

# The Experience of History in Mathematics Education

STEVE RUSS

Many mathematics teachers have never had reason to consider the history of their subject, and taking it up may seem a daunting additional burden of doubtful value. For, of course, there now exist lively new schemes for learning mathematics, which promise greater understanding and engagement with concepts and their applications than the traditional methods which most teachers themselves experienced. Under such circumstances, it is easy not to be enthusiastic about the past. But this line of thought misses the point, which is that the concepts and their applications are what they are as a result of historical process. The history is not so much an additional burden as a spelling out, or opening out further, of what is already contained in the concepts and methods we learn about and use today. And of course it compels us to consider the erratic, complex but often fascinating human and creative origins of these ideas. It can come as a surprise to realize that these ideas are no more "fixed" now than they have ever been, and that we and our pupils are in the midst of their continuing evolution.

This part of the opening session of HIMED 90 consisted of short contributions in which several mathematics teachers spoke briefly out of their own experiences of using history. They had been asked to address the question, *How has the history of mathematics mattered to me in my mathematics teaching?*—and to condense their years of experience into no more than 7½ minutes. This served the purpose of showing that the fruits of research in the history of mathematics—that is, what we know of the past—can have practical relevance in the classroom. The range and variety of experiences represented here suggest that this relevance can show itself in as many, and as varied, ways as are the styles and personalities of teachers. The emphasis was not so much on courses or lessons on the history of mathematics, though that was included, as on the usefulness to the teacher of a general and specific awareness of the historical origins of mathematics. As such background becomes second nature, it will permeate and influence the teacher's introduction to some topics. Furthermore, it can colour the teacher's account of the material itself and can stimulate and enliven the teacher's enthusiasm for the subject by providing new and instructive insights into the development of ideas. It is a source of large areas of useful resource material. It enables pupils to gain a more accurate picture of mathematics and their own role in the learning process. And it ensures that a more human face is put on a subject which for too long has been regarded as cast in concrete, impenetrable and rather frightening. All these

benefits should make for more enjoyable and effective lessons for pupils.

The following contributions show clearly the quite disproportionate benefits that a little history can bring to the busy teacher and the needs of pupils. And it is agreeably surprising to learn how wide is the possible range of pupils' circumstances. History has been found to enrich, in different ways, the teaching of mathematics all the way from primary to university level. The responses of the six teachers given here range widely (and was even wider on the day—a seventh contributor to this session, the primary school teacher Helen Gardner, has expanded her talk into a longer paper appearing elsewhere in these pages, and the eighth contributor, Jack Astin, described a history of mathematics course he has given as a liberal studies course for sixth formers). Peter Ransom and Patricia Perkins are secondary school teachers, and Gary Brown and David Fowler teach mathematics to university students. All have reflected on the place of history in their experience of understanding, learning and teaching mathematics, and they have used this occasion to pass on a vivid sense of the practical relation of history to mathematics teaching.

## Whys and hows

PETER RANSOM

I teach in a 13-18 mixed comprehensive of 850 pupils in Northumberland, in the north-east corner of England. My own interests lie in mathematical puzzles and recreations, the local branch of the Mathematical Association, colour supplement crosswords, industrial archaeology, and looking after the childrens' guinea pigs, who incidentally are called Pythagoras and Euclid (the guinea pigs, that is). I'm going to talk about my whys and hows.

The benefits I see for using history of mathematics in my mathematics teaching come under five heads.

### By broadening general knowledge

It concerns me that pupils appear to do little background reading in mathematics. Many books about its history or cultural context do not really address the right level and tone for many of my pupils, but a journal published by the Mathematical Association, *Mathematical Pie* (Figure 1), provides good digestible articles for 10- or 11-year olds onwards, on history of maths as well as practical and



# MATHEMATICAL PIE

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## LADIES IN MATHEMATICS - 6 EMMY NOETHER

Born in 1882, Emmy Noether made important contributions to modern algebra, which included studies on abstract rings. This is an area which you will not meet unless you go on to study mathematics beyond A level, or read *Mathematical Pie!* Trying to put it simply, a RING is a set of numbers in which you can obey the normal rules of addition and multiplication without leaving the set. If we deal with the set of numbers

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

and call it  $S$ , then  $S$  is a ring with normal addition and multiplication since if we add and/or multiply normally, we still end up with a number in  $S!$

e.g.  $(-3) + (7) = 4$  which is in  $S$   
 $((-2) \times ((3) + (-1))) = -4$  which is in  $S$ .

You cannot get out!

If we take the odd numbers

$$\{1, 3, 5, \dots\}$$

then we do not have a ring with normal addition and multiplication. Can you see why not?

What happens with even numbers?

Some mathematicians like to find out what they can about things like this - they are generally referred to as "Pure Mathematicians" and Emmy Noether was one of these.

Unfortunately she was sacked from her post at the University of Göttingen in Germany in 1933 during the Nazis' rise to power, as neither her politics nor her religion were tolerated at that time. She moved to the USA where she became a professor at Bryn (Mawr) College, dying in 1935.

She was an inspired teacher, often getting sidetracked into new avenues of exploration with her pupils. The problems (both mathematical and social - women had to be exceptional in Germany to obtain any post of significance) she met and overcame serve as an example to us all!

