

“THIS IS HOW WE DO THIS AND THIS IS THE WAY IT IS.” TEACHERS’ CHOICE OF MATHEMATICAL COMPASS

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In our work with two fifth-grade teachers, we were initially interested in investigating what might happen when they choose to rely less on teaching mathematics as a set of rules and what might happen if students were provided with more opportunities to use alternative methods, either self-developed or modelled by the teacher. While the teachers explored alternative activities in focus group sessions and in their classrooms, throughout the study we noticed that each teacher consistently deferred back to the textbook as the ultimate authority on curriculum and pedagogy. Thus, we were drawn to refocus on the factors that influenced this deferral.

Others have shown that textbooks have been viewed by teachers as comprehensive conceptualizations of curriculum awaiting enactment (Dowling, 1996; Johansson, 2007; Lloyd, 1999; Remillard, 2000), a view that has resulted in tensions and contradictions in teaching practices. Teachers have been observed adopting content in the mathematics textbook “as is,” despite having conflicting views of the textbook (Johansson, 2007; Lloyd, 1999; Ponte *et al.*, 1994; Remillard & Bryans, 2004; Wilson & Goldenberg, 1998). Content knowledge, parental pressures, conflicting beliefs, to name a few, have been described as resulting in pressure points for teachers, pushing them towards adherence to the textbook (Herbel-Eisenmann *et al.*, 2006; Lubienski, 2004; Peressini, 1998; Ponte *et al.*, 1994).

We draw on Bakhtin for our interpretive lens of “textbook as authority.” According to Bakhtin (1935/1981), the negotiation between self and other, teacher and curriculum, teacher and student, teacher and policy, and so forth, is a dialogue between *authoritative discourse* and *internally persuasive discourse*. As we probed deeper into our own results and the existing literature, we concluded that neither label was adequate for describing the role of the textbook. We found the teachers used the textbook at times as a *cover story* (Connelly & Clandinin, 1999) – a way of superficially addressing what they perceived to be the expectations of the mandated mathematics curriculum.

Mathematical compass

We examine those factors that situate a mathematics textbook as a central authority, thus the primary *mathematical compass* in teachers’ pedagogical practices – in spite of instances where such an adoption appears to be counter to the needs of students and to a teacher’s own understanding of mathematics pedagogy. We use *compass* in two senses:

first as an instrument for determining direction, which can be situated internally (*e.g.*, within the teacher) and/or externally (*e.g.*, in a book, policy statement, mathematical object, *etc.*); second as a range within limits. [1] The phrase *mathematical compass* is meant to imply the navigation of pedagogical direction within a perceived range of options in mathematics education, based upon understanding, beliefs, and, in particular, a perception of authority. For example, one pedagogical approach to number sense may focus on the learning of standard computation procedures, while another may focus on students’ invented procedures. These approaches may include a range of pedagogical directions, including inventing computation procedures and learning standard procedures.

In addition, the range of a textbook’s pedagogical compass may vary and consequently be inconsistent. For example, a textbook may approach some topics more procedurally and others more conceptually. A textbook as a mathematical compass does not necessarily point towards a single, consistent direction (Remillard & Bryans, 2004). Further, ideas in textbooks can be interpreted and used differently by different teachers (Lloyd, 1999; Remillard & Bryans, 2004; Wilson & Goldenberg, 1998). Finally, textbooks are often perceived by teachers as “authorities” of curriculum and pedagogy (Dowling, 1996; Johansson, 2007; Lloyd, 1999; Remillard, 2000).

The fifth-grade textbook used by the teachers in this study was adopted in a district-wide initiative. We do not single out a particular textbook as being more or less of an appropriate mathematical compass. Rather, we identify and discuss factors *outside* the textbook that possibly contribute to the establishment of a textbook as a seemingly central authority for teachers.

