

Pronouns in Mathematics Talk: Power, Vagueness and Generalisation

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Some years ago, I wrote (Rowland, 1992a) about the function of one pronoun in mathematics talk. Since then, drawing on a more extensive corpus of transcribed mathematics talk, I have become even more convinced that the study of pronouns is a significant topic for mathematics education. As it happens, linguists have taken little interest in mathematics discourse, so the mathematics education community has something to offer back by way of 'variation study' - the study of language phenomena in different registers or genres. I shall show, in the context of mathematics, that pronouns serve to code some transactional and interactional functions of language, and of speech in particular. These dimensions include social positioning, interpersonal power and the communication of generalisations.

Reference and ambiguity

Pronouns are lexical items which can be used as substitutes for noun phrases (including nouns). While there are several categories of pronouns (personal, possessive, demonstrative, and so on), my focus here is on the *function* of personal pronouns in talk about mathematics.

A pronoun is commonly co-referential with a noun phrase to be found elsewhere in the text or discourse, sometimes in the same sentence. In the case of both *anaphora* (referring backwards) and *cataphora* (referring forwards), the use of the pronoun is economical, and lends interest by avoiding repetition. Successful pronominal reference is achieved by the speaker if the hearer (as interpreter of the utterance) is able to recover the intended referent(s). The referents of pronouns are *potentially* vague, in the sense of being ambiguous or indeterminate, if they cannot easily be associated with a co-referential proper noun or noun phrase. The literature to date provides examples of how speakers may exploit such ambiguity for a variety of interpersonal and communicative purposes.

'We'

In the course of some post-Cockcroft (DES, 1982) research into mathematics teacher-talk, Hilary Shuard (1986) recorded and transcribed a number of (ostensibly mathematical) classroom interchanges between elementary school teachers and their pupils. One fragment of 'discussion' seems eerily familiar. It derives from a task for the pupil, who must subtract 25 from 42, and responds by 'setting out' the usual layout for column arithmetic. The teacher breaks in on the silence (my italicisation of certain pronouns).

Teacher: You've only got two ones haven't you?
You haven't got enough. Do you remember, when *we* went through these sums last week, what *we* said you had to do, if you hadn't got enough?

Pupil: .errr

Teacher: What did *we* say you had to do?

The teacher's utterances point to two players in this situation: a 'you' and a 'we'. It is improbable that the child is included in the 'we' in phrases like 'what we said you had to do'. The phrase could be intended to imply 'What I said in your presence'. Mühlhäusler and Harré (1990, p. 129) propose that when academics use 'we' in exposition, the addressee is trapped in tacit agreement, and thus inhibited from voicing opposition by the special relationship that has been artificially forged between expositor and audience. [1] I believe that the teacher's 'we' has much of this quality. It offers a classic case of the teacher's power over the pupil masquerading as solidarity with him/her.

More recently, I convened a group of teachers with the common aim of collecting and discussing classroom interactions. Here, one of them (Ann) is talking to ten-year-old Charlie. Ann is asking Charlie to share 2, 3, 4, 5, 6, ... sweets among five people, and shows how to enter the number 'left over' in a table. Ann's use of pronouns frequently associates herself with Charlie's progress in quite a personal way:

77 Ann: All right. You do that line *for me*. Well done.

79 Ann: *We* put a nought when there is nothing left over, didn't *we*.

The 'we' traps Charlie into complicity with Ann. At the very end, Ann assesses, with further reference to 'me' and now 'us', whether Charlie has understood - or at least remembered.

88 Ann: So can you tell *me* what the nought means? What is it telling *us*?

89 Charlie: That, um, you can share them out and have the right amount.

In his exploration of the social significance of the pronoun 'we', David Pimm (1987) drew on a classroom 'discussion' between a teacher and a ten-year-old pupil, based on the problem $26 - 17$. When the pupil hesitates over the decomposition procedure [2], the teacher prompts:

Teacher: We take a ... What do we take from the tens column? We take a ten, don't we

Pimm (pp. 69-70) suggests that the teacher, in using the plural pronominal form, is associating herself with some other (un-named) person or persons, thereby appealing to an anonymous 'expert' community to provide authority for the imposition of a certain kind of classroom practice. Like the editorial 'we' (Wales, 1980, p. 27), the effect is to associate the speaker with a select and powerful group, from which the audience is clearly excluded. The result is to discourage and devalue any sense that the child might make of the situation, and to urge acquisition of the 'proper way' of doing such 'sums'.