P.H.R.

Emmy Noether - Adam's rib -  
Antedating women's lib.

from Mathematics and humor by Katherine O'Brien.

investigational work. I want to encourage all my pupils to read around the subjects, to become aware of how the maths they study has developed.

Each year we enter a team of three pupils (which is changed for each match, to encourage a wider participation) in a sixth form context organized by our local branch of the Mathematical Association. Each match contains some questions on the history of mathematics, which encourages pupils by this stage to read around the subject, and to listen carefully whenever I drop some historical remark into their lesson. The kind of questions put into the contest are ones such as:

*Can you name an inventor of a calculating machine before the 20th Century?*

*Describe one of Zeno's paradoxes.*

### By providing alternative methods of solution

With the lower sixth, one of the first problems we encounter on the course arises from a practical situation: we want to find the minimum amount of card (area  $A$ , say) needed to produce a box of given volume subject to various constraints. This yields a formula for  $A$  that needs to be minimized. Since no calculus has been studied at this stage graphs are plotted (on graph paper or with graphical calculators) and used to obtain the minimum. This year we

looked at Fermat's method (described in chapter 4 of *History in the mathematics classroom*) and then used it to find the minimum area in this and related situations with different constraints. This practical situation (each sixth former brought in a carton to use) gave me the opportunity to do some history of mathematics in a relevant situation, and was seen by the pupils as being appropriate and helpful.

This had an unexpected bonus in January. The British Mathematics Olympiad contained a question which Fermat's method can be used to solve (since all questions can be solved without calculus, the challenge is not to use any!). It would be nice to report that our candidates used Fermat's method, but honesty compels me to confess that they didn't.

### By providing a source of interesting problems

When starting Pythagoras Theorem with 14-year olds this year, I used a 1747 Latin version of Euclid to look at the theorem there (Book 1 Proposition 47). This led on to our examining a number of different proofs, and various dissections of the accompanying diagram.

We then looked at examples from Bonycastle's *Mensuration*, in the 1827 edition which I found on a market stall.

## INTRODUCTION

TO

# MENSURATION

AND

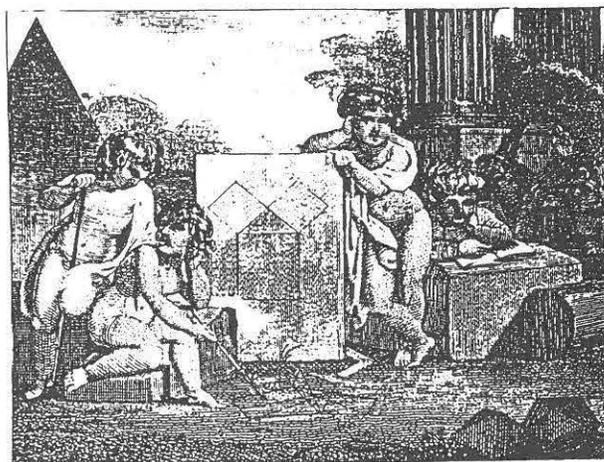
## PRACTICAL GEOMETRY;

WITH NOTES, CONTAINING THE REASON OF EVERY RULE.

BY JOHN BONNYCASTLE,

PROFESSOR OF MATHEMATICS IN THE ROYAL MILITARY  
ACADEMY, WOOLWICH.

*The FOURTEENTH EDITION, corrected and greatly improved.*



LONDON:

Printed for C. and J. Rivington; J. Nunn; Longman and Co.; T. Cadell;  
John Richardson; J. Mawman; Baldwin and Co.; Sherwood and  
Co.; G. B. Whittaker; Hamilton and Co.; J. Duncan; Hurst  
and Co.; J. Souter; and Simpkin and Marshall, and  
Stirling and Kenney, Edinburgh.

1827.

6. It is required to find the length of a shore, which strutting 12 feet from the upright of a building, will support a jamb 20 feet from the ground.

*Ans. 23.3238 feet.*

7. The height of a precipice, standing close by the side of a river, is 103 feet, and a line of 320 feet will reach from the top of it to the opposite bank: required the breadth of the river.

*Ans. 302.9703 feet.*

How *did* they obtain answers like this, to four decimal places, without a calculator? Amongst other methods such as logarithms (alternate methods again!) by extracting square roots by hand. Next year I intend to use the traditional method as described in Robert Record's *Whetstone of Wit* [1557] to do this!

#### Can raise issues of change and development

Why was Euclid written in Latin? We looked at the situation in schools at the start of the 19th Century—my pupils very much enjoyed learning about the situation in which their predecessors learned maths, or didn't. In many schools maths was either optional (if available at all), or if compulsory, was not considered when promotions were made. Schools which awarded marks weighted them 1 to 3 or 1 to 4 in favour of the classics. At Rugby a Mr James taught a little algebra to his fifth form on Saturday afternoons, but "it wearied his body to excess, and made it hot, or at any rate, perspire too much" (quoted in Howson [1974]). Some things don't change—I feel just like this, many afternoons!

