (hence ethnomathematical interest in the history of mathematics). However, most ethnomathematical work points to potential areas of mathematical interest.

As an ethnomathematician, for example, I am interested in Pinxten's anthropological investigation of Navajo spatial concepts not only because the concepts are interesting in themselves, but also because the investigation makes me reconsider my own mathematical knowledge. I learned about points and lines and circles through a Euclidean tradition where, for instance, points are given and circles are related to ellipses and hyperbolas If I take a Navajo perspective on geometry as move- ment, then circles become special cases of spirals, and points and straight lines are limiting values of spiral movement (Barton, in press). It is these sorts of attempts to shift perspectives from those of conventional mathematics that makes ethnomathematics, for me, an exciting field.

Alangui (2003) expresses this process as *mutual interrogation*. Starting with a rich description of a mathematical practice (in his work, that of rice-terracing in the Philippines), and then using his conventional mathematical knowledge of modelling and networks, Alangui puts himself in the position of being able to draw parallels between the two practices, and of using one system to "ask questions" of the other:

- why does the mathematical model not represent this part of the practice?
- what base variables are used in each system and why are they different?
- what becomes a critical point in each system and are these apparent in the other?

His position, as both a member of the indigenous group (therefore having an insight into the nature of this specific indigenous knowledge) and as a conventional mathematics lecturer at a university in Baguio, ideally situates Alangui to conduct this interrogation. He epitomises the ethnomathematician who is constantly struggling to bring together a practice that is not mathematics, but is mathematical, with a tightly defined (and controlled) academic field

This tension is present in Castro's article On the one hand there is the iron artisan who has a deep knowledge of working with iron as a practice; on the other is Castro himself who identifies within this practice an exemplar of his conventional and formalised mathematics knowledge. Bringing them together runs the risk of trivialising each: the iron artisan may be perceived as not really understanding the mathematics, just participating in a practice learnt, possibly, by apprenticeship; Castro may be perceived as perpetuating de-contextualised knowledge that is impractical (and unnecessary) in its formalised form. Both perceptions are wrong, of course. However there are things to be learned by bringing the two together – potentially things to be learned by each side.

For the mathematician, part of this issue relates to what is to be considered mathematics, and this brings us back (suitably) to D'Ambrosio. What are the criteria by which knowledge is to be admitted to mathematics? Leaving aside D'Ambrosio's wide conception, let us give ourselves the challenging task of trying to describe what would be acceptable to the community of mathematicians Thinking of practices that contain what I recognise as mathematical parts, there are some things that I would want to see before feeling comfortable about calling aspects of the practice 'mathematics'. The knowledge should be systematised, should be formalised and should relate to quantity, relationships, or space.

It must also be sufficiently abstracted to be removable from its practice By that I mean that practitioners should be able to discuss aspects of the system being considered, and hypothesise and convince each other about aspects of the system being considered, when they are physically removed from the site of the practice. This description would allow into mathematics much, much more than is usually admitted – there is no requirement here for a written formalisation, for example, so that orally recorded and formalised systems are acceptable. I am curious to know what other criteria people would like to insist upon.

References

Adam, S., Alangui, A. and Barton, B. (2003) 'A comment on: Rowlands and Carson "Where would formal, academic mathematics stand in a curriculum informed by ethnomathematics? A critical review"', *Educational Studies in Mathematics* **52**, 327-335.

Alangui, W. (2003) 'Menkabiti et binnanes: connecting indigenous knowledge of stone walling and water management and distribution to mathematics and its implication to indigenous peoples' education', paper delivered to the *Doctoral Students' Conference of the Association of*

Pacific Rim Universities, Mexico City, Universidad Nacional Autonoma de Mexico. (Wilfredo Alangui can be contacted at alangui@math.auckland ac nz.)

Ascher, M (2002) 'The Kolam tradition , *American Scientist* **90**, 57-63

Ascher, M and D'Ambrosio, U (1994) 'Ethnomathematics: a dialogue', *For the Learning of Mathematics* 14(2), 36-43.

Barton, B (in press) 'Mathematical discourse in different languages: implications for mathematics teachers', *Proceedings of the Midsummer World Mathematics Education conference*, Göteborg, Sweden, National Centre for Mathematics Education (NCM)

Berggren, J (1990) 'Proof, pedagogy, and the practice of mathematics in medieval Islam', *Interchange* **21**(1), 36-48

Carraher, T, Carraher, D and Schliemann, A. (1985) 'Mathematics in the street and in schools', *British Journal of Developmental Psychology* **3**, 21-29

Contreras L , Morales J and Ramírez J (eds) (1999) 'Ethnomathematics and mathematics education: building an equitable future', *Proceedings of the First International Conference on Ethnomathematics* (ICEM1), CD Rom, Granada, Spain, Universidad de Granada.