With an eye to a discussion to come later in this article, I think it is useful to add one further observation. The teacher's 'we' refers to rules and procedures of an invariant (perhaps non-negotiable) kind. 'We' find the mean of a set of numbers by dividing their sum by their cardinality. 'We' divide by 10 by fixing the decimal point and shifting all the digits one place to the right. 'We' carry out subtraction by executing an algorithm like decomposition. In effect, these 'we's all prescribe generalised procedures.

'It'

Pimm (1987, p. 22) also remarks on the prevalence of "half-finished and vague utterances" in spontaneous discourse about mathematics, and illustrates the point by drawing attention to the use of another pronoun in an exchange (about enlargement) between a teacher and secondary pupil

Teacher: [...] Its width is only a third as long as that one so - [...] how many of the smaller squares can you fill in? - nine, right

Pupil: Is it that you square it - every time?

Pimm comments on the ambiguity of the referents for the occurrences of 'it', particularly in the pupil's question. He suggests that in saying 'you square it', the pupil is making a *generalisation*. He picks up on the 'crucial expression ... every time' in evidence. My own analysis of mathematical conversations with an articulate nine-year-old (Rowland, 1992a) demonstrated the deictic [3] function of 'it', i.e. its function as conceptual variable, a linguistic pointer to a shared idea, to an understood but un-named mathematical referent. For example, in a long conversation about multiplication and division, Susie says:

Susie: I'm not sure if five does do it or doesn't do it. So could I find out if five does do it?

It emerges that the substance of Susie's remark, and others like it, is a generalisation concerning the inverse relationship of the mappings $x \rightarrow ax$ and $x \rightarrow x/a$ (here, with $a = 5$). This generalisation is never made explicit, but is effectively taken-as-shared by the participants.

The notion of a (possibly tacit) object of attention is captured by the term 'focus' (Chafe, 1972; Garrod and Sanford, 1982). Moxey and Sanford (1993) connect pronoun use to 'focus', in a way that resonates with my view of 'it' as a linguistic pointer to concepts which occupy the attention of the speaker:

Focus can be inferred through ease of pronominal reference. Because personal pronouns such as 'it', 'she' and 'they' carry only minimal information to recover the referent, [...] it is clear that in practice things in a discourse that can be referred to by pronouns must be a small subset of the possible previous antecedents, otherwise ambiguity would be rife. [...] The point is that pronouns are good for referring to things in focus, while noun-phrases of a more complex and informative kind are best for things not in focus. In this way ease or acceptability of pronominal reference can be used as an index and a probe for the state of focus, other things being equal. (p. 58, my emphasis)

Thus, the use of 'it' as a conceptual deictic enables the pupil to say what s/he could perhaps not say otherwise, to draw attention to mathematical entities whose name s/he does not know.

Caleb Gattegno (1981) seems to have anticipated these findings and this significance in his questions:

What is implied in the proper use of pronouns? Do children recognise them early and integrate them in their own speech with ease and total comprehension? (p. 5)

The answer to Gattegno's second question seems to be 'yes'. For children, the answer to the first certainly entails generalisation and indication of focus. Use of pronouns enhances the ability of speakers (particularly novice speakers of mathematics) to refer to aspects of their own mathematical thinking. The remainder of this article develops the argument in support of this claim, with particular reference to the pronoun 'you', drawing on transcript data from a number of sources

'You': power and solidarity

It is not unusual for speakers to use pronouns in an irregular, somewhat anarchic fashion, with the pragmatic effect of conveying a range of social dimensions and attitudes to themselves and their audience. A recent British Prime Minister inadvertently illustrated this rather well:

When I got there [Oxford], I think the first thing I learned was that for the first time in my life you were totally divorced from your background. You go there as an individual. So what did we learn? (Margaret Thatcher, in a television interview, 29 March 1983, quoted in Rees, 1983)

In this example, the speaker uses singular and plural, first and second person pronouns co-referentially. There is a fascinating shift from very personal recollection ('I') to description of presumed shared experience conveyed in the second person ('you'), to reflection with hindsight ('we'). The pronouns code and convey aspects of speakers' personal identity and group association.