#### By making my pupils think

Present texts are becoming more attractive and friendly to pupils, but in places I feel that, in part *because of* their user-friendliness, they may not challenge pupils to think hard. To read and understand the Euclidean proof of Pythagoras' Theorem is challenging! A gratifying number of my pupils rose to and appreciated the challenge. The work on Pythagoras provided an opportunity to look at cross-curricular ideas in history, art, and languages as well as providing practical and investigational experiences.

The history of mathematics matters to me because it is a relevant channel through which I can introduce topics and motivate my pupils. After we had worked on some materials, I asked them to write down their responses frankly. One pupil expressed himself with more enthusiasm than I had ever discovered in him, which helps give me confidence that this is indeed a good thing to explore.

I found the sheets strange to do although they were very interesting and told us more than just the theory. I found the work educating and enjoyable to do, with a bit of history and a bit of maths.

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# Using History to Enrich Mathematics Lessons in a Girls' School

PATRICIA PERKINS

City of London School for Girls is an independent school situated beside the lake in the middle of the Barbican. It is a selective school taking girls aged from 7 to 18 from many parts of London. Most girls are entered for Higher Level GCSE Mathematics and at least half the Sixth Form take Mathematics at Advanced Level, an unusually high proportion as schools go. We do not at present offer courses in the history of mathematics as such, but I am beginning to use history to enrich my lessons. I have found over the years that when girls start to ask the "Why are we doing this?" type of question, the "Because it's useful!" style of response is not very helpful. The girl in my class planning to be an opera singer knows that she is unlikely to need anything beyond basic arithmetic in her future career. Setting mathematics in a historical context and "selling" it as part of our cultural heritage alongside art and literature has proved much more successful.

In the lower part of the school, topics such as  $\pi$  and Pythagoras' Theorem offer plenty of scope for historical input. At Sixth Form level, girls lacking confidence in dealing with probability, for example, can be very encouraged to find that they can fairly easily solve problems that were once thought difficult by famous mathematicians and thinkers in history. Some of these problems once found their way into a A-level examination question.

22 The following are three of the classical problems in probability.

- Compare the probability of a total of 9 with the probability of a total of 10 when three fair dice are tossed once (Galileo and the Duke of Tuscany).
- Compare the probability of at least one six in 4 tosses of a fair die with the probability of at least one double-six in 24 tosses of two fair dice (Chevalier de Méré).
- Compare the probability of at least one six when 6 dice are rolled with the probability of at least two sixes when 12 dice are rolled (Pepys to Newton).

Solve each of these problems

(AEB 1978)

Difficult ideas about limits and infinity can be introduced using historical references to Zeno and Cantor. One book on this theme which is attractive and accessible to students is *Images of infinity* published by Leapfrogs in 1984.

Next we come to the problem of gender, that is, the "girls and mathematics" debate, which as you can imagine is particularly lively in my school. The girls are very conscious that some perspectives in society have devalued possible contributions by women to mathematics. Here his-

tory again provides a viewpoint in the form of two 16th-century illustrations reproduced in Robert Lawlor's excellent book, *Sacred geometry: philosophy and practice*.



Lawlor's comment on the first picture provokes considerable interest and discussion:

Geometry as a contemplative practice is personified by an elegant and refined woman, for geometry functions as an intuitive, synthesizing, creative yet exact activity of mind associated with the feminine principle. But when these geometric laws come to be applied in the technology of everyday life [the smaller and less significant figures in the picture] they are represented by the rational, masculine principle.

The girls in my Sixth Form groups—especially those studying mathematics with “arts” subjects such as history, English, art or history of art—very much appreciate this notion of male and female roles in the world of mathematics. However, class discussion of the symbolism of the second picture could be dangerous! Of this, Lawlor comments:

*Arithmetic* is also personified as a woman [...] On her thighs (symbolizing the generative function) are two geometric progressions [...] the left thigh, associating the even numbers with the feminine, passive side of the body [...] the right thigh, associating the odd numbers with the masculine, active side.

However, students are still reassured and find their morale raised by considering that the powerful female figure towers over the rather miserable-looking men, Boethius and Pythagoras, who appear to be holding a computational competition between abacus and Hindu-Arabic numerals. (Teachers may like to know that there is a most interesting and rather fuller discussion of this illustration by Dick Tahta [1991].)

Ideas about logic and elegance in mathematics can be further developed by more specific references to the history of mathematics. For instance, comparison of Euclidean geometry with the practical investigational approach encouraged for GCSE can give able pupils an introduction to the idea of proof. The topic even provides the opportunity to mention Hypatia, the first recorded woman mathematician, and the legendary accounts of her murder in AD 415.

The work I have done with pupils has convinced me that the history of mathematics has a great deal to offer, especially to those teaching girls. It provides opportunities to discuss with pupils creativity in mathematics and good style in solutions to problems, but above all it makes my lessons more interesting and therefore more successful.