Jones, M. (1998) *Multicultural mathematics*, York, UK, QED Publishers

Joseph, G (1992) *The crest of the peacock: non-European roots of mathematics*, London, UK, Penguin Books

Knijnik, G (1999) 'Indigneous knowledge and ethnomathematics approach in the Brazilian landless people education', in Semali, L and Kincheloe, J, (eds), *What is indigenous knowledge? Voices from the academy*. New York, NY, Falmer Press

Krause, M (1983) *Multicultural mathematics materials*, NCTM

Lave J (1988) *Cognition in practice. mind, mathematics and culture in everyday life*, New York, NY, Cambridge University Press

Miller, J (2003) *Earliest known uses of some of the words of mathematics*, Website http://members aol com/jeff570/mathword html accessed 27th November, 2003.

Millroy, W. (1992) 'An ethnographic study of the mathematical ideas of a group of carpenters', *Journal for Research in Mathematics Education Monograph No 5*, Reston, VA, NCTM

Monteiro, M de (ed) *Proceedings of the Second International Conference on Ethnomathematics* (ICEM2), CD Rom, Ouro Preto, Brazil, Lyrium Comunacacão Ltda

Rowlands, S and Carson, R (2002) 'Where would formal, academic mathematics stand in a curriculum informed by ethnomathematics? A critical review', *Educational Studies in Mathematics* **50**, 79-102

Semali, L and Kincheloe, J. (1999) *What is indigenous knowledge? Voices from the academy*, New York, NY Falmer Press

Trentacosta, J (ed) (1997 Yearbook) *Multicultural and gender equity in mathematics: the gift of diversity*, Reston, VA, NCTM

Combining pyramids and staircases

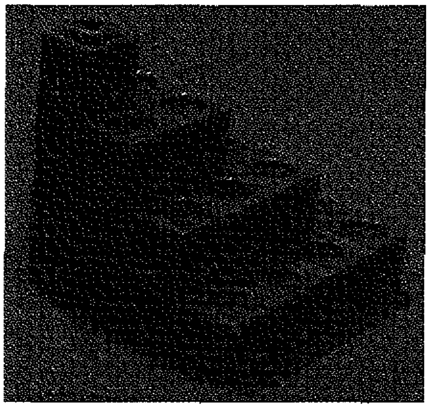

Figure 1A: Pyramid

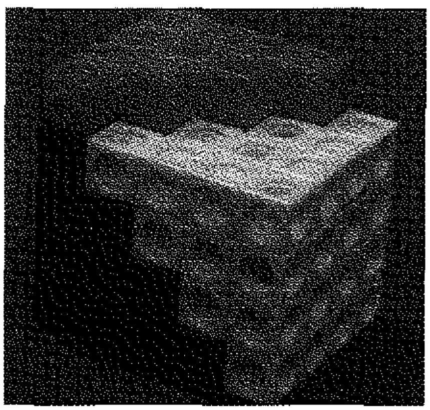

Figure 1B: Quasi-box – "not really a cube", "almost cube"

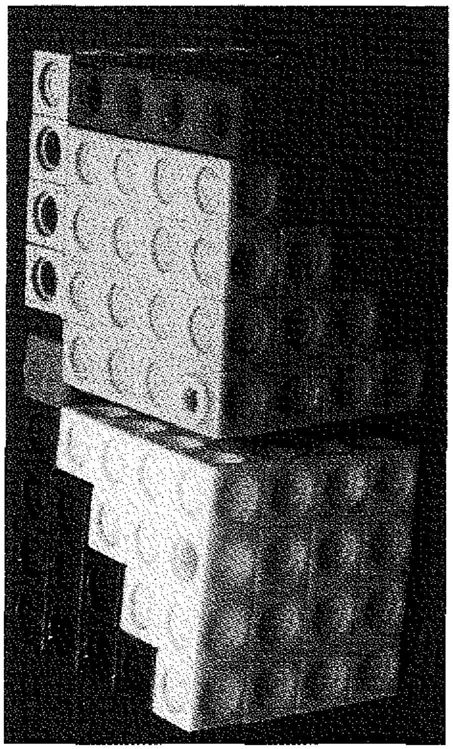

Figure 1C: Box – "cubic rectangle", "double-almost-cube"

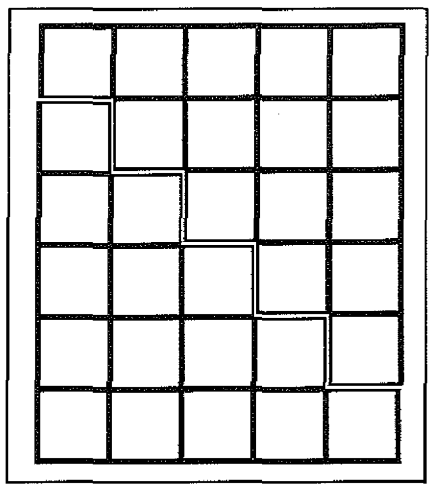

Figure 2: Staircases

I see the process of combining pyramids to find the sum of squares as analogous to the process of combining staircases to model the Gauss procedure for finding the sum of the positive integers. (p 26)

Figures 1A, B and C: Combining pyramids and Figure 2: Staircases illustrate the article by Vicki Zack and David A Reid that starts on the next page