Children's Experience of Shape in Space

ELISABET DOVERBORG, INGRID PRAMLING SAMUELSSON

Children's experience of shape and space is a question of living in a world where, as soon as they can move themselves, they begin to discover and develop concepts of shape and space. When babies begin to crawl, they have already developed a perception of depth, which can be viewed as one aspect of 3-D vision. Babies demonstrate this by refusing to crawl to their mothers when the latter call from a position where the child has to crawl over an area (glass) which looks like a chasm (Gibson and Walk, 1960)

Bunkholdt (1994) demonstrates that a child's perception of shape develops rapidly as the child is trying to see coherence and as its interest in finding meaning in the world increases. The child concentrates on recognizing people and objects in its surroundings. The child continuously explores its surroundings and finds different ways to deal with shape in different situations. When the child, for example, builds a house of blocks or sand, it creates a 3-D model of reality. This gives completely different information from the 2-D shape that a child can express in a drawing (Fosse and Munter, 1997). Therefore, shape and space are patterns necessary to perceive and develop right from birth.

It has been discussed by Askew (1998), among others, whether 2-D shape is to be explored before 3-D. But children are living in a world that is 3-D and therefore they only experience 2-D configurations in their surrounding world through pictures, TV, mirrors, etc. Seen from this point of view, one can agree with Haylock and Cockburn (1989) that 2-D and 3-D experiences need to go hand in hand.

In the Swedish school curricula and in psychological tests of children's development, shape and space are central notions. For a long time, the concept of shape has been a category used in a variety of tests. As early as the 1940s, Wigforss (1946) developed a number of tests to measure maturity and readiness for formal schooling. The child's perception and motor level was stated, based on the use of different shapes. It becomes a question about both conceptualizing and modeling. These kinds of tests are not used by the Swedish school system today. However, the medical health service uses similar tests with a similar purpose to test all children at four and at five-and-a-half years of age.

Within the pre-school setting, 'Logiblocs' became quite popular to work with during the 1970s in Sweden. During the same time frame, 'books-before-school' also became common. In them, children worked on recognizing and differentiating different geometrical shapes. The focus on shapes as something of importance for the child's development resulted in teaching children the names for the different shapes, which children often had difficulty remembering (Pramling, 1983)

This meant that shape during this period became focused on 2-D figures in books or specific objects representing the basic geometrical shapes and, because of this, learning about shapes involved little reference to the surrounding world. However, to have children understand the geometric features of different shapes, an important principle must be to give them opportunities to discuss the properties of the different shapes at the same time as they learn the names of the shapes (Hopkins, Gifford and Pepperell, 1999).

The Swedish curriculum for pre-school children (aged 1-5 years) states:

> The pre-school should try to ensure that children develop their appreciation of the basic characteristics of the concepts of number, measurement and form as well as the ability to orient themselves in time and space (Ministry of Education and Science in Sweden, 1998a, p. 15)

A similar aspect of mathematics is formulated in the curriculum for compulsory school, the pre-school class and the after-school centre (Ministry of Education and Science in Sweden, 1998b): however, it becomes more and more complex at each level of schooling. The importance of children developing an understanding of real shape in space is a generally known fact.

Perspectives on learning

To become better at organizing the world around you, in your own consciousness, is the core of learning. This means that learning is tied to one's awareness of different and specific aspects of one's surroundings. The perspective argued for here is based on the view that learning is an internal relation between that child and his or her world (Marton, 1994). This also implies that learning means that something new appears to the child when he or she has learnt something and, finally, it implies that learning is both content- and context-dependent and not related to different stages of maturity (Sommer, 1997)

Marton (1999) claims that variation is the source of all learning, but argues that the child has to experience the variation, in terms of different meanings, in order to distinguish between variations and similarities. How we experience similarities is always a question of earlier experiences. What a child experiences in the 'here and now' depends on that child's previous experiences. Understanding appears in the contrasts of variety: without variation, there is no discernment or learning.

The teacher's role is to help children to experience patterns of variation. And the less we know about what children will make use of in the future, the larger the experienced variation has to be. This means that, for example, children will learn about shapes by becoming aware of both similarities and differences. Children become aware of what a square is by contrasting it with objects that are not squares. But the child must also experience a variation of

similarities, such as squares of different sizes, to transfer an understanding from one situation to another.