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# Two Benefits of Using History

ABRAHAM ARCAVI

In mathematics education much stress is laid on understanding the nature and role of notation and symbolism. Alfred North Whitehead discussed at length the "enormous importance of a good notation" and the benefits of symbolism in mathematics:

[...] by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain. It is a profoundly erroneous truism, repeated by all copybooks and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them. Operations of thought are like cavalry charges in a battle—they are strictly limited in number, they require fresh horses, and must only be made at decisive moments. (Whitehead [1911] p.61)

In other words, we may say that mathematics instruction strives for meaningful automatisms, or in Skemp's formulation

If we are to make progress in mathematics it is, indeed, essential that the elementary processes become automatic, thus freeing our attention to concentrate on the new ideas which are being learnt—which in turn must also become automatic. (Skemp [1987] p.61)

But there are problems—this automating of thought may not be an unalloyed good. Hans Freudenthal, for example, writes

I have observed, not only with other people but also with myself [...] that sources of insight can be clogged by automatisms. One finally masters an activity so perfectly that the question of how and why is not asked any more, cannot be asked any more, and is not even understood any more as a meaningful and relevant question (Freudenthal [1983] p.469)

Following Freudenthal, there may be advantages to directing our attention precisely in the opposite direction to that pointed by Whitehead and Skemp: the undoing or unpacking of the automation process, the reviewing of those "sources of insight" which somewhere in the past were incorporated into our mathematical background as automatic. History seems to be a wonderful way to do so, providing excellent raw material for engaging in unclogging exercises. By *unclogging* exercises we mean activities based on historical sources which may help to re-examine known and taken-for-granted mathematical ideas. In our experience, we have found that mathematical understandings gain strength and scope from discussions of this sort.

Consider for example the following argument taken from a letter written by the French mathematician Arnauld (1612-1694) to his colleague Prestet (1648-1691)

(Schrecker [1935]): How can one reconcile *1 is to -1 as -1 is to 1* with the idea of proportion? On the one hand we have a proportion which is correct, if considered as the fractional equation  $1/-1 = -1/1$ , but on the other hand it does not seem right to assert that a big number is to a small number as a small number is to a big number. The argument bothered mathematicians in the past, because the intuitions from two areas of mathematics seemed in conflict. In our teacher courses we ask the participants how they would respond to this (and similar) arguments. This request always stimulates lively discussions ("unclogging") of ideas which teachers seldom make the subject of re-inspection, out of the belief, that nothing else could be said about them. With this stimulus, however, they suddenly realize that there are important aspects which they never thought about before.

Another potential benefit of using history of mathematics lies in sensitizing the teacher to possible difficulties of students' understanding; it may indeed yield clues on how to respond and help the student over them. History can provide a feeling, for example, for how standards of rigor evolved. What today may be regarded as a non-rigorous mathematical argument was widely accepted two centuries ago. In this way, one may grasp the social context of proof by looking at it as a communication device for members of a certain mathematical community at a certain period of time. This realization may be very helpful to us teachers in developing our sensitivity and willingness to listen to our students' mathematical arguments, which are indeed shaped by their feeling of what may be convincing for themselves or their peers, rather than by today's mathematical standards of rigor. (A helpful recent discussion of this area is the special issue of the journal *Interchange* edited by Hanna and Winchester [1990]).

I believe history provides an authentic and valuable setting from which to illustrate the two points above, which are only two among the many I could choose to point to as an answer to "how history mattered to me in my teaching practice". For a more detailed account of the rationale, materials and experiences of using history in preparation courses for mathematics teachers, see Arcavi, Bruckheimer and Ben-Zvi [1981,1987]).

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# The Reading of Original Texts: How and Why to Introduce a Historical Perspective

EVELYNE BARBIN

In France today, there is a revival of interest in the history of mathematics among mathematics teachers. It is particularly true for the studies and researches undertaken by the Institutes for the Research on the Teaching of Mathematics (IREMs). For around fifteen years now, these institutes have formed research and training teams devoted to the epistemology and the history of mathematics. They have organized many symposia and summer school sessions, and have promoted and written many books and booklets about history of mathematics. One of these books is translated into English and published by The Mathematical Association under the title *History in the mathematics classroom*.

In this book, the reader will see how much importance the French institutes attach to the study of original texts. This is true for the training courses of mathematics teachers, and also for the classroom. To consider this use of the reading of original texts is a good way of asking the questions: why and how to introduce a historical perspective into the teaching of mathematics?

## To change the image of mathematics

The reading of original texts allows the teacher or student to study the nature of mathematical activity in its various facets: to analyse the role of problems, of proof, of conjecture, of evidence, of error in the construction of mathematical knowledge. The reading of old texts is also a way to gain access to the epistemological and philosophical concepts which permeate mathematical texts. It is possible to study the scientific, philosophical, cultural and social context in which the mathematical knowledge was elaborated, and to see the cultural aspects of the mathematical knowledge by an interdisciplinary approach.

So the reading of original texts is a way to change the image of mathematics in pupils' and in teachers' heads, to see mathematics not as a language or as a spectacle but as an activity—a human activity. This role is very important in the context of mathematical education in France, because engaging with the history of mathematics is a way to struggle against a formalist and dogmatic view of mathematics.

I give now one example which concerns mathematical proof. The history of mathematical proof was the subject of our last symposium. We studied the mathematical activity or proof through original texts, which made possible the examination of questions such as: what is the object of

proof? what are the forms of proof? why are there such varieties of proof? why are there controversies about proofs? We have furnished many original texts which can also be studied in the classroom.