These pronominal variations can be associated with delicate shifts of social positioning of the speaker in relation to his/her audience. In the context of pedagogy, the school teacher's 'we' conveys - albeit ambiguously - extremes of distance and mutuality.

Brown and Gilman (1970) give an account of pronouns as linguistic devices for expressing social distinctions in non-British European languages, to convey messages of power over or solidarity with others, with complex situational rules for their use. French, for example, retains a pronominal social semantics in its 'T-V' (*tu, vous*) system. *Tu* is the marked singular, expressing intimacy or certainly informality; on the other hand, it may also express superiority or condescension (and is used to children by default) *Vous*, the unmarked plural, is also the singular for public or formal conversation, occasionally used as a marked pronoun of respect; on the other hand, it indicates coolness when used to a friend.

With the decline and disappearance of 'thou' as a T-alternative to V-'you', English is almost unique among western European languages in having neither plural nor honorific distinction in second person pronouns. [4] The complexities and instabilities of the functioning of this system in seventeenth-century English are elegantly analysed and illuminated by Aers and Kress (1981) by reference to Shakespeare's play *King Lear*. Aers and Kress argue that pronouns are used throughout the play as a precise indicator of personal self-esteem and power/mutuality in relation to others. Thus, in the opening scene, Lear patently *demand*s a declaration of love from his daughters, using the royal 'we' of himself and addressing them formally as 'you'.

Lear: Tell me, my daughters,
(Since now we will divest us both of rule,
Interest of territory, cares of state),
Which of you shall we say doth love us best?

Once his two elder daughters have passed his 'test', however insincerely, and have been raised in power by conferment of land, Lear expresses his mutuality with them by addressing them as 'thee/thou'.

Lear: Of all these bounds even from this line, to
this,
[...] We make thee lady.

The youngest, Cordelia, though she truly loves her father, will not be drawn to diminish him by empty flattery. Lear is blind to her sincere love: he addresses her also as 'thou', but this time it is not mutual but contemptuous.

Lear: Let it be so, thy truth then be thy dower: [...] The barbarous Scythian [...] shall to my bosom
Be as well neighboured, pitied, and reliev'd,
As thou my sometime daughter.

The challenge of the outraged Kent, hitherto a faithful courtier, is marked by his show of disrespect to Lear in the T-form of address

Kent: be Kent unmannerly,
When Lear is mad, what would'st thou do
old man?

The ambiguities and social complexity of the early seventeenth-century English T-V system are such that Lear is unable to reconcile his affection for his daughters and his friends with his demand for respect as their father and

monarch: nor can he understand that love may express itself as resistance to his temporal power.

Referents of 'you'

The use of 'you' to refer to generalities is familiar in mundane language use, but has attracted little analysis in the literature. Simons (1981, p. 39) records the following fragment of an interview with a 15-year-old girl. The subject was participation in class discussions.

- HS: Did you feel ... that you did have things to say?
P: Yes. But often other people said them ...
HS: And that put you off saying something another time did it?
P: Umm. If you say something you sometimes think that if you say something wrong people are going to think it is funny

Whereas Simons uses 'you' to address the pupil, she (the girl) refers, not to Simons, but (presumably) to herself and other pupils. Simons makes no comment on this – whilst it would be out of place in a formal text, it is a commonplace and acceptable use of 'you' in speech:

The pronoun of the second person may be used vaguely to denote someone (often the speaker himself) to whom something happens, or may happen, in the ordinary course of events (Zandvoort, 1965, p. 128)

Now this kind of 'you' is vague, in that it is unclear who is meant to be included by it, the only certainty being the speaker 'himself' (by implication, since s/he does not choose a third person pronoun). This exemplifies a kind of referential indeterminacy associated with generality. [5]

