Communications

Who/what is the more knowledgeable other?

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A comment on "Counting in threes: Lila's amazing discovery", from FLM 34(1). In their article, Graven and Lerman (2014) used Vygotsky's notion of the zone of proximal development (ZPD) to explain Lila's interaction with her mother, Mellony, as Lila used the television remote control to count in threes. Referring to Goos, Galbraith and Renshaw's (2002) bi-directional and collaborative conceptualization of the ZPD, Graven and Lerman noted:

> Initially, Lila is the activator of the emergence of the ZPD. In this respect, and given that she is holding the artefact which mediated her discovery, Lila is the more knowledgeable other of the discovery and her mother is the learner connecting the relationship between the structuring of the numbered buttons on the artefact and Lila's counting in threes. Once Lila has shown and explained her discovery Mellony becomes the more knowledgeable other, affirming Lila's discovery and confirming the correctness of naming the arrangement. (p. 30)

Whilst I support the collaborative conceptualization of the ZPD, its bi-directional view, as employed by Graven and Lerman, raised questions for me. In their analysis, Graven and Lerman employed this bi-directional conceptualization of the ZPD to explain Lila and Mellony's interactions, in which the role of being the more knowledgeable other is alternating between the mother and the child. Later in the article, they suggest that Lila's attention was caught by the layout of the remote, which initiated the emergence of the ZPD. The ZPD was then "*sustained* as Lila rushed to her mother to show her what she had found" (p. 30). Throughout this writing, a series of questions lead my thinking. To begin, my questions are:

- What was the role of the artefact (*i.e.,* the remote control) in the initial emergence of Lila's ZPD and, all along, in the interactions between Lila and Mellony?
- If the ZPD emerged when Lila's attention was caught by the remote, considering the bi-directional conceptualization of the ZPD, who/what was the more knowledgeable other?
- If the ZPD "is sustained" (Graven & Lerman, 2014, p. 30) as Lila rushed to her mother, then is it possible that the ZPD is not bi-directional and is rather multi-directional?

One of Vygotsky's assumptions in developing his theory is that the tie between tools and signs creates artefacts. A remote control sitting on a coffee table is a tool; it is a means of labour. It can be used to change the TV channels, can be a toy car, or can perhaps be used to count in threes. The remote becomes an artefact when signs are attached to it. Examples of signs include language (verbal and non-verbal), numbers, and symbols. The signs, however, are tied to a tool (*i.e.,* a tool becomes an artefact) based on how the tool is perceived by the person who is using it: by Lila in this case. Now I have more questions.

When did the remote become an artefact for Lila? When Lila first used the remote control to get to her favourite channel, she perceived the remote control as a "channel-changer" tool. Later, by noticing the layout of the numbers, Lila developed a new perception of the tool, as she explained: "Mommy, I've worked out how to count in threes!" (Graven & Lerman, 2014, p. 29). Lila created an artefact for herself by tying signs to the tools; pointing to the buttons on the remote, she said: "look Mommy, It's 3, 6, 9 [...] Cause everyone is 3" (p. 29). The remote control was an artefact for Lila and its "artefact-ness" was related to both Lila's perception of the tool and the physical structure of the tool. Consequently, there was a system of relationships among the layout of numbers of the remote, the task of counting in threes, and the knowledge Lila was gaining by using the remote. I will focus on the knowledge that Lila was gaining as she interacted with the artefact, paying special attention to the emergence of the ZPD from this interaction.

To conceptualize the ZPD and its related aspects (*i.e.,* knowing and the "more knowledgeable other"), I use Radford and Roth's (2010) approach. Radford and Roth's conceptualisation of the ZPD rests on a "non-individualistic conception of the participants" (p. 301), in which the question of the more knowledgeable other emerges from the interaction, collaboratively but not necessarily bi-directionally. Within the ZPD, Radford and Roth conceptualized knowing as "the possibilities that become available to the participants for thinking, reflecting, arguing, and acting in a certain historically contingent cultural practice" (p. 301). They further argued that the emergence of these possibilities is made possible by "language [and] forms of perception" (p. 302). Lila's perception of the physical structure of the remote control (that is, the layout of the numbers in relation to counting in threes) created a possibility for her to think, reflect and act. Lila was participating in an interaction with the remote control.