Within early childhood education, the above perspective has been developed under the heading of 'developmental pedagogy' and used with great success in many early intervention studies (Ahlberg, 1995; Neuman, 1987; Pramling, 1990, 1994, 1996; Doverborg and Pramling Samuelsson, 1999a, 1999b, 2000).

There are two aims in the present study: one is related to making sense of the above perspective of learning in practice with children, while the other is to evaluate the teacher's work in terms of what is of benefit to children from being involved in this approach to learning. This means that the *aim of the teachers' work is to develop the ways six-year-old children experience real shape in space*.

Our role as researchers is to follow how the work proceeds in a group of children during the school year and *to evaluate the outcomes* by comparing children's experiences in the group with a reference group of children of the same age. The entire idea of this study is similar to what Brown (1992) calls 'design experiments', where theoretical and methodological challenges are used to create complex interventions in classroom settings.

Method, design and procedure

The present study is primarily an exploratory, qualitative feasibility study of children's learning, consisting of two parts:

(1) to follow two teachers in their classroom work;

(2) to evaluate children's learning about the concept of shape in space.

Data for the classroom process and the tasks of the teachers with *the variational learning group* are summarized in the following forms:

- the teachers' diary - entries have been made on a continual basis and detail what tasks have been carried out with children as well as the children's reactions and the thoughts they have expressed about the tasks;
- observations - classroom observations were conducted by the researcher once every second week;
- visual documentation - photographs of the work in the group and objects and drawings created by the children were collected.

The data collected in the *reference group* consisted of an interview with the teachers about their approach to children's learning and how they worked with the concept of shape in space. Both the variational learning group, where the children had worked with shape in space, and the reference group consisted of six-year-olds in two pre-school classes from the same school. Participating children had the same socio-economic backgrounds. All the teachers had the same formal and in-service training. The variational learning group consisted of sixteen children and the reference group consisted of fifteen children.

The researcher made an evaluation of the children by conducting two tasks with them: drawing a house and drawing a cube. If someone did not know what a cube was, it was explained as a die. The tasks were conducted on two different occasions and approximately six months after the initial work with shape was performed with the variational learning group. At the same time of the year, children in another matched group performed the same task with the same researcher.

Children were asked to carry out the same task a second time, sixteen months later, just as they started their second year in primary school. On this occasion, children were asked to perform both tasks by the same researcher.

Finally, an analysis of the children's drawings was made to find qualitative differences: that is, how the features of children's pictures could be distinguished and grouped into qualitative categories.

Approaches to learning

The variational learning group

The teachers' work observed had a theoretical base in 'developmental pedagogy': that is, it is not the child's level of development as such that is in focus, but an effort to develop the child's experience by means of educational principles (Pramling, 1994, 1996). Experience is viewed here as the child's understanding of how he or she conceptualizes, discerns or relates him- or herself to the world around. 'Learning', in other words, means to become aware of different aspects of the world and to create personal meaning (Marton and Booth, 1997). All children have an unreflective way of experiencing different phenomena and education becomes a question of challenging and changing the perspective of the child - that is, their unreflective way of thinking.

The principles employed by the teachers were as follows:

(1) To focus children's attention towards different aspects of the world that they wanted them to develop their experience of.

(2) To encourage children to imagine, think and reflect about content related to shape in space.

(3) To get children to express themselves and, by means of this, to make their own ways of thinking visible: in other words, to work at a meta-cognitive level (see Pramling, 1983, 1990). In order to make the children's ways of thinking visible, they were asked to draw pictures, to create objects and to talk about them.

(4) To make the children aware of the diversity of ways of thinking, as expressed in the drawings, the objects or verbal descriptions of them were then used as content. This means, for example, that if children had a round figure on a piece of paper and had been asked to make something out of this figure, then the children's various drawings were discussed and compared.

To use diversity helps each child become aware of his or her own way of thinking or creating meaning, since their own unreflected ways of thinking are challenged (Doverborg and Pramling, 1995; Doverborg and Pramling Samuelsson, 1999a, 1999b, 1999c).

The work on shape in space began immediately after the summer holiday, building on a request by their teachers to write a postcard to the pre-school during their vacation. The card which the children became most interested in was one with the house of Pippi Longstocking on it (Lindgren, 1996), which is a very old, erratic house.