In effect, 'you' is being deployed in place of the more formal indefinite pronoun 'one' which might be regarded as somewhat affected in English speech. There is some indication that there is a corresponding trend in French. Laberge and Sankoff (1980), in a study of Montréal French, remark that:

Tu and *vous* [...] are now locked in combat with *on* for indefinite champion, a title *on* thought it had locked up. (p. 271)

The issue is that of *generality* in relation to that which is being asserted

a detailed study of the contexts of use of indefinite *on* shows that *tu* and *vous* can be used in virtually all of them. Perhaps the most central element unifying these various contexts is the theme of generality or generalization. [...] It is important to note that the indefinite referent here is always vague as to the possible inclusion of speaker and hearer: *anybody* 'means' just that – possibly you, possibly me, or anyone else in like circumstances. (p. 275)

This linguistic trait is very evident in spoken child language, for example in the description of standard procedures such as 'rules' of games. The following examples from the Fawcett Corpus (Fawcett and Perkins, 1980) are from the speech of ten-year-olds:

[Rules of Monopoly] You buy lots of property and houses and then when someone comes along they land on you collect money that way as well.

[Game called 'Bull-dogs'] You have to get across the field and someone's got to try and stop you [...] When they tackle you you can brake [*sic*] your arm.

[Rules of Bagatelle] You roll a ball down a chute'n it's got to go in one of the holes

This matter seems to have attracted very little analytical attention with regard to English speech. In fact, this is fertile territory for mathematics education

'You' and generalisation

'You' can, of course, be used to address the person or persons to whom one is speaking. Such use is typically deictic but generally unambiguous. The following anaphoric examples come from transcripts in Brissenden (1988) of two groups of children using a computer program, *Trains*.

Craig: Nine add seven, that's sixteen, it's one ten and six. [he enters 6 first and then 1]

Steven: Craig, you've got sixty-one now, it's the wrong way round

Gavin: Seventy-two plus seventy-two, that's a hundred and forty-four

Teacher: Gavin, that was quick. How did you work it out?

Steven addresses his peer ("Craig, you've got sixty-one now"); the teacher addresses one of the children. Use of the pronoun 'you' as a means of address can be interpreted as a message of power (Brown and Gilman, 1970; Hodge and Kress, 1988), inappropriate for the child in conversation with a teacher. In adult-child mathematical conversations, where the power relationship is asymmetrical, my transcripts indicate that the teacher/interviewer frequently uses 'you' to address the child, whereas the reverse is relatively rare. I shall show that these same data strongly suggest that the majority of instances of 'you' by children in mathematics talk can be seen to be indicative of things that happen "in the ordinary course of events" (Zandvoort, 1965, p. 128). Such things are *generalities*.

This observation has clear pedagogical significance: in speaking with a teacher about mathematics, a child's 'you' is likely to refer to something rather than somebody. Recall once more the pupil in Pimm's transcript:

Pupil: Is it that you square it - every time?

Recollect Pimm's observation that one pointer to the fact that the pupil is offering a generalisation is the expression 'every time'. Another, I suggest, is the use of the vague, unmarked 'you', functioning as a vague 'generaliser'. Here are four examples from my transcript data. [6]

Anna

Here, I am talking to two ten-year-old girls about ways of 'making' twenty. Anna has proposed 'minus one add

twenty-one'. In my 'bookend' questions [turns 54, 58], probing for Rokšana's position, she (Rokšana) is the referent of my 'you'. But now look at Anna's 'explanation speech' [turn 57], and consider "Who is 'you'?" for Anna

54 Tim: Minus one add twenty-one. What do you think, Rokšana? [pause] Right, explain to us why that would give us twenty, Anna.

55 Anna: Cos nought add twenty equals twenty ...

56 Tim: ... right

57 Anna: ... so if you're going into the minuses you've got to ... em ... you've, instead of saying twenty, that would equal nineteen, instead of twenty-one. And, minus, if you're doing minus ... one add, add minus one, something equals twenty, you go minus one add twenty equals, it equals nineteen. So you need to go minus one add twenty-one equals twenty

58 Tim: Are you convinced by that, Rokšana?

In choosing the impersonal 'you' in preference to 'I', Anna has 'de-centred' and become, in some sense, detached from what s/he is asserting. Personal confidence - albeit tentative - in the general application of some process or proposition enables the speaker to offer it for others to appropriate. When Anna says [turn 57] "you go minus one add twenty equals, it equals nineteen", it could be that she is sharing some kind of number line imagery that she believes should be accessible and convincing to others. In any case, she is tacitly saying that "anyone can do this".