In France teachers have been taught to see proof as synonymous with deductive proof, and they are very much attached to the perfect form of such proof, even with young pupils (aged 13-14). So the reading of original texts can change the conceptions of teachers and their image of a mathematical proof: to elaborate a proof also means constructing the sense and the meaning of the mathematical concepts, while envisaging a proof as something that can only be purely deductive hides the questions, the doubts, the way of invention.

To illustrate this, I compare two texts about the concept of angle and the proof of the basic geometrical result that the sum of the angles of a triangle is two right angles.

## The sum of the angles of a triangle

The first text is Book I of Euclid's *Elements*. Here the angle is defined as an inclination between two lines. But when Euclid wants to compare two angles, he closes them and uses the figure of the triangle. To compare the angles  $A$  and  $D$ , he takes  $AB$  equal to  $DE$  and  $AC$  equal to  $DF$ , and he compares  $BC$  and  $EF$  (Figure 1). Situating the angle in a triangle allows him to prove (Prop. 16) that the exterior angle is greater than either of the remote interior angles: he takes  $AE$  equal to  $EC$  and  $BE$  equal to  $EZ$ , and constructs the triangles  $BAE$  and  $ECZ$  to obtain the equality between the angles  $BAE$  and  $ECZ$  (Figure 2).

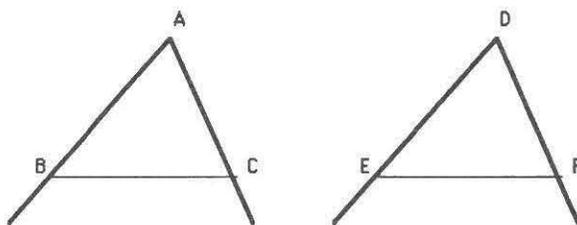


Figure 1

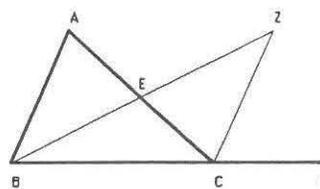


Figure 2

But to prove that this exterior angle is equal to the sum of the remote interior angles, Euclid uses a new way of comparing angles arising from the concept of parallel lines. He takes  $CE$  parallel to  $AB$ , and concludes that the angles  $BAC$  and  $ACE$  are equal, and that the angles  $ABC$  and  $ECD$  are equal (Figure 4). So he uses a proposition (Prop. 29) whose proof needs the famous Euclidean postulate (Figure 3).

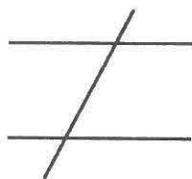


Figure 3

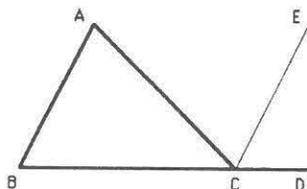


Figure 4

So we see that after each of these proofs, the concept of angle changes because we have a new way to use it and to understand it through its relations with the triangle, with parallel lines and so on.

Now there is a difficulty with Euclid's proof. Of course, we are convinced *that* the sum of the angles of a triangle is equal to two right angles, but we no longer know *why*. So how can we approach more closely to an intuitive feeling for why this magical conception of parallels gives rise to the result?

We gain in understanding through comparing Euclid's approach with that of Clairaut, in his *Elements of geometry* of 1765. This text does not have the deductive structure of Euclid's *Elements*. Clairaut does not like it because, as he writes, he wants to interest and to enlighten his reader, not only to convince him. He wants to give to the reader the mind of invention, the possibility of progressing creatively himself. Clairaut defines the angles as an inclination, like Euclid, but unlike him he uses this definition within the proofs. Before proving the result about the sum of the angles of a triangle, Clairaut proposes to his reader to imagine a triangle  $ABC$  and to turn the straight line  $BC$  (Figure 5). We see that the angle  $B$  opens and the angle  $C$  closes, and Clairaut writes that we *feel* that the sum of these two angles stays the same. Questions? Doubts? We want to know. Clairaut explains that this induction contains his proof.

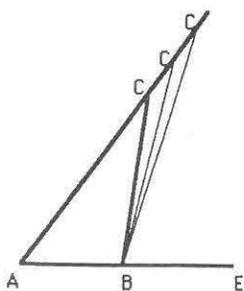


Figure 5

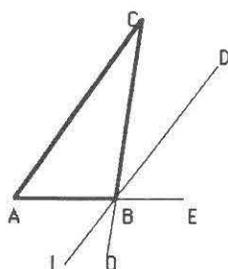


Figure 6

If we continue to turn  $BD$  until  $BD$  becomes parallel to  $AC$  then  $AC$  and  $BD$  have the same inclination to  $AB$ , so the angles  $BAC$  and  $EBD$  are equal, and  $AC$  and  $BD$  have also the same inclination with  $CB$ , so the angles  $ACB$  and  $IBO$  are equal, and the same for the angles  $CBD$  and  $CAB$  (Figure 6).

So from Clairaut's *Elements of geometry* not only do we know the result, we are in a better position than a reader of Euclid's *Elements* to know how we know it.

