OUR LIVES AS PERFORMANCE MATHEMATICIANS

GEORGE GADANIDIS, MARCELO BORBA

We are neither mathematicians nor performance artists. On the other hand, we are performance mathematicians – and so are you. We want to blur the boundaries of what we are or can be – and what you are or can be – so we can see our work in mathematics education through a new lens, that of mathematical performance. In doing so, we will explore the questions:

- Are we mathematicians?
- Are we performance mathematicians?
- What does performance help us see and understand about mathematics education?

We first take a look at what it might mean to use the labels of *performance* and *mathematician* in the context of mathematics education. Then we analyze a digital mathematical performance through Boorstin's (1990) performance categories of voyeuristic, vicarious and visceral. Our goal in this paper is exploratory. We want to start conceptualizing what might be seen when we look at mathematics education though the lens of performance, and what difference this lens might make.

Are we mathematicians?

Are we mathematicians? Our research is on mathematics education, and not on mathematics. We do have undergraduate degrees in mathematics, but by typical scholarly standards we would not be considered to be mathematicians. However, the thinking done in mathematics teaching situations is mathematical and not strictly pedagogical (Ball, 2003; Ball & Bass, 2002; Ball, Bass, Sleep & Thames, 2005; Davis & Simmt, 2006). Classroom situations create mathematical problems that teachers (and students) need to solve. These problems are not the same as the ones typically tackled by research mathematicians, but they are nonetheless mathematical problems. Perhaps we might qualify as *teacher* mathematicians.

What about the students we teach? Are they mathematicians? Papert (1980) suggests young children enter school mathematically curious, imaginative and capable, and that they have to learn to be otherwise. Ginsburg (2002) notes that, although mathematics is "big," children's minds are bigger. He argues that "children possess greater competence and interest in mathematics than we ordinarily recognize" and we should aim to develop a curriculum for them in which they are challenged to understand big mathematical ideas and have opportunities to "achieve the fulfilment and enjoyment of their intellectual interest" (p. 7). Perhaps we are not made into mathematicians by the graduate degrees we hold. Perhaps all humans are natural mathematicians. Perhaps we can call students young mathematicians. In fact, such a label is not uncommon in the mathematics literature (*e.g.*, Fosnot & Dolk, 2001a, 2001b; Fuis & Huinker, 2000; Sharp & Hoiberg, 2001).

The once-mathematician Ubi D'Ambrosio (2006), now a mathematics educator, brought the notion of ethnomathematics as a means of legitimating the mathematical expressions of different cultural groups. In this perspective, kids from a slum, workers or chemists can produce valuable mathematics. For Borba (1990) mathematics developed by mathematicians is just one form among others that can be produced by different cultural groups, even though he recognizes that such mathematics plays a differentiated role in society. In this perspective we are all *ethno*mathematicians – or for short, we are all mathematicians.

There is an advantage to applying the label of 'mathematician' to all humans, as mathematics then is seen as a human endeavour rather than one for the elite few. In terms of a cultural group, we could consider ourselves 'mathematics educators.' In this sense we can have this unique experience of talking to academic mathematicians and to many who hate mathematics and who are very surprised when we claim that they are doing mathematics.

Are we performance mathematicians?

Are we performance artists? There are similarities between art and mathematics. Dissannayake (1992) pushes the boundaries of what is art well beyond the cold walls of a museum or an art gallery. She suggests that art is normal, natural and necessary for humans and that we are biologically predisposed to art, to a desire to *make special.* Higginson (2004), responding to Dissannayake, suggests that there are many similarities between art and mathematics both in terms of

> primordial predispositions, actions and sensitivities – pattern, rhythm, symmetry, coherence, fit, balance [and in] the higher-order similarity in the parallels between the 'modern fates' of art and mathematics. Both have become alien sets of artifacts – rarified, commodified, ultra-abstract, elitist entities; the domain of specialists, things costly, hard-edged and puzzling. (p. 80)

If art is normal, natural and necessary, then perhaps mathematics art (or performance) is also normal, natural and necessary. Rodd (2003) makes the connection between performance art and mathematics teaching by looking at

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university mathematics lectures through a theatre arts lens. She suggests that a good mathematics lecturer shares some of the qualities of a good theatre performer when she or he "draws new people to his or her world" (p. 18).