Simon

I began the first of three mathematical conversations with 12-year-old Simon with an enquiry along the lines "Give me two numbers whose sum is ten". I have begun undergraduate courses in Number Theory with the same question - intended, in that situation, to explore what 'number' might mean to the students. After similar preliminaries with Simon, he proceeded to determine the number of ways that any positive integer could be 'made' as a sum of two positive integers, and progressed to consider making a given integer as a sum of three integers.

First, he worked on 20 and came to see that the number of ways is the 18th triangular number. Picking up the conversation at that point:

205 Tim: Right. Suppose instead of twenty, right, I said how many different ways are there of adding up three numbers to make fifty

206 Simon: I'd do forty-nine times [pause] twenty-five

207 Tim: [pause] You'd better explain that

208 Simon: Um, no, I wouldn't. I'd do forty-nine times twenty-four.

209 Tim: Explain it.

- 210 Simon: Well, it's the triangular number of, em, it's the ... working out the triangular number of ... the forty-eighth triangular number.
- 211 Tim: Mm-hm. Why do you know that?
- 212 Simon: I'm going on the assumption that it works the same for twenty.
- 213 Tim: What happens with twenty?
- 214 Simon: It, em, I found the triangular number for eighteen, because ... the second number before twenty.
- 215 Tim: Right, right. So what you do with fifty, you say ...
- 216 Simon: Make, work out the triangular number forty-eight
- 217 Tim: Right.
- 218 Simon: And to do that, I times it by ... so I do forty-eight times ... no, I do forty-nine times half of forty-eight, which is twenty-four.
- 219 Tim: Right. Can you see why it's forty-nine times half of forty-eight?
- 220 Simon: Yeh.
- 221 Tim: Why?
- 222 Simon: Because, to work out a triangular number, you get the first and the last, and the second and that ...
- 223 Tim: and multiply it by how much?
- 224 Simon: Um, the num ... a half of the number ... of ... half the number of numbers you've got. So it's like from nought to forty-eight, so half of that, cos you've only got half the numbers to work out.

Observe in passing Simon's expression "it works" in turn 212, to point to a general relationship or procedure (Rowland, 1992a). In turns 206-212, he uses 'I' to describe what he did for 20 and what he predicts for 50. Whereas I use the vague generaliser 'you' in turn 215, Simon persists with the personal 'I' in turn 218 to describe how he would calculate a particular triangular number. In turns 222 and 224, however, he adopts the pronoun 'you' himself: here, he is formulating a *general* procedure for such a calculation, and explaining why it works - Gauss' method, in fact (pairing from the beginning and end of the list), for the purpose of finding the 48th triangular number.

This reading of Simon's use of pronouns interacts nicely with consideration of his use of different genres and tenses. A distinction can be made between narrative - the natural genre for accounts of personal experience - and the impersonal, report style which is characteristic of mathematical exposition (Morgan, 1998). Solomon and O'Neill (1998) highlight the complexity of this distinction by reference to particular letters and notebook entries of William Rowan Hamilton on the subject of quaternions. Superficially,

Hamilton's communications appear to present mathematics in narrative genre, whereas his subsequent publication in the *Philosophical Magazine* conformed to the impersonal stereotype of formal mathematical writing. Solomon and O'Neill suggest that this is to miss the subtlety of the earlier texts, for in each case (letter and notebook), a *mathematical* text can be found to be seamlessly embedded in a narrative text [7]. They argue that the two genres are most readily distinguished by reference to tense: the narrative recounts a 'train of thought' largely in the past tense (e.g. "it was natural to assume the product ..."), whereas the mathematical subtext is *tenseless*, with indicative sentences in the 'timeless present'. An example from Hamilton's letter (quoted in Solomon and O'Neill) is:

$\sqrt{-1}$ is in a certain well-known sense a line perpendicular to the line 1

Whereas the cohesion of the narrative text is achieved by temporal order, it is the logical order that holds together the mathematical text.