Authoritative and internally persuasive discourse

Authoritative discourse, according to Bakhtin, is dogmatic discourse eliciting adherence often in a subversive and compelling way. In contrast, internally persuasive discourse is conceptually reflective of autonomous thought and action. Discourse, as used by Bakhtin, is intended to represent a broad perspective of discourse as “socially accepted associations among ways of using language, of thinking, valuing acting, and interacting, in the ‘right’ places and at the ‘right times’ with the ‘right objects’” (Gee, 1999, p. 17). Discourse,

then, is intended to signify a plurality of perspectives (Moschkovich, 2007).

The distinctions between authoritative and internally persuasive discourse are not straightforward. Developing internally persuasive discourse involves a process of distinguishing between our own and someone else's discourse – it is a process of developing an individual consciousness, of “ideological becoming,” to use Bakhtin's term.

In the case of mathematics education, and this discussion of textbooks, ideologically becoming would involve an understanding of textbooks as compasses with mathematical directions, parts of which may or may not be pedagogically appropriate, important, or useful. Ideologically becoming, for a teacher, would be reflected in autonomous choices to follow certain mathematical directions over others – particularly those directions that would allow students to engage in deeper mathematical meanings and relationships. Entering into a dialectical relationship with a mathematics textbook means that a teacher *can* use the textbook to learn and grow pedagogically and mathematically. However, such directions involve engaging in more sophisticated mathematical relationships (*e.g.*, optimization) and complex relationships among various forms of mathematical communication (*e.g.*, graphs, equations, tables of values, diagrams, and problem statements).

We question whether the labels *authoritative* and *internally persuasive* are appropriate for all teacher-textbook discourses. Consider the case where the textbook provides opportunities for students to develop internally persuasive discourse about a certain mathematical topic. Now suppose that the teacher does not have a matching internally persuasive discourse. We can imagine the teacher entering into an internally persuasive discourse with the textbook, reflecting on his or her own pedagogy and mathematical knowledge, then engaging in an ideologically becoming experience in which they themselves develop an internally persuasive discourse about this topic. However, we have difficulty imagining an authoritative discourse in this circumstance: without an ideologically becoming experience, the teacher does not have the requisite mathematical understanding to teach the topic. In this case, we can imagine the teacher partially or superficially adopting the direction of the textbook, giving only the appearance of an authoritative discourse. Also, if the teacher implements the direction of the textbook partially or superficially, can we call this an internally persuasive discourse between teacher and textbook? We think not, given the avoidance of a dialectical relationship.

Our experience as researchers and educators tells us that teaching situations are complex and the use of either/or labels is sometimes problematic. Also, our involvement in the professional lives of the two teachers was an attempt to engage in a dialectical relationship, an attempt at fostering an internally persuasive discourse rather than an authoritative discourse. Various other authoritative discourses affected our relationship, or rather, the web of relationships.

Marika and Joli's mathematical compass

Data were collected over a single year in two fifth-grade classrooms. The school was in a large urban setting in an area with a relatively high population of families of higher

socio-economic status. The teachers, Marika and Joli, taught the fifth grade previously, and both had over ten years teaching experience. Neither was a “mathematics specialist.” Both reported that the textbook was their primary mathematics teaching resource, used to direct their program at least 90% of the time. Each indicated that they closely followed the textbook, chapter by chapter. Each also reported they used the tests provided in the teachers' guide as means of assessment, almost exclusively.

Before we joined Marika and Joli in their classrooms, they took part in a full-day focus group session. One goal of this session was to explore program areas they might be interested in improving through our work together. We also engaged teachers in mathematical activities that modelled the exploration of mathematical relationships. Although we did introduce alternative mathematical compasses through our collective explorations, it was not our intention to simply provide specific strategies or activities. We were ready and willing to follow their mathematical interests, whether this resulted in trying our ideas or their own.

During the focus group session, we also reviewed Marika and Joli's long-range teaching plans. We discussed which parts of the curriculum they found challenging, pleasurable, unnecessary, or most important. Marika and Joli identified areas of concern within their own classes and within their pedagogical practices. These discussions formed the basis of the mathematical compasses explored in subsequent sessions. For example, both teachers identified numeracy as an area of need, in that students' lack of “basic skills” prevented them from engaging in more sophisticated mathematics. In follow-up discussions we suggested a focus on alternative algorithms and offered a resource. Our intent was to center the research direction, as much as possible, on the teachers' perceived classroom needs.