How would one say if the ZPD has emerged in this interaction? Radford and Roth argued that one of the most crucial aspects of the ZPD is "the emergence of a new form of collective consciousness, something that cannot be achieved if we act in solitary fashion" (p. 306). Lila's knowing (her possible thinking, reflecting, and acting) could not have been achieved individually. It was in her interaction with the layout of the remote that she claimed to be able to count in threes. The subsequent references of both Lila and Mellony to the remote also confirm this collaborative achievement. Examples of these references include:

- Lila's utterances such as " Because everyone you count in one threes" (p. 29) or "Three, six, nine, twelve" (p. 29) while holding her fingers over each row of three buttons (p. 29).
- Mellony's utterances such as: "Have a look on the buttons" (p. 29) or "Can we pretend that they are three" (p. 29).

I therefore argue that a ZPD emerged in Lila's interaction with the artefact, as she perceived it.

In this collaborative ZPD, who/what was the more knowledgeable other? To systematically think about this question, I use and possibly extend Radford and Roth's (2010) view of ZPD. Radford and Roth used the notion of language and other semiotic resources to explain how participants position themselves in zones of proximal development and "to tune to others in conceptual and affective layers to collectively reach interactional achievement" (p. 307). As mentioned above, I argue for the emergence of the ZPD as Lila interacted with the artefact. Participants in the ZPD are Lila (with her own perception of the remote) and the physical layout of the remote. I argue that there is a "collectively reached interactional achievement" (p. 303). Lila's gestures and utterances such as: "Three, six, nine, twelve" (p. 29) as she held "three fingers over each of the number sets 123; 456; 789, then over the three buttons below 789 as she calls out 3; 6; 9; 12" (p. 29) show that Lila's achievement in counting in threes was reached through her interaction with the artefact. Yet there is also a question that might be an extension to Radford's conceptualisation of the ZPD.

What are the resources (semiotic and possibly not semiotic) that Lila used to position herself in the ZPD that has emerged from her interactions with the remote? This question leads me to more questions, with which I end this writing:

- Is it possible that the more knowledgeable other in Lila's interaction with the remote is the remote control itself?
- If the remote is the more knowledgeable other, what is the *language* as Lila interacts with the artefact?
- Where is the language of the artefact? Is it in the social origin of its design? Is it in the perception of Lila holding the remote? Is it in both?
- If the ZPD could emerge from Lila's interaction with the remote control, then could the "language and other semiotic resources" (Radford & Roth, 2010, p. 307) be extended to include resources related to physical structuring of the artefact? (For example, had the remote control been organised in rows of four, would Lila be counting in fours?)

Further explorations and research may shed some light on the role of physical properties of the tools as *resources* in the emergence of the ZPD.

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Unthought knowns

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With their communication in the hundredth issue of FLM, Maria Bartolini Bussi and her co-authors (2014) strongly caught my attention. There are three things I would like to engage with:

- regularities without complete uniformity in the manner of saying and writing fractions (the latter both in words and in symbols);
- the nature of connections between fraction words and ordinal words in a language (including their question: "Why is the denominator expressed in ordinal numbers?");
- languages as variable preservers of traces of mathematical thought.

With fractions written by hand, the composite symbol is produced in a given, temporal order. How might that gestural order relate either to what is said or to how what is said is conventionally written? In English, the first word spoken in time is the numerator: is this so for any language? When a fraction word is written down in English, left to right, the numerator is again the first word to be written. (Ditto the question about other languages.) But when the composite symbol for the fraction is produced, there are variations possible, as their terrific vignettes from China and Burma attest. But both examples point to the *arbitrary* nature of manual symbol formation (in Hewitt's, 1999, use of that word) and to the fact that, once made, the symbol retains (almost) no trace of its making.