This example illustrates the theme introduced by my opening question. If the IREMs are committed to the reading of original texts, it is because to introduce a historical perspective in the teaching of mathematics in this way is a profound means of grasping the sense and the meaning of mathematical knowledge and mathematical activity.

## Integrating the History and Philosophy of Math into Core Curriculum Math Courses from a Cultural and Humanistic Viewpoint

GARY BROWN

I teach at a women's college in central Minnesota, USA, that has a unique cooperative arrangement with the men's college, St. John's University. We have a joint mathematics department and cooperative math courses, but separately male and female institutions.

During the academic year 1986-7, our joint faculties decided upon a new core curriculum package. The courses we teach that were designated as core curriculum math courses are: Calculus I, Calculus II, Finite mathematics, Essentials of calculus, Statistics, and Math appreciation. Courses designated as having core curriculum credit had to satisfy certain guidelines, of which the first appertained to the history and philosophy of mathematics.

The question became, how should we implement the core guidelines in these courses? I decided to teach these courses from a *cultural* and *humanistic* viewpoint. To explain what is meant by these buzz words, I must go back to my mathematical roots. I was originally trained in the foundations of mathematics and still enjoy dabbling in it. I studied the foundations of set theory, and once thought that all mathematics ought to be taught from a set theoretical viewpoint.

#### Core curriculum guidelines for mathematics

- (1) The course will address the history, philosophy and contemporary role of mathematics. It will stress mathematics as a conceptual discipline and will demonstrate to students the aesthetics and the value of mathematics.
- (2) The course will be structured so that students are actively involved in doing mathematics and are asked to demonstrate in various ways their understanding of appropriate concepts.
- (3) The specific content, the questions, and the examples used in class should begin with ideas familiar to the students.
- (4) The course will assist students to understand mathematical symbolism and the need for it. It will address the power and limitations of mathematical modeling as a tool for solving problems outside of the classroom.
- (5) The course will focus on student involvement, understanding and appreciation for mathematics rather than on computational rigor.

In recent years, however, I have come to believe that the philosophy of mathematics should not be centered on the foundations of mathematics, but instead should focus on *mathematical practice*. This viewpoint is strongly advocated in the work of Thomas Tymoczko, especially in the book he edited called *New directions in the philosophy of mathematics*, which includes articles by Imre Lakatos and Philip Kitcher. Reuben Hersh's contribution to this book includes the following helpful definition, which I have begun to work with.

By "philosophy of mathematics" I mean the working philosophy of the professional mathematician, the philosophical attitude towards history that is assumed by the researcher, teacher, or user of mathematics.

Once we assume that the philosophy of mathematics teaching ought to be centered on *mathematical practice* and the working process of those actually doing mathematics, then it becomes readily apparent that mathematics is a humanistic discipline—that is, a cultural product.

The eminent mathematician Raymond Wilder wrote about mathematics as a cultural endeavor in his book, *The evolution of mathematical concepts*. Here, drawing on anthropological work, he wrote that "a *culture* is the collection of customs, ritual beliefs, tools, mores, etc, which we may call *cultural elements*, possessed by a group of people...Generally it is not a fixed thing but changing with the course of time."

If mathematics is an humanistic discipline that changes over time, then one ought to study the history of mathematics in order to better understand the nature and philosophy of mathematics.

There is a network of mathematics teachers in the U.S.A. formed in 1986 that calls itself the Humanistic Mathematics Network (contact Alvin White from Harvey Mudd College in Claremont, California for more information). A shared objective of this network is to teach mathematics humanistically. This involves, for example, arrang-

ing classroom practice so that the student is put into the position of an inquirer, while acknowledging the emotional climate of the activity of learning mathematics. An excellent way to understand mathematical activity is to compare classroom practice with how the activities of mathematicians have changed and evolved historically. Hence I view the mathematics classroom as a microcosm of the cultural endeavor we call the practice of mathematics. We ought to be aware of all the customs, rituals, mores, etc., that Wilder calls *cultural elements*.

I will close with three examples of my classroom practice in trying to teach these core curriculum math courses from a humanistic and cultural viewpoint.

#### Mini-lectures

These are episodes in the history of mathematics which occupy five minutes at the beginning of a class. I have used Fred Rickey's unpublished mathematics calendar as a reference for possible ideas. A mini-lecture may comprise a famous mathematical story, mathematical folklore, a short biography, controversial quotes, or occasionally poetry. At the end of many mini-lectures I include a timeline that places the mathematical episode within its period in history.

Eventually, students have the opportunity to give mini-lectures of their own and are required to write short papers using at least three distinct sources. Students can enhance their grades with these short oral presentations if they wish.

#### Mathematical dialogues

Here two or three students form a group to write and perform a mathematical dialogue. The students find some theme that has dramatic content and write a play about it. Most plays last for 5-10 minutes. Examples of plays mounted by my students include the debate between Fermat and Pascal about probability theory, and a debate over the proper place of the epsilon-delta definition of limit in a first year calculus course. I have described this activity more fully in Brown [1986].

#### The centrality of mathematics

This article [Grabiner, 1988] is the best I have found for beginning math students that encourages excellent discussion about the nature of mathematics. It discusses the Platonist and Aristotelian viewpoints of mathematics, how mathematics has evolved, especially in the last 200 years, and the question of where the philosophy of mathematics belongs in the mathematics classroom.