Figure 1 The house of Pippi Longstocking

In a way, it became evident and natural to talk about shapes, and in relation to that to use several appropriate, basic notions of mathematics such as: large, small, long, length, numbers, height, but also to make all kinds of comparisons.

Some of the activities during the school year undertaken by the teachers and their class were:

- exploring the neighborhood (looking at different kinds of houses);
- finding fences, gates, balconies, porches, doors, windows, etc. (looking for distinguishable features);
- observing objects from different angles;
- making drawings of explorations in the neighborhood and discussing their findings;
- making models of fences, doors, windows and cubes;
- developing a house plan by sketching and constructing a cabin in a tree in the children's garden;
- drawing the pre-school from a bird's eye view (both before and after they had studied the building from different angles);
- working with tessellation;
- designing pictures using pre-cut shapes, completed by the children;
- designing gingerbread house models;
- baking a real gingerbread house based on the models (see Figure 2);
- making drawings of the completed gingerbread house;
- studying pictures with reference to size and distance;
- matching pictures of houses cut in half.

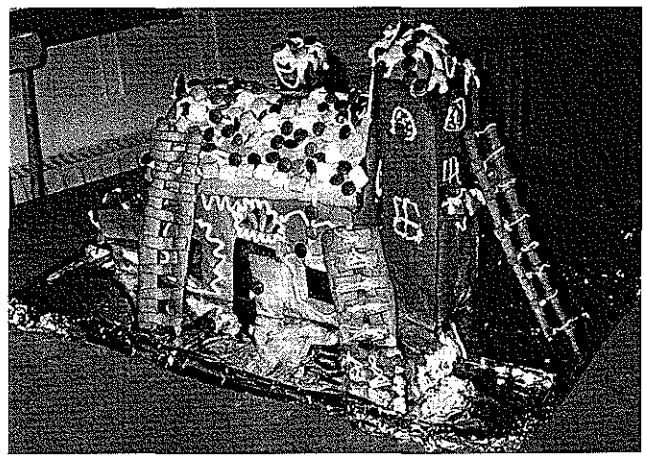


Figure 2 The children's gingerbread house

The tasks highlighted above must be viewed in the light of the principles earlier developed in this article.

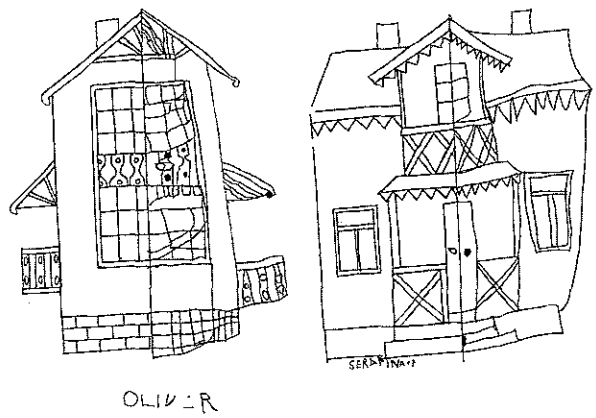


Figure 3 Synchronic picturing

Each task focused the children's attention on the concept of shape in space. This was also done in dialogue between the teachers and the children, when the teachers asked the children to tell a story or describe an object and then extended the children's ideas with comments or questions (Doverborg and Pramling, 1993), showing their support and enthusiasm about all ideas expressed by the children. They did not confirm or correct the children by saying that something was right or wrong, but helped them to explore differences in different children's ways of experiencing something.

A sample variational learning group lesson

At group-time, the children and the teacher talked about what Pippi Longstocking's house looked like.

Christoffer: The house is yellow, green and pink. The veranda is pink and the rest of the house is built of yellow boards

Serafina: It looks like there are crosses on the window panes

Oliver: There is a garden and a veranda where a horse is standing

Anna: Pippi has a fence around her house.

Teacher: Have you seen any house that looks like Pippi's house here in our neighborhood?

Jenny: I haven't looked for one but maybe we could do that.

In the afternoon, the children and the teacher went out for an expedition into the residential district nearby to find out what it took to make a house look like Pippi's house.

