History of Mathematics: Resources for Teachers

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1. Introduction

For someone beginning a search for historical material, it is often quite difficult to know where to start. What are the most likely and easily accessible sources which will give us something we can use? While there is much more being published today in the history of mathematics than even a few years ago, it is still the case that it tends to be a fairly specialist activity, aimed mostly at the higher education level, which make the books expensive.

The purpose of this article is to suggest some of the more easily accessible material that might be useful to teachers. Any such attempt is inevitably selective, and I offer my apologies in advance to those who feel I have neglected their favorite items

The criteria I have tried to use here are that the sources are reliable, cheap, and easily available from bookshops or reference libraries. In addition, many of these sources themselves contain references that can lead the reader who so wishes on to more detailed investigation.

Readers confident in their handling of historical information may like to move straight to section 4, where the bibliographical part of this survey begins.

2. Getting started

Besides knowing where and how to look, it is helpful to preserve for future use records of the information garnered from books, articles, encyclopaedias and other material consulted, noting their relevant contents—and any that might become relevant later—and where they may be found again. Although this seems to slow explorations down initially, it saves a lot of time and effort in the long run!

How one preserves and catalogues this information is a matter of choice. Many readers will have their own favoured system, from jottings on old envelopes to computer notebooks. Card indexes are an old favourite, and browsing through a stationers will reveal other possibilities a system constructed from the leaves of small duplicate books, indexed and arranged in envelopes, is quite feasible.

The categories into which you sort information will depend on the purposes of your investigation, but a standard first indexing would be

(a) Author, title, publishing date and library reference

Then if you were investigating quite a lot of material you might need to keep a track of it by

(b) chronology
(c) historical topic
(d) historical period

Or it might be more useful to arrange your material by the kind of source, for example

(e) encyclopaedia articles
(f) mathematics textbooks containing historical material

Further categories will depend on your specific interests For example, "books for primary schools/children" may be important to you, or you may be interested in a particular topic such as "Vedic mathematics".

Once you start into your area of investigation, it is important to explore the accounts available to you and *compare sources*. You will find different books have different approaches and tell different stories. On the whole, the more recent the book the more likely it is to be informed by good modern scholarship, but you should stay alert to the possibility that an author has uncritically copied from a previous author, or in some other way introduced or perpetuated a myth, misunderstanding or mistake. The value of comparing sources is that it helps you to see where there are inconsistencies, enabling you to be critical and to beware of taking things at face value. In short, do not believe anything automatically! You may not be able to resolve any discrepancies, but at least you know there is disagreement.

You may feel there is little scope for originality in the history of mathematics — that it is either straight facts or rather high-powered stuff — but your originality may come from a different approach or synthesis of ideas that has not occurred before Even when getting material together for pupils, where originality is not the most or even a desirable criterion, making the story you own is important in order to make it come alive. Crudely, in history of mathematics one has to tell a good story according to two main sets or dimensions of criteria: those of historical development, and those of the context of the people at the time.

History has many aspects, and no one approach can give a complete picture. A fascinating thing about mathematical history is the wide range of different things that affect it: biographical details, time periods, questions of education, different civilisations, social and cultural aspects, learned societies, publications and other means of communication are all part of the creation and dissemination of the mathematics whose history we are exploring And behind all this, of course, lie one's own background, philosophical and political leanings, and indeed perception of the nature of mathematics.

The raw material of history is chronology, the simple ordering of events in time. This is useful as a broad framework or indicator, but cannot be taken to tell the full story. The task of history lies not in merely recording facts, but in placing them in a meaningful context. Too many pupils have been decisively turned off a historical perspective by an overzealous

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recounting of boring facts at the expense of context and understanding! Simple-minded interpretation of historical facts can lead to a number of what might be called *cardinal sins;* there is a more extensive discussion of these problems in Ken May's 1973 *Bibliography and research manual* mentioned below, but it is worth mentioning some pitfalls here.

3. Historiographical pitfalls

3.1 Priority claims are usually meaningless

Nothing happens "out of the blue"; if someone discovers, invents or proves something it can be difficult to decide either what precisely the something is, or what discovering, inventing or proving it really means.