Consider now Simon's contributions from these perspectives of genre and tense. Initially, Simon addresses a hypothetical situation "Suppose ... I said" [turn 205] by indicating what he *would* do [turns 206, 208]. Asked to justify his intentions, he refers back [turn 212] to another case of which he has direct experience, and his narrative summary of this case [turn 214] is in the past tense ("I found"). In the remainder of the excerpt given above, Simon is struggling to articulate his awareness of the role of triangular numbers, and his Gaussian perception of how to compute them. This mathematical text is indeed in the timeless present, and personal/impersonal to varying degrees - "it's the triangular number" [turn 210], "I times it" [turn 218], "you get the first" [turn 222]. The second person pronoun seems to convey some detachment from the procedure. The issue of tense and mood in teachers' questioning is also considered towards the end of this article.

Susie

My fifth and final conversation with Susie was memorable; elsewhere, I have given the remarkable *mathematical* content the attention it deserves (Rowland, 1997). In this extract, Susie is developing a highly idiosyncratic method of dividing 100 by various fractions, five-sevenths in this instance.

- 62 Tim: OK. Now, you said that it wouldn't work for seven-ninths didn't you, this method. Right? Now, I'd just like you to write down five-sevenths, just here.
- 63 Susie: I'm going to have to think though, very well. Um, I'll try ... [pause]. Ahh, of course [interrupted].
- 64 Tim: You have a think while I push the door up.
- 65 Susie: ... you can't ... I don't understand. It's definitely a hundred. So that means two ... Ahh, ahhh [*big moment*] you've got two left, and you need five each time. So if you have two hundred ... um ... divided

by five. How many times does five go into two hundred? Well, it goes into one hundred twenty times

66 Tim: Mm-hm

67 Susie: Must go into forty times. So that's a hundred and forty.

Notice how Susie's 'I' [turns 63, 65] becomes 'you' [turn 65] after the 'ahh' which seems to mark the moment of insight.

Again, the pronoun 'you' serves as an effective and non-trivial pointer to a quality of thinking. Susie's shift from 'I' to 'you' signifies her reference to a mathematical generalisation. The generality expression 'each time' occurs in turn 65 as part of an account that has the quality of a generic example, for the exposition of the division method that Susie is developing

In my corpus of 25,000 transcribed words of mathematical conversations with 9–11-year-old children, 'you' is the most frequently-used word (744 occurrences), followed by 'and' (662), 'to' (400) and 'a' (394). [8] In the 'dividing by fractions' conversation with Susie above (2450 words), Susie and I each use 'you' forty times. *Every* single time I use the word, I am addressing Susie, whereas she uses 'you' to address me *only twice* – once to make sure that she's not leaving me behind!

187 Susie: And it has to be two hundred. So you would have two hundred [writes] divided by five. Do you understand that?

188 Tim: Yes, thank you.

On the whole, Susie reserves 'I' to mark her feelings and beliefs, or accounts of her personal actions, whereas 'you' indicates a kind of detachment from her strategy and computational methods. The same 'personal *versus* general' markers are evident in my final example.

Katy

In an undergraduate supervision with me, Katy is talking about her progress with a project on continued fractions:

17 Tim: And what have you proved?

18 Katy: Um, I've proved that ... I think, now I want you to have a look at it, 'cos I'm not sure if it's right, but I did this [] yeah, I was trying to, I've had a look at, I said that, right, the root of a squared plus one is equal to a plus one over alpha, like we did before

19 Tim: Oh yeah, yeah, yeah, go on, yes.

20 Katy: Um, and it's also equal to a plus one over two a plus one over alpha because you can, because alpha goes on forever you can start it whenever you want.