The textbook used by the teachers did address the issue of alternative algorithms by providing opportunities to use more flexible “mental math” methods when adding or subtracting. For instance, in a section about adding four-digit numbers, the textbook introduced the concept by showing the first steps of two different solutions – one that first adds the one's column, while the other first adds the thousand's column – then asked students to explain and complete each method. However, the student exercises, chapter review exercises, and chapter test did not explicitly mention that a variety of solution strategies, when solving problems, could be used. Consequently, students could be successful while focusing on a narrow range of strategies. Our pedagogical suggestion, in pursuing a focus on alternative algorithms, was to make the focus on alternative algorithms much more explicit.

In addition to the first focus group session, four planning/interview sessions were held throughout the year, each immediately followed our joining the teachers in their classrooms. Transcribed recordings and field notes were collected during observations of lessons on number, algebra, measurement and data management and probability. We also made observational notes on daily conversations with Marika and Joli. Often these conversations reflected the underlying processes they used as they navigated through their mathematics program.

A content analysis was conducted of the data collected

(Berg, 2004) and themes or categories identified. In the second stage of the content analysis, the data were analysed once more with the final set of categories to verify the integrity of the data organized under each category. Descriptive statements were added to capture the essence of the themes identified. Our summary is not intended to speak for our teachers. We present, rather, an account of *our* interpretations and conclusions of our dialogues with the teachers.

Factors influencing authoritative discourse

We knew from Marika's and Joli's estimations of their use of the textbook that they viewed the textbook as an authoritative mathematical compass. One obvious factor was their individual mathematical identities, shaped by their personal histories with mathematics and mathematics education (Gadanidis & Namukasa, 2009; Ponte *et al.*, 1994). Our time spent with Marika and Joli revealed four additional factors to their own mathematical identities that situated the textbook as the authoritative discourse: mathematics textbook as privileged discourse, students' privileging of mathematical discourses, curricular demands and time constraints, and parental pressures.

Teacher's mathematical identity

Marika and Joli recounted fairly negative experiences as students of mathematics. Lack of success contributed to anxiety about teaching mathematics. Teaching mathematics was not comfortable for either; both required additional planning and resources to help them meet curriculum expectations. Indeed, each expressed dependency on the textbook and supporting materials (*e.g.*, teachers' guide; black-line masters).

Marika and Joli described having always been taught to follow rules and procedures in mathematics. Marika recalled her experience as a student, "I was never taught why math works, we were just given formulas, rules, and laws." Joli suggested that she fared relatively well in math until the later grades. Memorizing rules and formulas was key to survival in mathematics. When Marika and Joli tried to shift their teaching along the spectrum from offering rules to supporting understanding, they realised their own needs for better mathematical understandings. Joli explained: "If I felt competent and talented in math, I could teach with the ease and freedom that I teach art. I see things in math page-by-page because I don't have another vision."

With respect to mathematics, both Marika and Joli, like many elementary school teachers, had low personal self-efficacy and they lacked positive, personal learning experiences. These factors influenced their adopting the textbook (or at least part of the textbook, as we elaborate) as an authoritative discourse and the primary mathematical compass, to the exclusion of mathematical compasses that might be developed internally. In Bakhtinian terms, there did not seem to be a dialectic relationship between the teacher and the textbook.

Mathematics textbook as privileged discourse

By *privileged discourse*, we refer to instances of discourse that exert power, subversively or otherwise, over the potential for autonomous practice by teachers. Similar to other

documented cases, Marika and Joli saw the textbook as a faithful interpretation of both curriculum and sound pedagogical practice (Dowling, 1996; Remillard, 2000). Teachers' attempts to move toward more autonomous practice, or alternative mathematical compasses, may be significantly influenced by the extent they are able to navigate past this perception.