Neither of us has a formal education in the performing arts. Gadanidis has written and produced songs, music videos and other performances about mathematics, which are used as starting points for mathematics explorations for teachers and for students. He has also participated in these mathematical performances. In his university days, Borba was involved in political theatre as a form of struggle against the dictatorship in Brazil in the late 1970s. Later, as an educator, his interests in constructivism, active learning, group work, collaboration and other ideas resonated with the ones sponsored by political theatre activists such as Boal (1985) regarding the blurring of actors and spect(actors). Both authors are intrigued by Boal's political theatre and the agency afforded to spectators, and they are collaborating to develop digital performances that help disrupt traditional assumptions in mathematics education. These assumptions include:

- mathematics is a cold science – rather than an human, aesthetic experience (Sinclair, Pimm & Higginson, 2006; Sinclair, 2001);
- mathematics is about learning procedures for getting correct answers – rather than attending to and gaining insights about the complexity of mathematical ideas (Gadanidis 2004);
- a good teacher makes learning easy – rather than creating situations where students have to think hard (Jonassen 2000);
- teaching should start with what a child already knows and understands – rather than with what a child can imagine (Egan, 1997a, 1997b; Greene, 1995).

Perhaps we might be classified as *aspiring* performance mathematicians.

Mathematical identity and performance

Labelling ourselves as performance mathematicians touches on issues of mathematical identity. How we see mathematics and ourselves as mathematicians (whether we are professional, student, or teacher mathematicians) helps determine to a large extent what takes place in mathematics education. For example, consider the following statement made by a teacher after engaging with the mathematics of spherical geometry that we discuss later in this paper: "I learned that math can be discussed with your family and friends just like you would a favourite book or new movie." What difference would such an image of mathematics make in mathematics education? We can imagine one difference being that we would need mathematics that is interesting enough to talk about with others. This might lead mathematics educators to start looking for parallels between what makes for "a favourite book or movie" and what makes for "a favourite math idea or activity." Another difference we can imagine is that students would need to develop ways of telling (performing) the story of the mathematics they are learning to others (like family and friends), so as to motivate them to engage in discussion. This might lead mathematics educators to look at the performing arts for developing students' repertoires for organising and expressing the ideas they seek to communicate.

Seeing mathematics as something that "can be discussed with your family and friends just like you would a favourite book or new movie" means seeing mathematics as something that can be brought to the world, the world that exists outside the mathematics classroom and outside the community of mathematicians. Such a shift in mathematical identity has tremendous implications for mathematics education. Our world, unlike the traditional mathematics classroom, is not only about paradigmatic thinking but also about narrative thinking (Bruner, 1986). And some would suggest that our world is "a performance-based, dramaturgical culture" (Denzin, 2003, p. *x*).

Through the performance lens

Let's suppose that we can apply to ourselves the label of "(aspiring) performance (teacher) mathematicians." Can we point to a mathematical performance and can we say what difference it might make to call it a performance, rather than a lesson or an activity?

In this section, we turn our attention to an artefact that might be classified as a digital mathematical performance called *Parallel Lines: The Movie,* which we will refer to as *Flatland* (see Fig. 1). [1] *Flatland* is a collection of video (and text) organized under four headings:

- "Music Video" – a music video from the (imaginary) movie;
- "Interviews" – interview clips of two people who saw the movie;
- "About the Math" – explorations/performances of ideas from the movie (geodesics, parallel lines, and triangles on a sphere);
- "Rehearsals" – mathematical performances created by students after they investigated concepts of spherical geometry.

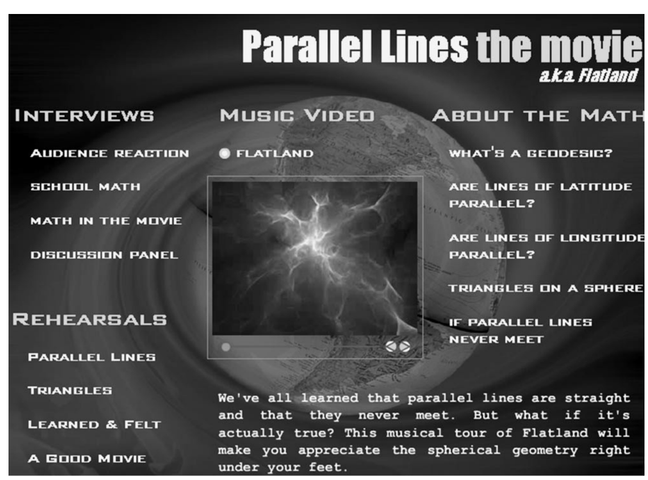

Figure 1. The digital mathematical performance of Flatland

Flatland is not offered as an exemplary performance, but rather as an object to think about and to think with as we explore mathematical performance. We analyze the design of this performance through the three performance categories developed by Boorstin (1990) for analyzing film. Boorstin suggests, "We don't watch movies one way, we watch them three ways … [we] derive three distinct pleasures from watching a film" (p. 9) – which he calls the voyeur's pleasure of "seeing the new and the wonderful" (p. 12), the vicarious pleasure of savouring "the empathic pleasures of the moment" (p. 80), and the visceral pleasure of experiencing "the gut reactions of the lizard brain" p. 110). But what do categories aimed at analyzing film have to offer to mathematics education? We chose to use Boorstin's categories for five reasons.