Serafina: The house must be yellow

Lena: There has to be a veranda

Benjamin: There should be a fence around the house

Oliver: And there has to be a garden and a horse

On their expedition, the children walked past many different houses, but finally they found a yellow house which had both a balcony and a veranda, as well as a fence. As they walked, the children got excited about the variety of fences around the houses. They each made a drawing of the fence they liked the most. Back at the pre-school, the drawings were displayed on the wall and each child told about his/her picture. They described the fences' different heights and picket widths; some had a varying pattern, one picket with a pattern and the next without, or one picket turned up and the next turned down.

Next, the children worked in two groups constructing fences, some from cardboard, other from clay. The eight children who were going to make their fences out of cardboard were asked to use a rectangular piece of cardboard: 'How do you make a fence out of one piece of cardboard?' The dialogue which follows between the children and the teacher describes how they approached the problem.

Christoffer: It is square. Yes, it is!

Lisa: The length of the sides is not the same.

Christoffer: No, there are two long and two short sides.

Teacher: What did the fence look like?

Christoffer: A lot of short pegs

Petra: Some tall ones too.

Teacher: But how can we get both long and short slips out from one piece of cardboard?

Lena: We can make all of them short and then we will have to join some of them to make the long ones

Oliver: Or we could get one more piece of cardboard

Teacher: No, you only get one

Christoffer: We can cut long ones first and then short ones.

Each child set about completing their task, cutting a different number of slips with different lengths. They then laid them out in a row before pasting them together as a fence panel. The children then stood back and looked at each other's cardboard fences. They discovered that they had not each made the same kind of fence. There were three different kinds of fences and three different heights. They also noticed that the length differed and started to count how many pickets there were in each fence.

Anna suggested that they make an exhibition where they could gather the fences in different groups: for example, one group for those of the same height and one for those of the same length. The question of how to make the fences stand upright arose. Anna knew that if you pasted one or a couple of slanted slips on the back of the fence you could make it stay upright, like paper dolls do.

The children who had made fences out of clay had also constructed a variety of fences. To make these fences stay upright was no problem, however. This fact opened up a conversation about similarities and differences between clay fences and cardboard fences. Questions such as: "How come that the clay fences can stay upright without any prop?" and "What do the fences look like?" were discussed. The children once again set out on an excursion, bringing paper and pencils to make a sketch of the fence they had chosen as their favorite one and to try to find out how the fences can stand upright.

The children were divided into three groups and given the task of sketching three different fences, to look at, to touch and feel the posts, the pickets and the different patterns on the pickets. From the constructions and the drawings, it became evident that the children discovered many

building techniques; some pickets were dug into the ground, others were nailed, etc

The work in the reference group

The theme during the school year was nature. The teachers said that they had not planned to bring mathematics into the theme, hence no discussion or tasks aimed at developing the children's understanding of shape in space were brought into the theme. However, what the teachers did recall (and also could find in their diary notes made during the school year) was that they actually had worked with some tasks where the aim had been to develop the children's understanding of shape in space. These tasks were seen as ordinary and not particularly connected to this year's theme

Some of them were:

- discovering different shapes in their surroundings;
- discussing traffic and the different shapes of the traffic signs;
- creating 'shape figures', where the head is made in the shape of a circle, a square, a rectangle or a triangle;
- collecting different materials from nature, sorting them according to their different characteristics;
- singing a song called 'Square man';
- playing a game based on shapes (Shape bingo);
- playing games such as European football (soccer) and *bandy* (a game with sticks and a small ball).

The teachers claimed that the children's world was full of mathematics and that shapes, for example, were to be found everywhere in their surroundings if they took a look around them. You could find shapes in rooms, the interior and furnishings, among the toys, jigsaws, party games, etc., and some songs and games contain different shape words. Since the children meet shapes in many different contexts, they also get many opportunities to learn about geometrical shapes. The teachers' opinion was that since the children are taking part in so many different activities where shapes are involved, they consequently develop their understanding of shape and, because of that, it is not necessary for the teachers to make this content a content in itself

A comparison between the teachers' work

The difference between the teachers' work in the two groups was whether there was a specific focus on shape as a content or not and the approach to learning. This meant that the teachers in the variational learning group used the surroundings and the children's experience for teaching them shape in space. They also worked on focusing children's attention on critical features and objects constituting different shapes in space, while the teachers in the reference group took for granted that children learn about this if they come across it in their everyday life

The approach to developing children's understanding of shape differed in that the teachers in the variational learning group systematically created situations where children could think and reflect: they also tried to focus the children's attention on the variety of ways they had solved different problems. The teachers in the reference group named and talked about shape with the children as a part of daily life and communication.