21 Tim: Umm ... yeah, go on

22 Katy: And so I put those two equal to each other.

The occurrences of 'I' are all set within accounts of

personal actions located in time and space. This is also true of the 'we' in Katy's first turn [#18]. I suggest, however, that the 'you's in her second turn [#20] are located in some informal object language which is appropriate for the specification of procedures and algorithms – because 'you' is not any particular, actual person; rather, it is anyone. Again, the pronoun functions as a vague 'generaliser'

Summary

When pronouns are deployed conventionally – to address another individual or to refer to some perceptible person or item – the act of reference is normally successful, the referent being clear and unambiguous. Surprisingly, this is rarely the case in teacher–pupil talk about mathematics, when the referents of some personal pronouns may be intangible groups or entities, yet whose meaning is, nevertheless, effectively taken-as-shared. The most universal mathematical factor is the clear relationship between pronouns and the articulation of generalities. Our understanding (who are 'we?') of this role of pronouns in mathematics talk is partial, but a framework of ideas from sociology, linguistics and mathematics is emerging to support an appraisal of their function in relation to the varied intentions of teachers and learners.

Teacher talk

The teacher's 'we' asserts the authority of the speaker and the obligation of the hearer to conform to practices that are the hallmarks of 'our' community of learners, giving priority to conformity over comprehension. Mathematical ingenuity and originality threaten the security of accepted, normative practices of which the teacher is 'master' and trustee. As Askew (1993) has observed:

Teachers tend to listen to children's explanations to see how closely they match their own explanations, rather than really wanting to hear what they have to say. (p. 22)

These teachers' own explanations tend to be the received and conventional ones, the common property of their peer group of pedagogues. In identifying with this group, teachers have a distinct preference for 'we' rather than 'you' in the articulation of generalities.

Pupil talk

Analysis of the use of pronouns in the mathematical discourse of novice users of the mathematical register reveals, by contrast, a genuine desire to communicate ideas, notwithstanding limited command of the register of mathematics. In particular, deictic use of 'it' serves to enable pupils to refer and point to mathematical concepts and generalisations which have not (or, for various reasons, cannot) be named in the discourse; the pronoun 'you' functions as an effective pointer to a quality of thinking involving generality; the shift from 'I' to 'you' commonly signifies the deployment of a mathematical generalisation.

The use of 'it' as a conceptual deictic enables the pupil to say what s/he could not say otherwise, to draw attention to mathematical entities whose names s/he does not know. This pronoun (typically as object of the verb 'to do') is effectively

added to the mathematics object language as a vague variable. The notion of 'focus' as locus of attention is important here, for the teacher who is sensitive to the pronoun/focus connection can be made aware of the presence of a cognitive focus involving generalisation – elsewhere (Rowland, 1992a), I have discussed Susie's comment: "times can do it can't it, and add, and take ... no, takeaways can't do it" This teacher-awareness opens up the possibility of further investigation of that focus through appropriate, contingent questioning

Such questioning could be of two kinds, which might be labelled 'conspiracy' and 'confrontation'. The conspiratorial approach is for the teacher to take up and use 'it' in the discourse as though her/his 'it' were intended to be co-referential with the pupil's. For example, "Why can't takeaways do it, then?" would enable the confirmation or formation of hypotheses about the referent on the basis of further information about 'it' – rather like a game of 'twenty questions', the only question not permitted is asking for the name of the mystery object. Confrontation, on the other hand, amounts to an 'on record' request for the object to be revealed: for example, "Wait a minute, what is this 'it' you're talking about?" Clearly, the choice of the conspiratorial or the confrontational approach must depend on a range of contextual and inter-personal factors, and there is scope for more research here

The second person pronoun 'you' is a prevalent and effective pointer to generalities in mathematical discourse. This is perhaps especially true in children's discussion with their teachers, a context in which children rarely address the adult participant as 'you'. I have shown how 'you' serves as a pointer to a generalised procedure or relationship. The subtle shift from 'I' to 'you' to mark a tendency towards speaker detachment is an important cognitive indicator. Oscillation between first and second person pronouns indicates a switch between action and knowledge, possibly with regard to different processes or generalisations.

The *I-you* contrast can be related to the process-object distinction (Sfard, 1991), the detachment associated with the conception of a mathematical notion as an object independent of the action of the speaker being marked by 'you'. [9] The pedagogic significance of the pupil/student's use of 'I' or 'you' might be in the recognition of a pre- or post-object cognitive state with regard to the mathematical notion being discussed. There is far greater ambiguity in the teacher's use of 'you', in that s/he may either be addressing the pupil (meta-language) or referring to a mathematical notion (object language). In practice, the referent is pragmatically determined, and/or determined by tense and mood. Compare the present "How do you find the area?" or the modal "How would/could/might you find the area?", with the perfect "How did you find the area?" The first solicits a generalised procedure, the second a particular but hypothetical intention, the third an account of an actual, particular, historical course of action.