I thought there was something slightly awry when the authors write of "writing fractions in Burmese" (p. 31) or claim "All European languages now share the top-down process of fractions and the consequent naming order" (p. 32), as these practices have to do with the mathematical writing system rather the orthography of specific languages (see Pimm, 1987, Chapter 6): "3/7" is not part of English, whereas "three-sevenths" (or is it "three sevenths"?) is. If the symbol 3/7 were to be produced in a manner that actually reflected the spoken word-order in English, it would be numerator (three) → denominator (seven) → fraction line (-ths), whereas the practice in the vignettes would be seven -ths three. The construction and development of fraction number words in any language has a history but a limited number of options, options that have a geographic spread around the world.

Which leads me to my next comment related to possible reasons for ordinal words being close to (or identical to, as in English) fraction words in a given language. The only place I have come across a history of mathematics saying anything remotely about this is in van der Waerden's (1954) *Science Awakening*. Speaking of Ancient Egyptian fractions, he observes:

> Worthy of notice is the verbal expression for $^2/_3$ which means literally "the two parts". The complement, necessary to make a whole out of the two parts is "the third part".
>
> In Greek one also speaks of
>
> "The two parts" $^2/_3$ —"the third part" $^1/_3$
> "The three parts" $^3/_4$ —"the fourth part" $^1/_4$.
>
> It presents quite naturally a concrete image: three parts and then a fourth part combine to make the whole. Analogously we can explain our use of the words third, fourth, fifth, etc. In this representation the fifth part is the last part, which combines with the four other parts to complete the unit. Philologically it does not make sense to speak of two fifths, because there is only *one* fifth part, viz. the last. (pp. 19-20, italics in original)

Traces of this idea can be seen in the Ancient Egyptian notation for what are frequently if misleadingly called "unit fractions" (whose very term suggests that this was but one type of fraction, rather than the only other numbers that existed apart from whole numbers). Chace (1928/1979) uses the term "reciprocal numbers" (denoted symbolically as the related whole number with an oval symbol above it—fascinating to wonder whether the scribe wrote the number first and then added "reciprocal" or conversely).

Ancient Egyptian arithmetic relied upon the scribes' ability to double and halve any number, and therefore the so-called "2 divided by *n*" table on the Rhind Papyrus is actually the double reciprocal table (which individually solves each problem "What is the double of reciprocal *n*", for *n* odd between 3 and 101) by listing useful equivalences: for example, double reciprocal 35 is (among other things) reciprocal 30 (plus) reciprocal 42.

For us, nowadays, $a \div b = a/b$ can be seen as either a notational equivalence or a general solution to the problem of whole-number division (provided a/b is taken as an object, an answer, a number). Topologist William Thurston:

> I remember as a child, in fifth grade, coming to the amazing (to me) realization that the answer to 134 divided by 29 is 134/29 (and so forth). What a tremendous labor-saving device! To me, "134 divided by 29" meant a certain tedious chore, while 134/29 was an object with no implicit work. I went excitedly to my father to explain my major discovery. He told me that of course this is so, a/b and a divided by b are just synonyms. To him it was just a small variation in notation. (1990, p. 847)

Lastly, I wish to mention the way in which languages (writ large to include gestural practices) can act as preservers but also rigidifiers of thoughts-and-gestures. Language is a shared experience, but not just with those in our immediate present. In Bartolini Bussi *et al.'s* communication, I was delighted to come across François Jullien's notion of "unthought", which they use as "the arbitrary that one is not aware is arbitrary". I remembered the "unthought knowns" that psychoanalyst Christopher Bollas discusses in his book *The Shadow of the Object.* The final sentence of Bollas's book is: "In thinking the unthought known we ponder not simply the kernel of our true self, but elements of our forebears" (p. 283). It got me wondering about the cultural shadow of the fraction object that falls upon our mathematical egos, through its traces in language, conventional notation and culturally encoded gestures, reaching back over five thousand years.