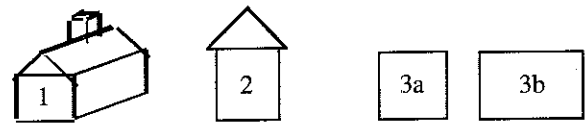
There were, so to speak, different approaches to learning as well as different views of what the teacher's role was in the two groups. But also the content as such appears differently. In the variational learning group, the content, that is, the teachers' intention of what they want children to develop an understanding about, was very clear and every opportunity to make this visible to them was utilized. The teachers in the reference group believed, in a way, in children's spontaneous learning – that is, discovering and discerning shape in the flow of their everyday experiences

Results

After a sixteen-month interval, both groups were asked to draw a house and to draw a cube

The house drawings

The drawings from both the variational learning and the reference groups can be placed in one of three categories. First of all, we can see children drawing a 3-D house where a gable is visible and at times it is accompanied by a side view of the roof (1). We also have a 2-D house with a roof made like a triangle (2) and two alternative single shapes, (3a) square and (3b) rectangle.



Category	1	2	3	*n*
First occasion				
Variational learning group	14	1	1	16
Reference group	5	7	3	15
Second occasion (sixteen months later)				
Variational learning group	15	1	0	16
Reference group	3	9	3	15

Table 1 House drawings

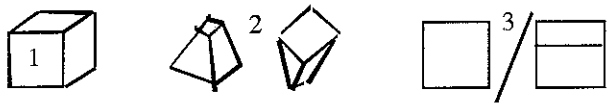
When we analysed the children's drawings on the first occasion, we noted a large discrepancy in the category of drawings completed by the variational learning group and the reference group. Fourteen of the children in the variational learning group drew a 3-D house, while only five of the children from the reference group drew a house in that way. Only one child from the variational learning group

drew a 2-D house, while seven of the children from the reference group drew this kind of house. And only one child from the variational learning group drew a single shape depicting the house, while three of the children from the reference group drew this type of house.

No significant change was shown between the results from the first and second occasion, conducted sixteen months apart.

The cube drawings

The second task was to draw a cube: we can also distinguish different qualities here. First of all, we can see children drawing a 3-D cube (1), but we can also see attempts to drawing a 3-D cube where the lines are not parallel (2), and a third way, which only represents the cube as a 2-D square (which, in some cases, has a line straight across it) (3)



Category	1	2	3	*n*
First occasion				
Variational learning group	13	2	1/0	16
Reference group	0	0	11/4	15
Second occasion (sixteen months later)				
Variational learning group	15	1	0/0	16
Reference group	2	0	11/2	15

Table 2 Cube drawings

Looking at children's ways of solving this task, a trend similar to the house drawings was noted: that is, thirteen of the children in the variational learning group drew a 3-D cube, while none of the children from the reference group drew a 3-D cube. Two children in the variational learning group drew figures with non-parallel lines. In the reference group, all the children drew a flat 2-D shape. Eleven of them just drew a plain square, while four drew a straight line through the square. This could be seen as an attempt to make a 3-D drawing.

Also here, as can be seen above, there is no significant change within the groups. Most of the children in the variational learning group could already draw the 3-D cube on the first occasion. The children in the reference group did not continue to develop in their skills of expressing themselves in 3-D on their own during this sixteen-month period.

Discussion

It is important to start the discussion by making it clear what kind of knowledge this article provides (cf. Brown's (1992) 'design experiments'). First of all, it has to be viewed as a feasibility study: that is, a description of what possible development *could* be attributed to the group we name the 'variational learning group' (without claiming that there is a 'cause and effect' relationship in absolute terms). A specific approach to learning has been focused on in the variational learning group. The reference group consists of 'comparable' children and teachers, but does have to be viewed as representing a common pre-school class of high quality. This means that the reference group is used as a reference against which the work in the variational learning group is laid out. With an awareness of the exploratory approach in the present study, we will discuss the results.