In one of his novels, R. F. Delderfield (1969) gives this portrait of Evan Rhys-Jones, bank manager and landlord, described by his clerk and lodger, Charlie Pritchard

He had ... a gravity that *you* could mistake for dignity until *you* adjusted to the maddening deliberation of his movements. It was this characteristic that fascinated *me* on that first occasion, so that *I* found myself wondering how long it would take him to select a stick of celery, bring it up to his chubby jaws and produce the soft, carefully modulated snap, in contrast to his wife's regular volleys from across the table. *You* had the feeling that if *you* asked him to pass the salt the meal would falter to an uncertain halt, so in the end *I* compromised, watching him but listening to his wife's coy exploration of *my* non-existent love-life (p. 13, emphasis added)

This alternation between first and second person pronouns, 'I/me/my' and 'you', has the effect of distinguishing experiences and feelings from detached observation and generalised objective comment. Like Delderfield's bank clerk, pupils in mathematics classrooms need ways of using language to distinguish, for their audience, the personal and particular from the detached and general. These two dimensions co-exist in mathematical activity, and both are essential

Notes

[1] On the other hand, Wales (1980, p. 33) accounts for the prevalence of 'we' in scientific discourse in terms of the egocentric force of pronouns in English. Thus, the choice of 'we' in preference to 'I' is made in order to achieve rhetorical distancing of the speaker/writer from the content of what s/he says/writes, in order to achieve muted egocentricity

[2] This particular algorithm (decomposition) seems to be a particularly rich source of teacher 'we's

[3] Deixis is concerned with aspects of meaning that are inaccessible without the provision of context: for example, the use of a word or phrase whose referent is determined by the context of its utterance. Deictic features of speech and writing correspond to what philosophers call 'indexicals', which reveal attributes of place or person. Deictic forms such as 'you', 'now', 'here' are effectively context-dependent variables, or 'shifters' (Mey, 1993, p. 90). Divorced from the context of utterance, their meaning may be ambiguous or obscure

[4] Vestiges of the singular 'thou' survive in speech in northern England, and non-standard English includes plural forms of 'you' in various dialects e.g. 'y'all' (USA) and 'you'se' (Ireland).

[5] In his 1905 paper 'Issues of pragmatism' (reprinted in Peirce, 1934), C. S. Peirce distinguishes between two kinds of indeterminacy: *generality* and *vagueness* (para. 5.447). The indeterminacy of the former lies in the fact that it refers, not to this or to that, but to anything (in a given class). The indeterminacy of the latter has more to do with class boundaries, so that its field of reference is indeterminate

[6] These data include several extended interviews with Susie (aged 9-10) and Simon (12), and a number of one-off interviews and teaching episodes with students spanning the age-range 10 to 25. I am the teacher in some of these cases, but not in all

[7] Given the increasing recognition of a human presence in mathematical text, and the eagerness of authors to assert it, a number of contemporary texts (including Rowland, 1992b) exhibit this same embedding of mathematics in narrative. Solomon and O'Neill's main point (*contra* 'process writing' proponents) is that mathematics cannot be adequately conveyed (solely) in narrative terms.

[8] For comparison, 'you' is ranked 5th in the Fawcett corpus (Fawcett and Perkins, 1980) of 10-year-olds' speech, averaging 25 occurrences per 1000 words, compared with 30 per 1000 in the 'Susie' corpus. The most frequent words in the same Fawcett corpus are 'the' (38 per 1000), 'I' and 'a' (both 32), and 'and' (29). 'It' comes sixth (23), as it happens

[9] Sfard's (1991) process-object formulation also nicely embraces the notion of 'it' as a conceptual deictic, when she writes that:

Seeing a mathematical entity as an object means being capable of referring to it as if it was a real thing (p. 4)

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