A key example of the privileged position of the textbook occurred in the number operations unit. Marika and Joli deviated initially from the textbook. They added a much greater emphasis on alternative procedures through a series of lessons on alternative computation algorithms and a culminating activity, referred to as the "Math Olympics." In the Math Olympics, students worked in groups and accumulated points by using their knowledge of different algorithms. The activity itself was developed in the focus group sessions, outside of the textbook.

As noted, both teachers previously used tests provided in the teachers' guide as the summative assessment tools for units of study. In light of the current emphasis on alternative algorithms in their classes, they decided during the focus group session to create different summative assessments. We initially saw this decision as important. Both teachers appeared to be engaged in internally persuasive dialogues, demonstrating the navigation of alternative mathematical compasses.

Later we were surprised to hear from Marika and Joli that, despite this decision, because of the different mathematical compassing in which they and their students were engaged, in consensus, they had administered the test from the teachers' guide. This choice was made even though they felt that they had not spent enough time on the textbook material for students to be successful on its unit test, nor was the expanded focus on alternative algorithms reflected in the test. Marika and Joli explained this in another way: they were concerned that they had not met the "required math curriculum expectations," which they perceived as being most genuinely interpreted by the textbook and its test. They also worried that the students would not be prepared to write other tests from the textbook since, in their opinions, one chapter intrinsically linked to another - regardless of content.

Upon reflection, the teachers did see the limitations and implications for students by using a prescribed test. Joli commented,

The test presented the questions differently than we presented the material. The test was structured differently and it was somewhat unfair for the students. We really should have written our own test for this unit if we are going to change the way the material is presented.

Marika's and Joli's potential for following an internally persuasive discourse or alternative mathematical compass was circumvented. Their decision was unexpected - but perhaps should not have been surprising, given that each viewed the textbook as representing "the curriculum."

Students' privileging of mathematical discourse

Marika and Joli noted that students - particularly high achievers - appeared to revert to traditional procedures on

tests. For example, for the test on number operations, only a few students used the alternative methods they were taught. (As noted, the test did not set explicit expectations to use or compare methods.) During the number operations unit students were introduced to and assigned homework on a variety of strategies. For example, strategies for addition of whole numbers included:

- partitioning
 $49 + 34 = 49 + 30 + 4 = 79 + 4 = 83$

- compensating
 $19 + 34 = 20 - 1 + 34 = 54 - 1 = 53$

- expanded notation:

$$\begin{array}{r} 49 \\ 34 \\ \hline 83 \end{array} = \begin{array}{r} 40 \\ 30 \\ \hline 70 \end{array} + \begin{array}{r} 9 \\ 4 \\ \hline 13 \end{array}$$

- left-to-right column addition

$$\begin{array}{r} 145 \\ + 136 \\ \hline 200 \\ 70 \\ + 11 \\ \hline 281 \end{array}$$

Additionally, students solved problems mentally and shared personal procedures with immediate neighbours and the whole class. We surmised that the low use of alternative algorithms on the unit test was partially attributable to the asymmetrical relationship between the mathematical compass of the unit and the mathematical compass in the test from the teachers' guide.

Although weaker students expressed enjoyment and seemed eager to try new strategies with alternative algorithms, some stronger students seemed more reluctant. One possible explanation is that successful students may naturally want to keep using the methods that led to their success and perhaps matched their mathematical compasses. Consequently, we question the extent to which the authoritative mathematics discourse, in the two teachers' practices, was further privileged by the perceived preference of "good" students for traditional procedures and attitudes (Lubienski, 2004; Ponte *et al.*, 1994; Wilson & Goldenberg, 1998).

Curricular demands and time constraints

Both teachers reported that time constraints influenced their mathematics teaching. According to Joli, time constraints often propelled her to "fall back on the *ready-to-serve* math program that is in the textbook and teachers' guide." She continued:

We revert back . . . when we are in a rush. When we are trying to finish things up we revert back to that, 'I am going to tell you how to do this, you are not going to figure it out or be part of it, I am going to tell you how to do it.'

Marika and Joli expressed that the mathematics textbook contained more "lessons" than they had time for. Since the

textbook was viewed as "the" curriculum, they felt pressure from the sheer volume of content. As Marika said, "One day per topic, and only one hour a day, it just isn't enough time."