Those children who have or have not developed the skill of drawing a 3-D house and cube are obvious in this study. To be able to experience objects as 3-D and to represent them in a drawing is obviously an effect of education. Ajdarova (1987) points out the importance of giving children opportunities to learn about and understand the relation between the abstract and the concrete - something which is achieved when children are challenged to make models of different phenomena they are exploring.

The teachers in the variational learning group let the children make models of things connected to the theme which they had explored or were about to explore, like the houses, ladders, fences, balconies, shelters, etc. In that way, the children were given the opportunity to learn about this content in both a concrete and an abstract way, as well as both 2-D and 3-D.

The educational approach and the focusing of children's attention towards real shapes in space have raised their awareness of these two aspects of reality. This picture becomes even more obvious when we look at the second occasion, when children have been in primary school for more than a year. It does look like there is very little development going on in this regard. This also gives proof of the fact that developing this skill is not a question of maturity, but is rather a matter for education.

What is it then that these children have learnt in the variational learning group? They have learnt to imagine and think creatively: to raise themselves above acting concretely and to use their imagination and by that to create their own meaning. They have worked in concrete situations, but all the time they have been challenged by their teachers' and by other children's points of view. This has been carried out by questioning, and by interest and support from the teachers, but also by means of the children's own documentation, introduced by the teacher.

Children have, in other words, started in concrete and, for them, meaningful situations; they have been challenged to think and to verbalize their thinking and, last but not least, they have documented their views giving them possibilities for viewing their own thinking (Pramling Samuelsson and Mårdsjö, 1997; Doverborg and Pramling, 1995). In this acting, it is the meta-cognitive approach utilized in education which has been used to great success in earlier studies (Brown, 1980; Pramling, 1990, 1994; Doverborg and Pramling Samuelsson, 1999a, 1999b, 2000).

What is it, then, that these children have learnt about the world around them? Our interpretation is that children's

ways of expressing themselves in the drawings are related to their experience of reality. This means that educational settings in the variational learning group have helped children create a relation between concrete everyday life and more abstract thinking, which is necessary for further theory building (Hershkowitz, Parzysz and van Dormolen, 1996).

Just as Neuman (1987) has shown in her teaching experiments on children's understanding of number, which developed into abstract mathematical thinking, our teachers have given children the opportunity to develop an initial geometrical understanding. And the base for this has been that the content and the approach are built upon the ways in which children experience different aspects of the world around them.

References

Ahlberg, A (1995) *Att möta matematiken i förskolan. Matematiken i temaarbetet*, Göteborg, Göteborgs Universitet, Institutionen för Pedagogik. (Encountering Mathematics in Pre-School: *Mathematics in Thematic Work*)

Ajdarova, L (1987) *Barnets udvikling i de første skoleå*, (Dansk översättning) Köpenhamn, Sputnik Progres (*The Child's Development during the First Years in School*)

Askew, M (1998) *Teaching Primary Mathematics, a Guide for Newly Qualified and Student Teachers*, London, Hodder and Stoughton.

Brown, A. (1980) 'Metacognitive development and reading', in Spiro, R., Bruce, B. and Brewer, W. (eds), *Theoretical Issues in Reading Comprehension*, Hillsdale, NJ, Lawrence Erlbaum, pp 453-481

Brown, A (1992) 'Design experiments: theoretical and methodological challenges in creating complex interventions in classroom settings', *The Journal of the Learning Sciences* **2**(2), 141-178

Bunkholdt, V (1994) *Utviklingspsykologi*, Oslo, Tano (Developmental Psychology)

Doverborg, E. and Pramling, I (1993) *To Understand Children's Thinking. Methods for Interviewing Children*, Göteborg, Göteborg University Department of Education.

Doverborg, E. and Pramling, I. (1995) *Mångfaldens pedagogiska möjligheter. Att arbeta med att utveckla barns förståelse för sin omvärld*, Stockholm, Utbildningsförlaget. (*Pedagogical Possibilities of Diversity. Working with Developing Children's Understanding of the World around Them*)

Doverborg, E. and Pramling Samuelsson, I (1999a) 'Apple cutting and creativity as a mathematical beginning', *Kindergarten Education: Theory, Research and Practice* **4**(2), 87-103.