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Nomadic ethics and regimes of truth

PETER APPELBAUM

I was delighted to see Margaret Walshaw take on the serious integration of Foucauldian concepts into mathematics education in FLM 34(2); she is someone well-positioned to do this, with her academic background and theoretical experience. Margaret summarizes her goals as follows:

> At the core of the discussion lies an interest in reconfiguring learning as a political and moral project—an interest that is couched within a disavowal of the possibility of utopian transformation. (p. 2)

I admire this as a fundamental standpoint for mathematics education; it is both refreshing and critical. Yet, more recent theories provide solutions to what has been thought of as a lack of agency within Foucault; ethics might replace moral dilemmas in mathematics education. At the very opening of the article, I fear an important point has been overlooked; she begins:

> Mathematics, it is commonly agreed, should be accessible to all. Democratic societies subscribe to the view that, for individual and societal progress, *all* students should have right of access to mathematical knowledge

> and that teachers should develop students' capacity to learn mathematics. (p. 2)

This rights discourse presumes many of the assumptions that Foucault and others are criticizing: it is predicated upon individual subjects who have rights, own them, should have access to them, *etc*. It creates an economy of rights within an economy of power. This is inconsistent with Foucault's understanding of power, typically characterized as a flow, or physics of power in a collection of social networks (Appelbaum, 1995). Power, for Foucault, is a property of relations, not a reified substance that can be bought, sold, traded, owned, or refused. Individuating the learners and the teachers, making them into selves that are rational, governable, acting or passive, but always subject to the wielding of power by others, as in a rights discourse, may feel "right" at some fundamental level, amazingly apt in describing our social reality. Why criticize rights when so many do not have them? However, that social reality is already informed by our regimes of truth, and these regimes of truth structure possible forms of interaction, social change, and the language used to think about them. In other words, the idea that rights are important is an example of ideology and power/knowledge relationships in the first place and, indeed, is an example of how power is a property of relations rather than a thing that moves from person to person or from groups to groups.

This is not to dismiss the reality that some people live lives of abuse, mistreatment, and so on. It is not to ignore the plight of those who have few opportunities. Nevertheless, the "reality" of an individuated rights interpretation of our life world is a regime of truth. Reframing this "reality" with a contrasting regime of truth is what Foucault actually intended with his work. He saw eras of epistemologies through history, significant shifts that led to this regime of truth that takes selves for granted, that places the individual self at the heart of one's life, so that the care of this self now becomes the life project of a human being. But this is only one way of understanding life, not a fundamental, biologically-determined "life." In fact, I would say, the self-based discourse and worldview is what makes it possible for people and groups of people to create worlds in which other "individuals" can be manipulated, used, abused, ignored, mistreated, overlooked, unrecognized, and so on. Those "other selves" are distinct objects when one lives in a world of individuated selves. It is necessary to bring about a new regime of truth not predicated on individuation in these ways.

Margaret writes:

> If an identity as a mathematical learner is relational and is able to tell us about the nature of an equitable mathematical experience, then it needs to be expressed as something dynamic rather than as a static process or as a property of people. (p. 3)

I ask, what is identity, dynamic or otherwise, if not a symptom of a regime of truth that has constituted individuated selves that are always striving to reify identity?

A problem with grounding mathematics education theory and practice in Foucault without a critical perspective on the self-based regimes of truth is the re-inscription of selves as mutually reinforcing regimes of truth, rather than the reconceptualization of mathematics education independent of those regimes of truth. The result is mathematics education practice that reinforces those regimes of truth, rather than working for social justice and social change. Foucault deconstructed the self as something that perpetuates the need to care for the self, another regime of truth that structures the processes of, in turn, structuring structures, as indicated by relations of power/knowledge. He was not working for selves to have rights—that project leaves us trapped in selves, in a network of power relations that maintain our selves and the belief in the self as a real thing that can then have characteristics such as rights. Again, this is not to deny the good intentions of those who care about human rights. Instead, it is to point out that rights and other attributes of individuals have their good and bad points, as indicated by the relations of power that can be observed. For example, rights can be taken away from individuals as much as given to them; this is an indicator of the power that is a property of the relationships.