In our view, the perception that the textbook contained more content (or sections) than can be covered in a fifth-grade mathematics program is correct. In fact, it is typical for textbooks to include extra topics to give teachers various ways to address and extend concepts. However, this perception also serves as a justification to avoid mathematically complex and/or time-consuming topics (Ponte *et al.*, 1994). As Marika noted, "we are always feeling that crunch."

Marika and Joli reported that addressing *all* the curricular expectations was challenging, and consequently following mathematical compasses outside of the textbook was seen as "play." The material of value, which would ensure students "can pass the test," was seen as that within the mathematics textbook. Marika said:

Unfortunately, sometimes you just have to say, "this is how we do this," and "this is the way it is," and that goes back to how we were taught, "this is how you do this, you don't have to get it, but we have to move on, and if you do it this way you will get the answer."

The "this" in Marika's comments referred to the mathematics in the textbook. Notably, the textbook did not simply present the traditional textbook style of describing a concept – provide examples, then offer practice questions. It also offered opportunities for problem solving and communicating mathematically in various ways. This discrepancy indicates that the textbook, as an authoritative discourse, at times served this purpose only superficially.

Returning to the question of whether the labels *authoritative* and *internally persuasive* are appropriate for all teacher-textbook discourses, although the textbook appeared to be an authoritative discourse for the teachers, it seems that at times it was a *superficially* authoritative discourse. For reasons noted above, some of which overlap with the reasons for accepting the textbook as an authoritative discourse, the teachers avoided or altered the mathematical direction provided by the textbook. Using the textbook as a cover story, in some cases the teachers transformed what might have otherwise been rich textbook activities into "this is how we do this" and "this is the way it is." For us, this personal interpretation of the textbook does not qualify as internally persuasive discourse. The teachers chose a narrower implementation of the textbook as a way of avoiding, rather than engaging, in a dialectical relationship.

We sensed that even with more classroom time devoted to mathematics both teachers would continue to struggle with: (a) being able to fully implement the directions of the textbook, even if they accepted it as an authoritative discourse, or (b) becoming more autonomous and moving towards internally persuasive mathematical compasses. The teachers expressed a lack of confidence in knowing or deciding how and why concepts in various lessons were linked or sequenced, which lessons might be the best to leave out, and which lessons might be merged into single lessons. In short, both teachers found it a challenge to navigate some of the mathematical directions of the mathematics textbook.

Parental pressures

Similar to findings from other studies (Herbel-Eisenmann *et al.*, 2006; Lubienski, 2004; Peressini, 1998), parental pressure significantly influenced pedagogical directions for Marika and Joli. Some parents in this school's community were reported to question and critique curriculum and curriculum implementation. Since the mathematics textbook was sanctioned and adopted district-wide, Marika and Joli reported feeling a sense of security against potential critiques and concerns of parents. Both teachers felt that parents, like themselves, tended to view mathematical textbooks as faithful interpretations of what ought to be taught.

According to Marika and Joli, following mathematical compasses outside the textbook potentially created additional parental problems. They were concerned that parents would be unable to assist their children in understanding non-traditional mathematics. Some parents, according to them, relied on the mathematics textbook as a means of learning the mathematics themselves to support their children's learning. In one case, Marika reported that she spent the better part of a few hours after school (over a couple of days) reviewing alternative multiplication strategies with parents of a student. They expressed concern that parents would see the alternative mathematical compasses that were not textbook-based as play and an additional stress on limited time resources in the home. As Joli noted, "Parents do not have the time to help their children at home, or play the games with their children."

Marika and Joli taught with the parental "audience" in mind. They were keenly aware of cases in which their practice did not match what they perceived to be the *parents' mathematical compasses*. The textbook thus offered a sense of security. Adherence to the textbook as the primary mathematical compass communicated to parents that, in Marika's words, "we are doing what we are supposed to be doing." Parental pressure further pushed the teachers to adopt the textbook as the authoritative discourse and their primary mathematical compass. However, their perception of parents having more traditional mathematical compasses provided another reason for superficially covering some of the more progressive directions provided by the textbook.