Doverborg, E. and Pramling Samuelsson, I (1999b) 'Hästar och äpplen i ett didaktiskt perspektiv: om begynnande förståelse för grundläggande matematik', *Didaktisk Tidskrift* **9**(4), 337-378. ('Horses and apples in a didactic perspective: about beginning understanding of basic mathematics')

Doverborg, E. and Pramling Samuelsson, I (1999c) *Förskolebarn i matematikens värld*, Stockholm, Liber (*Pre-school Children in the World of Mathematics*)

Doverborg, E. and Pramling Samuelsson, I. (2000) 'To develop young children's conception of numbers', *Early Child Development and Care* **162**, pp 81-107.

Fosse, T. and Munter, J. (1997) 'Geometri og små barn - hva er det?', in Johnsen Højnes, M. (ed.), *De små teller også: Matematik i förskolepedagogiken*, Oslo, Casper Förlag, pp. 21-38 (*The Young Ones also Count: Mathematics in Pre-School Pedagogy*)

Gibson, E. and Walk, R. (1960) 'The "visual cliff"', *Scientific American* **202**, 64-71

Haylock, D. and Cockburn, A. (1989) *Understanding Early Childhood Mathematics*, London, Paul Chapman Publishing.

Hershkowitz, R., Parzysz, B. and van Dormolen, J. (1996) 'Space and shape', in Bishop, A. *et al* (eds), *International Handbook of Mathematics Education*. Dordrecht, Kluwer Academic Publishers, pp. 161-204

Hopkins, C., Gifford, S. and Pepperell, S. (1999) *Mathematics in the Primary School. a Sense of Progression*, London, David Fulton Publishers.

Lindgren, A. (1996) *Pippi Longstocking*, London, Puffin Books

Marton, F. (1994) Världens bästa kunskapssyn, *Lärarnas Tidning* **14**, p. 4. (*The Best View of Knowledge in the World*)

Marton, F. (1999) 'Variatio est mater studiorum', Invited address presented at the *8th European Conference for Learning and Instruction*, Göteborg, August 24–28

Marton, F. and Booth, S. (1997) *Learning and Awareness*, Hillsdale, NJ, Lawrence Erlbaum Associates

Ministry of Education and Science in Sweden (1998a) *Curriculum for Preschool, Lpfö 98*, Stockholm, Fritzes.

Ministry of Education and Science in Sweden (1998b) *Curriculum for the compulsory school, the pre-school class and the after-school centre*, Lpfö 94, Stockholm, Fritzes

Neuman, D. (1987) *The Origin of Arithmetic Skills*, Göteborg, Acta Universitatis Gothoburgensis

Pramling, I. (1983) *The Child's Conception of Learning*, Göteborg, Acta Universitatis Gothoburgensis.

Pramling, I (1990) *Learning to Learn: a Study of Swedish Preschool Children*, New York, NY, Springer-Verlag

Pramling, I. (1994) *Kunnandets grunder: Prövning av en fenomenografisk ansats till att utveckla barns sätt att uppfatta sin omvärld*, Göteborg, Acta Universitatis Gothoburgensis (*The Foundations of Knowing: a Test of a Phenomenographic Effort to Develop Children's Ways of Understanding Their Surrounding World*)

Pramling, I (1996) 'Understanding and empowering the child as a learner', in Olsson, D. and Torrance, N. (eds), *Handbook of Education and Human Development: New Models of Learning, Teaching and Schooling*, Oxford, Basil Blackwell, pp 565-589

Pramling Samuelsson, I. and Mårdsjö, A.-C. (1997) *Grundläggande färdigheter och färdigheters grundläggande*, Lund, Studentlitteratur. (*Fundamental Skills and the Foundation of Skills*)

Sommer, D. (1997) *Barndomspsykologi. Utveckling i en förändrad värld*, Stockholm, Runa. (*Child Psychology: Development in a Changed World*)

Wigforss, F. (1946) *Barnens färdigheter i räkning vid skolgångens början*, Pedagogiska skrifter 191. Stockholm, Sveriges allmänna folkskollärareförenings litteratursällskap (*Children's Mathematical Skills when Beginning School*)