I am reminded of the Tom Lehrer song about Lobachevsky, who "published first"—but Gauss was exploring non-Euclidean geometry in his notebooks before Bolyai added *The science of absolute space* as an appendix to his father's textbook. And some claim the Arabs were there before any of them.... We may be able to identify distinguishing features or particular steps on the way to a discover, but nowhere does an idea spring fully fledged into existence.

3.2 We cannot judge past events by modern standards

We cannot generally say categorically who was "right" and who was "wrong", nor is it fair to claim that a particular mathematician had an idea which is now attributed to contemporary mathematics. While it is important to see how modern ideas may have originated, we cannot attribute these same ideas to past mathematicians

For example, Archimedes developed a method of finding by numerical approximation areas and volumes of shapes bounded by curves; variations of this process later became called "quadrature"; Leibniz used the long "s" to indicate a summation of infinitesimals—but not until Bolzano and Cauchy do we find the technique approaching what we today call "integration".

Again, the same technical term can mean different things at different times. "Quadratic equations" are not what the Babylonians were solving, although that's what we would use to solve similar problems. And the word "functions" has had many interpretations before arriving at the convention we use—even today, different books have different definitions and explanations of these ideas.

3.3 Speculations are not facts

Deduction or plausible inference from established facts can sometimes be confused with what really happened. Can we ever know what really happened? The confusion in a courtroom over people's recent memories and observations of the simplest events indicate just how hard it is establishing the truth.

The most blatant speculation-disguised-as-fact occurs with events very far back in the past. An obvious example is in the mythical accounts of the history of number, where early man is said to have invented counting because he needed to know how many spears or how many enemies he had, and the like.

4. Sources for general overviews

An overall perception of certain historical themes is vital to the background of any mathematics teacher. Such themes include getting a feel for the development of mathematical ideas, the practice of mathematics, the relativity of rigour, and the assumptions and arguments involved in different places at different times. Two very readable books which do this admirably are

Philip J. Davis and Reuben Hersh, *The mathematical experience,* Penguin 1988

Philip J. Davis and Reuben Hersh, *Descartes' dream,* Penguin 1988

The second volume is more philosophical, showing how the modern world is permeated with mathematics, and in particular with many of the mathematical attitudes that sprang up in the 17th century.

4.1 Encyclopaedias and dictionaries

Sometimes the obvious sources are missed. Up to date editions of good encyclopaedias and other reference works can provide much useful information.

Encyclopaedia Britannica
McGraw-Hill encyclopaedia of science and technology
Chambers encyclopaedia
The dictionary of national biography
The dictionary of scientific biography

Most of these should be available in a central public library, or a local educational institution. There are some other less expensive but very useful items in a dictionary format, all with quite a lot of historical information.

J. Dainteth and R.D. Nelson, *The Penguin dictionary of mathematics,* Penguin 1989

David Wells, *The Penguin dictionary of curious and interesting numbers,* Penguin 1986

E.J. Borowski and J.M. Borwein, *Collins reference dictionary of mathematics,* Collins 1989

H.E. Eiss, *Dictionary of mathematical games, puzzles and amusements,* Greenwood Press 1988

4.2 Chronological tables and charts

There are some chronological tables in the history of mathematics textbooks by Boyer and Eves (cited below), which are useful as a start but necessarily selective

A standard reference work is Steinberg's historical tables:

Steinberg/Paxton, *Historical tables,* Garland 1986

This contains a wide range of detailed information about world events, but, alarmingly, hardly any scientific events are mentioned until the industrial revolution, and even fewer mathematical events. This only reinforces the belief that if scientists know no history, historians certainly know nothing of science!

Wallcharts come and go, and it can be very difficult to get hold of attractive and useful charts. One which has survived for many years and is still available is the IBM publication *Men of mathematics.* Even today, the women are invisible, so a good exercise is to encourage students to fill in the gaps.