The dilemma, for those who want autonomy and freedom, is that this autonomy and freedom are the outputs of social reproduction systems that create the idea of identity, autonomy, freedom, rights, and so on, in order to govern and manipulate those individuals via their own self-perception of autonomy and freedom. This was one point of Walkerdine's (1988) *Mastery of Reason.* A child-centered pedagogy and its baggage of liberating the individual, reasoning subject actually constructs each child as a "subject." This "subject," where the word starts out as meaning something like "an active agent", turns into a grown person who is "subject" to the power properties of a governing state that uses "reasoning" to manipulate and control the now-understood "subject" in its rational thinking and behavior. Walkerdine was applying Foucault to the study of post-war, progressive education. Walshaw makes a similar point with the ways that students' participation in a classroom discourse may appear as creative contributions. Over time, however, and in a collectively powerful way, their contributions might instead be said to implicitly make clear to students, teachers, and all "educated people" that they are *not* agents of social change:

> As Radford notes, students' ideas are invited yet they can never be autonomous since they are "unavoidably engulfed in discourses and epistemes (*i.e.,* systems of thinking) that are not [the students'] own." (p. 3)

Students are being enculturated/acculturated into the regimes of truth. Walshaw makes this connection in the following paragraph:

> Knowledge about school mathematics is an *effect* of particular rules of formation. While these rules are often unknown to the actors involved, they circumscribe the possibility of thought concerning what exactly school mathematics is. (p. 3)

These ideas affirm the work of others in mathematics education who have also written about power (Ernest 2000; FitzSimons, 2002; Klein, 2007; Skovsmose, 1994, 2012; Gutiérrez, 2013; Valero & Zevenbergen, 2004), and those who take a philosophical or cultural rather than a psychological approach to contextualizing such issues (Swanson, 2005; Brown, 2011; Gerofsky, 2012).

However "regimes of truth" do not help us to reconfigure learning, but instead describe a social situation; they leave us in a quagmire with no agency to effect change (Appelbaum, 1995). I point instead to other theorists who have in the last several decades responded to Foucault, disagreeing with him during his lifetime and after, or offering nuanced critiques of his work. For example, de Certeau's (1984) concept of *la perruque,* and his theories of strategies and tactics, focused directly on the ways that agents might alter power relationships. Rancière (2011) provides, for me, a contrast to Foucault, a way out of the crises of guide-on-the-side thinking: in this work, the "schoolmaster" adheres to a radical equality, remaining ignorant of any knowledge of the students' ignorance; it is only by remaining true to this "ignorance" that the properties of the relationship, including power/knowledge, can be radically equal. Otherwise, potential learners, change agents, *i.e.,* active participants in society, are constructed as "needing to be taught" by those "with" knowledge. Such radical equality is, Rancière might say, fundamental, to avoid "education" and "telling," turning change agents into passive receivers of entertainment.

Inequities, inequality, and so on, are discourses of selves. They return us to the regime of truth, maintaining and reconstructing them. I imagined Deleuze (1992; Appelbaum, 2008) speaking to this; instead of toppling regimes of truth, we can live with them, coexist with them, but, at the same time, be independent of them. The strategy is analogous to the nomad, who is both homeless yet travels with her home, and neither homeless nor with a permanent home. Nomadic concepts in general defy dichotomies. It is possible to enact curriculum in "ordinary contexts" and in meaningful ways using nomadic epistemology. This very coexistence may seem to reinforce its invisibility, which has its own side-effects of feeling like you are not making enough of a difference. The critical feature of the "nomad," however, is how aggressively creative she is in this coexistence, always outsmarting the behemoths around which the fields of power coalesce, such as "the state," which is more of a passive consolidator. The state thrives by capturing nomadic innovations and transforming them with de Certeau-like strategies to fit its own needs, to consolidate a particular constellation of relationships as seemingly stable and unchanging. At the same time, the nomadic concepts and nomadic agents respond with de Certeau-like tactics, ever-newly invented ways of being in and out of the consolidation at the same time, which the state again is consolidating, to which the nomad more aggressively invents circumventions that need to be absorbed and adapted to the consolidating processes that in turn open up amplified forms of nomadic action.

Reconfiguring "learning as a political and moral project" (Walshaw, 2014, p. 2) re-inscribes the regime of truth, since morals are individual, internal principles or habits that govern conduct. Ethics is an alternative discourse to rights and equity, in which teaching can be an ethical stance one takes upon the world. When we see teaching this way, we can proceed independently of individuating relations; building on the emerging literature of ethics in our field (Atweh & Brady, 2009; Cotton, 2012; Stemhagen, 2008), ethics could be a nomadic epistemology of mathematics education. By coexisting with self-based regimes of truth, ethics can offer tactics of social change.

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