#### References

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# Perils and Pitfalls of History

DAVID FOWLER

Mine is a cautionary tale. I want to describe some of the perils and pitfalls for teachers in getting involved in research into the history of mathematics. Like everybody in this session, I'll do it from my personal point of view.

I work at the University of Warwick Mathematics Department, in Coventry (not Warwick!). It is a big and open department, with visitors from all over the world (it is staging-post for the international mathematical jet-set, as well as lots of other kinds of people), a very large graduate school, more than 400 undergraduates, some odd visitors like foreign exchange students, and so on. There are usually many different kinds of activity going on—this week, for example, there is a research workshop on differential equations, and 90 sixth-form girls are having some experience of university mathematics. All of this takes place in a building that is attractive and well-designed, though now over-crowded and cluttered. The department was set up by Christopher Zeeman about 25 years ago, and I used to have lots to do with organizing the visitor's programme, an experience which has had some influence on my idea of "the mathematical community". So I used to be around the Department most of the time: high profile, always accessible. I used to take a straightforward and uncomplicated attitude to teaching. It occupied a quarter or so of my time, and, to my surprise, I quite enjoyed it. I was just as accessible to undergraduates as I was to visitors and colleagues. They recognized this and responded to it.

I'd always been mildly interested in history; then, about 10 years ago, I got seriously involved, principally in Greek mathematics but with many ramifications. This involved my spending lots of time in libraries, lots of time holed up in my study, and generally being very careful and jealous in my use of time—the day just isn't long enough when you get engrossed in a project. So I became remote, neglected my family (not all a bad thing, since my adolescent children wanted to be left alone), neglected my students, and lived in my books and word-processor. That's the first effect. I must admit that it has been greatly aggravated by increased loads in teaching and administration—you run the risk of being swamped, if you let it happen—and by the evolution from a new and experimental department to a middle-aged, but I hope still experimental, one.

The second effect was a greatly increased consciousness of the paradoxically restricting effect of historical knowledge upon teaching. There are lots of different ways of looking at and using many bits of mathematics, and I became more and more aware of that. For example, a book I wrote on Greek mathematics [Fowler, 1987] revolves around possible meanings of " $a : b$ " for about 400 pages! So my teaching became more complicated and convoluted; I am now incapable of a straight answer to many a straight

question, but want to add commentaries, qualifications, glosses, and footnotes. For most students, this is less effective.

I became the historian in the Department and so, since there is always pressure from the students as well as colleagues and other interested parties for this kind of thing, I started a course on the history of mathematics. The students wanted to be told the story—a straightforward, uncomplicated, narrative history of the development of mathematics. I don't see things that way: history is perverse and convoluted in its detail, and I like that detail. So, instead, we look closely at very few episodes, starting by looking at texts without framework if interpretation, and then see how assumptions get loaded onto these texts. Some students don't like that approach—they repeat that they want the story, no complications.

Why not avoid my complications and tell a straightforward story? I think you may then get more of the students' attention, and you may be a more effective teacher in the sense that you get more material over, but—and this is the third pitfall—you may be creating a mythical and misleadingly simple past, and not doing justice to history. I'll give just one example from my field of interest. There has been a trend, during the last century and more, to examine critically the stories about Pythagoras and the Pythagoreans and reject many of them as unreliable and implausible, and so to dismantle this early episode in the history of mathematics (though not everybody accepts this line of interpretation). The most thorough and influential recent book on this topic is Walter Burkert, *Lore and science in ancient Pythagoreanism*. Here's a quotation from it, from page 406:

What is the origin of the firmly rooted conviction that Pythagoreanism was the source of Greek mathematics? This question is easy to answer: it came from the educational tradition. Everyone comes upon the name of Pythagoras for the first time in school mathematics; and this has been true from the earliest stages of the Western cultural tradition. None of the ancient textbooks which formed the basis of the medieval curriculum forgets Pythagoras. He is the companion of Arithmetica in Martianus Capella; and according to Isidore he was the first, among the Greeks, to sketch out the doctrine of number, which was then set forth in detail by Nicomachus. This takes us back to the origin of the tradition; Nicomachus [early second century AD, some six centuries after Pythagoras], who is himself called a Pythagorean, begins his *Arithmetica*, which was much used as a school-book, with praise of the master. Boethius' *Arithmetic*, [early sixth century AD], drawn largely from Nicomachus, also names Pythagoras in its first line. Likewise Gerbert of Aurillac [tenth century AD] mentions the name of Pythagoras several times in his geometry; and among the patrons of their *ars geometriae* medieval Freemasons include Pythagoras. The *Ars geometriae* bearing the name of Boethius, though obviously not composed before the High Middle Ages, even presents an early version of the Arabic numerals as an invention of the "Pythagorici," and describes the method of calculating with these *apices* on an abacus, called *mensa Pythagorea*—perhaps the most striking of the anachronisms in which the Pythagorean tradition is so rich. Finally, the early modern period derived the astronomy of Copernicus and Galileo from Pythagoras. The general belief in the Pythagorean scholastic tradition of late antiquity.