5. More detailed works

5.1 Bibliographies

These provide information about books and articles on specific topics. None will give a complete list of everything available, and many are limited by the choice of their compiler to periods or topics, or by the date when they were published. One that is of interest and easily available is

John Fauvel, *Mathematics through history: a resource guide*, QED Books 1990

This annotated catalogue of books in print is a most valuable inexpensive guide to much more material. Some larger books cover material whether in or out of print, and are well-worth consulting in a reference library

J.W. Dauben, *The history of mathematics from antiquity to the present: a selective bibliography*, Garland Press 1985

Kenneth O. May, *Bibliography and research manual of the history of mathematics*, University of Toronto Press 1973

Some recent titles of special interest provide much fuller information than the strictly bibliographical.

Louis S. Grinstein and Paul J. Campbell, *Women of mathematics: a biobibliographic sourcebook*, Greenwood Press 1987

M.B. Ogilvie, *Women in science, antiquity through the nineteenth century: a biographical dictionary with annotated bibliography*, MIT Press 1986

Ivan van Sertima, *Blacks in science, ancient and modern*, Transaction Books 1983

Several journals contain quite a number of articles on the history of mathematics, but knowing and accessing what has been published can be difficult. Few journal indexes sort out the articles of historical interest, and of course some authors include historical material in a more general article whose title may not suggest such inclusion. A series of bibliographies in the American journal *School Science and Mathematics* are highly recommended.

C.B. Read, "Periodical articles dealing with the history of mathematics", *School Science and Mathematics* 70 (May 1970), 415-435

C.B. Read, and J.K. Bidwell, "Selected articles dealing with the history of advanced mathematics", *School Science and Mathematics* 76 (Nov, Dec 1976), 581-598, 688-703

Even though some of the articles indexed in these bibliographies are somewhat dated now, they generally consist of short, easily accessible secondary source material on a wide variety of topics.

The journals covered in these bibliographies are *American Mathematical Monthly, The Arithmetic Teacher, Mathematical Gazette, The Mathematics Teacher, The Pentagon, School Science and Mathematics, Scientific American, Scripta Mathematica*, and *Mathematics Magazine* (which arose from two earlier journals, *Mathematics Newsletter* and *National Mathematics Magazine*). Even if your library does not take these journals, it may be possible to order through the inter-library loans service a photocopy of an article that you need.

5.2 Source books

These are collections of original works, translated into English, often with explanatory notes or commentary. Some of the most useful are

John Fauvel and Jeremy Gray, *The history of mathematics: a reader*, Macmillan 1987

John Fauvel (editor), *History in the mathematics classroom: the IREM papers*, Mathematical Association 1990

J.R. Newman, *The world of mathematics*, 4 volumes, Allen and Unwin 1960 (recently reprinted)

5.3 General histories of mathematics

All general histories of mathematics have to be selective. The best are quite clear about what they leave out, others may be biased unwittingly, or through the constraints of style, cost, or intended audience. Some of the large general histories published in English tend to omit chunks of non-European material, are sexist, and tell a story of mathematics as technical progress with little reference to its cultural contexts. An example of such an account, easy to read and popular but limited in scope and obviously Eurocentric, is

Morris Kline, *Mathematics in western culture*, Penguin 1972 (and many reprints)

To redress the balance, the following work is an important addition to the list of essential books.

George Ghevargese Joseph, *The crest of the peacock: the non-European roots of mathematics*, I.B. Tauris 1991 (a Penguin paperback is expected shortly)

Other "standard" history books are informative, yet should be used guardedly for they suffer in various ways from the problems mentioned above.

C.B. Boyer and Uta C. Merzbach, *A history of mathematics*, second edn, Wiley 1989

Howard Eves, *An introduction to the history of mathematics*, Saunders College Publishing 1983

Morris Kline, *Mathematical thought from ancient to modern times*, Oxford University Press 1972

The following is an excellent picture book for children from about eight or nine years old. It contains quite a good range of material, and should still be available in or through public libraries

Lancelot Hogben, *Mathematics in the making*, Macdonald 1960

6. Mathematics in its wider contexts

6.1 Evolution of mathematical ideas, and cultural history

Some modern historical work increasingly pays attention to the cultural and social settings in which mathematics develops. Some notable starting points are

Imre Lakatos, *Proofs and refutations*, Cambridge University Press 1976

R.L. Wilder, *Evolution of mathematical concepts,* Transworld 1974
R.L. Wilder, *Mathematics as a cultural system,* Pergamon 1981
Alan Bishop, *Mathematical enculturation,* Reidel 1989, pb Kluwer 1991

The latter gives a much wider setting to the problems of mathematics education in a social and cultural context. In hardback it is quite expensive, unfortunately, but worth searching for in libraries for its provocative ideas and extensive bibliography.