Developing an internally persuasive mathematical "compass"

We did notice some tendency towards changes in the mathematical compass used by Marika and Joli. As Marika noted, "I am certainly spending more time on my math program, thinking things through, and how to better present." Joli expressed similar sentiments:

I found strategies that I never had and that I am kind of excited about. I have always just tried to avoid math, in most contexts, even in basic life skills because I didn't really have a basic strategy.

Research suggests that the extent to which these changes in mathematical compasses might live fruitfully in their future teaching is questionable (Cohen & Ball, 1990; McGowen & Davis, 2001; Norton, McRobbie, & Cooper, 2000). We saw frequently that the pull towards the mathematics textbook,

as authoritative discourse, was compelling for both teachers, often resulting in little dialectical tension with their own internally persuasive discourse or mathematical compasses.

The issue of the textbook as an "authoritative discourse" is complex. Although the teachers deferred to the textbook, as the "authority" of what *should* be taught, they did not fully implement its pedagogical intent. In some instances aspects of the textbook that offered opportunities to investigate mathematical concepts and relationships were by-passed. Joli's and Marika's lack of mathematical confidence may have contributed to this predicament. Time constraints and perceived parental expectations were also factors.

The textbook, which provided a wider and a less traditional compass, could not be seen as *the* authoritative discourse for the teachers. The teachers could have utilized the textbook as an internally persuasive discourse by engaging in a process of "ideological becoming," but for the most part this was not the case during our experience in their classrooms. It seems to us that the teachers *used* the textbook as an authoritative discourse as a *cover story*, as a way of superficially addressing what they perceived to be the expectations of the mandated mathematics curriculum.

The teachers did deviate from the textbook because of their involvement in this research. They also deviated when they turned some of the potentially sophisticated mathematical activities of the textbook into what they referred to as "I am going to tell you how to do this, you are not going to figure it out or be part of it, I am going to tell you how to do it." However, the teachers did not deviate from using the textbook tests as assessment instruments. In this way, Marika and Joli used the textbook tests as a cover story.

There are compelling factors that draw teachers to mathematics textbooks. There are also compelling factors that draw teachers to superficially adopt some of the mathematical compasses of textbooks. Future research may explore how teachers might make sense of range of mathematical compasses *within* a textbook, perhaps generating alternative cover stories.

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Notes

[1] Compass also means: space within limits; area; extent; range; scope (e.g., the narrow compass of the strait; the broad compass of the novel)

[2] Compared to, for example, discourse as exclusively language

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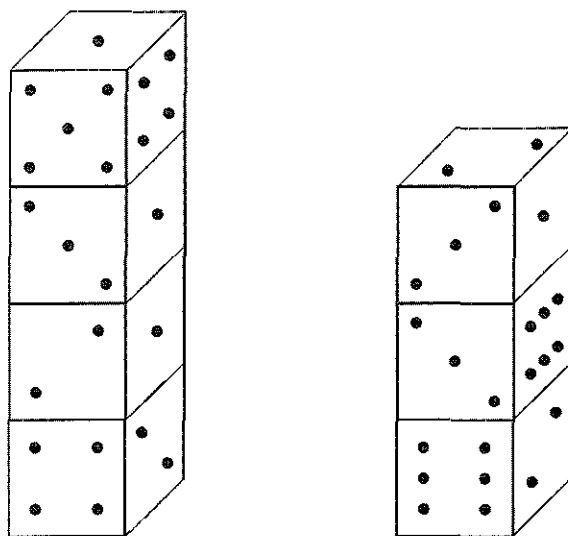
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Hidden Pips

Two stacks of dice are sitting on a wooden table. The dice are all identical standard dice.

Find the total of all the pips on the hidden faces - that is, the faces that cannot be seen from any side of the table. The hidden faces are all those that are horizontal, except for the top face of each stack.



(posed by Alistair Lachlan; selected by Malgorzata Dubiel)