So the Pythagorean story, some bits of which I suspect you all tell, may be a bit of pseudo-history created by teachers to ginger up the curriculum.

I ask you to reflect on what you are doing: are you using history, or creating a pseudo-history? While providing material for the budding mathematicians of today, you may also be creating or reinforcing phantoms that will take armies of historians to exorcise, if needed they can ever succeed! Mathematics has a comfortable characteristic: that the words "right" and "wrong" apply to it more easily

than to any other subject; history, however, is more complicated than that. The mixture can be very uneasy.

### References

W. Burkert, *Lore and science in ancient Pythagoreanism*, translated by E.L. Minar from *Weisheit und Wissenschaft: Studien zu Pythagoras, Philolaos und Platon* (1962). Cambridge, Massachusetts: Harvard University Press, 1972

D.H. Fowler, *The mathematics of Plato's Academy: a new reconstruction*. Oxford: Clarendon Press, 1987

### DIAGRAM OF THE DEVELOPMENT OF MATHEMATICAL EXPERIENCE IN THE RACE AND IN THE INDIVIDUAL

Time Axis runs from top to bottom through centre of Chart. Scale from 600 B.C., onwards, is roughly one inch = 2,000 years.

INTERPRETATION (see also chapter XVI).

The two elements of mental activity in the development of knowledge (excluding the will and the feelings) are:—

- (1) Thought-activity, centrally excited (physiologically), forming the conceptual element of knowledge: this is symbolized in the diagram by shaded space.
- (2) Sense-activity, peripherally excited (physiologically), forming the perceptual element of knowledge; this is symbolized in the diagram by unshaded or white space.

The conceptual (shaded) element of experience, postulating its existence, becomes ultimately unrecognizable in the chart, on the rough scale of ratio adopted.

The diagram is to be considered both vertically and horizontally, with a view to the suggested parallelism between racial and individual developments, and in respect of (i) the kind, quality, and quantity of the experience or knowledge, and (ii) the external factors stimulating the development.

Only the external factors have been summarized in the diagram. There remains the internal factor, the aesthetic-scientific interest impelling the mind to a study and perfection of its own creations—the pursuit of science for its own beauty.

#### ADDITIONAL EXPLANATIONS.

We may roughly group the main Primary and Derivative Occupations thus:—

##### I. Primary Occupations.

Miner.	Shepherd.
Woodman.	Peasant.
Hunter.	Fisher.

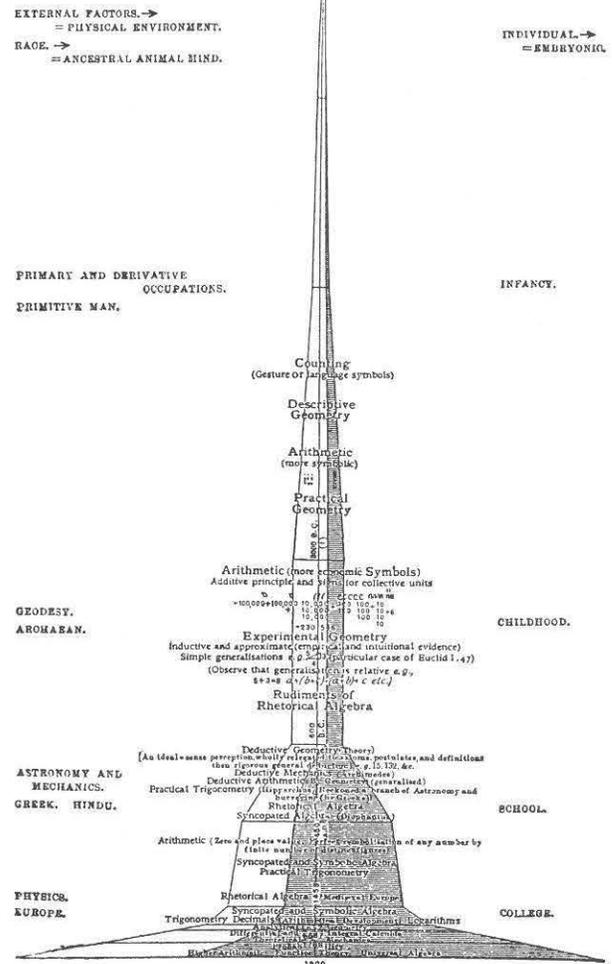
##### II. Derivative Occupations.

<i>Trades.</i>	<i>Professions.</i>
Arts and Crafts (male and female).	Scribe (Lawyer, Teacher, &c.).
Smiths and Wrights (Wheel- and Ship-wrights, &c.)	Musician and Poet.
Builder.	Warrior.
Engineer.	Architect and Engineer (e.g. Pyramid Builders).
Farmer.	Priest (Doctor, Teacher, Astrologer and Mathematician, &c.).
Trader and Merchant.	

##### Rulers and Administrators.

Under Geodesy would fall the Surveyor (of the Nile Districts) and Cartographer.

Under Archaean would fall Egyptian, Babylonian, Phoenician, &c. (Pythagoras was a Phoenician; possibly Thales too). Along with Hindu would come Arabian and mediaeval European mathematics.



Schema by Benchara Branford, 1908 (reference on page 3)