Recently the term "ethnomathematics" has been invented to cover the modern sub-cultural as well as the historical aspects of indigenous mathematics. Work in this area is mostly available in journal articles at present, especially in previous issues of *For the learning of mathematics* and, for example, in Claudia Zaslavsky's article in the current issue.

A book which exemplifies new approaches to relevant social and cultural material in the context of a particular mathematician is

John Fauvel et. al , *Let Newton be!,* Oxford University Press 1988

This is the unfolding of the fascinating background to the scientific work of Britain's greatest mathematician, with chapters on his religious beliefs, his work as an alchemist, and his views about matter, magic, and the wisdom of the ancients.

6.2 Open University materials

The former OU course on *The history of mathematics,* AM289, had a number of useful general units, and some special units on the history of the calculus and the evolution of numbers, counting practices and calculation. These are still work searching for, if a local library has them.

The new course MA290, *Topics in the history of mathematics,* has a similar collection of excellent materials, and a more sophisticated and challenging historiographical approach. It is possible to use both sets of materials without access to the videotapes, which are expensive and may pose problems of compatibility.

7. Into the classroom

7.1 Special topics

For older children or teachers in training, a wide range of material is available on all sorts of subjects. Sadly there is very little in print accessible to children of primary age, however, and teachers of children up to about fourteen will often have to adapt material they find in other books.

Some local education authorities or school districts have developed multicultural material for use with younger children which uses aspects of the history of mathematics. Much of this material also covers aspects of equality and social justice. Local teacher resource centres are the best places to go for this sort of detail, but two publishers are active in the field: Jonathan Press (England), and Creative Publications (USA). Both these are developing their lists, and have some useful and interesting material. They will also know of other publishers of primary material with similar interests.

The history of aspects of number, old number and numeral systems and the like, is a very suitable topic for younger children. The classic text on numbers, notation, and calculation is

Karl Menninger, *Number words and number symbols: a cultural history of numbers,* MIT Press 1969

This is a translation of an earlier German edition. Three more recent books are

Graham Flegg, *Numbers through the ages,* Macmillan 1989
Thomas Crump, *An anthropology of number,* Cambridge University Press 1990
J.M. Crossley, *The emergence of number,* World Scientific 1987

Flegg's book is an accessible and interesting text, using quite a lot of the previous Open University history of mathematics course. The Crump collects information on many aspects of number, containing many gems which are not to be found in Menninger. Crossley's book is probably the best recent account of the origins and development of integers, fractions, non-rational and complex numbers.

Interestingly, it doesn't seem that anyone has produced a history of geometry which is accessible to non-specialists in the way that Menninger and Flegg are. But a useful book which covers many aspects of elementary arithmetic, calculation and geometry is

Lucas N.H. Bunt, Phillip S. Jones and Jack D. Bedient, *The historical roots of elementary mathematics,* Dover 1988

7.2 Classroom topics

If you are ever caught with questions like "why do we call them sin, cos, and tan?" or "who invented matrices?", then a good book which is a mine of information on particular topics in the standard school syllabus is

A. Hallerberg (ed), *Historical topics for the mathematics classroom,* NCTM 31st yearbook, Washington 1969

The new 1989 paperback edition has been improved and updated.

8. The best buys

Finally, this is where I stick my neck out by suggesting the five most useful books, bearing in mind the criteria I stated at the beginning of this article. This is no doubt, where there is most ground for disagreement with my choice, but here goes. They are all paperbacks and in English, recent, reliable, and most likely to be easily accessible either through bookshops or libraries.

Philip J. Davis and Reuben Hersh, *The mathematical experience,* Penguin 1988
John Fauvel, *Mathematics through history: a resource guide,* QED Books 1990
John Fauvel and Jeremy Gray, *The history of mathematics: a reader,* Macmillan 1987