First, our task is exploratory. This article is a first step in conceptualizing what might be seen when we look at mathematics education though the lens of performance, and what difference this lens might make. We are purposely taking a less travelled path as we seek to see "the new and the wonderful" in mathematics education. We do not expect to find a perfect match between the tools used to analyze performances in the arts and performances in education. However, not having a perfect match does have the advantage of putting us in a position of "understanding" through "seeing as" (Zwicky, 2003, p. 3). To say that we are "performance mathematicians" is to make a metaphorical leap, to see ourselves as something else, and to "see, simultaneously, similarities and dissimilarities" (p. 4).

Second, we did not choose Boorstin's work randomly. We came across it while reading Norman (2004). Norman is a prominent researcher and author in the area of design, and we have used his work when analyzing the design of digital interaction in online mathematics learning. Norman has developed three categories for analyzing design – namely, reflective, behavioural, and visceral – which he suggests "bear perfect correspondence" (p. 123) to Boorstin's categories. This claim gives us some confidence that Boorstin's categories may transcend the spectre of movies and may be useful in analyzing the design of digital mathematical performance – and that "patterns of meaning" in the disparate worlds of film and education might "intersect and echo one another" (Zwicky, 2003, p. 6).

Third, there is a case to be made that by engaging in a movie experience, or more generally by engaging with the "entertainment" of a story or performance, we are engaging with an educational experience. McKee (1997) asks: "But what, after all, is entertainment? To be entertained is to be immersed in the ceremony of story to an intellectually and emotionally satisfying end" (p. 12). McKee claims that we do not engage with the "story arts" simply to be entertained: "We do not wish to escape life but to find life, to use our minds in fresh, experimental ways, to flex our emotions, to enjoy, to learn, to add depth to our days" (pp. 4–5).

Fourth, it has already been suggested that mathematics education is story based (Gadanidis & Hoogland, 2003). We think in terms of stories, we understand the world in terms of stories that we have already understood, we learn by living and accommodating new stories and we define ourselves through the stories we tell ourselves (Bruner 1990, 1996; Schank 1990; Wilson, 2001). We "live most of our lives in a world constructed according to the rules and devices of narrative" (Bruner 1996, p. 149). We are actors in real-life stories where "we walk on stage into a play whose enactment is already in progress – a play whose somewhat open plot determines what parts we play and toward what denouements we may be heading" (Bruner 1990, p. 34). Our lives make sense when shaped into narrative form (MacIntyre 1984, p. 39). Film is a storied experience because story stains our human experience, whether it is an experience through the arts, or through the sciences:

> Science, like the rest of culture, is based on the manufacture of narrative.… We all live by narrative, every day and every minute of our lives.… By narrative we take the best stock we can of the world and our predicament in it. (Wilson, 2001, p. *xiv*)

Fifth, Rodd (2003) has already made a case for using "the interpretative perspective of theatre studies" (p. 15) for analysing the quality of the mathematics teaching and learning experience. As she notes, the "perspective of theatre studies offers descriptors for performance and more delicate explanations for the performance's success or failure" (p. 17). This performance lens allowed her to "draw attention to the idea that something inspiring and marvellous can happen in a [mathematics] lecture" (p. 15) and to see lectures "not as information-delivery venues, but as a place where the 'awe and wonder' of mathematics can be experienced" (p. 20).

The voyeur's eye

Boorstin describes the voyeur's pleasure as three-fold. First, it is the "joy of seeing the new and the wonderful" (p. 12), "watching out of a kind of generic human curiosity" (p. 13). "People love to be taken to a place that's like nothing they've seen before" (p. 16). This joy appears to correspond to Rodd's "awe and wonder." Second, the "new and the wonderful" must make sense, as it is seen through "the mind's eye, not the heart's" (p. 13). Events in the movie cannot happen randomly or without connection to other events. The plot line must be believable, otherwise the voyeur thinks "this can't happen" or "there is no reason for this to happen" and stops watching. Events must fit into a larger, rational scheme, and in some movie scenes this is done visually through the use of the wide-angle shot. Third, "[b]ecause the voyeur's eye bores easily, it demands surprise – so long as surprise comes with a rational explanation" (p. 13). Boorstin continues:

> Audiences want their overall expectations fulfilled – they want the hero to triumph and the lovers to be united – but moment to moment they want to be wrong. The voyeur in us wants to be surprised.… For the writer, this means constantly creating expectations that (for the right kind of reasons) aren't quite fulfilled. (p. 50)

Let's "watch" a typical classroom performance, on the topic of parallel lines, through the voyeur's eye. When middle school students and teachers are asked to relate what they know about parallel lines, the response is that "parallel lines are straight and they never meet." [2] Some also mention parallelograms and the properties of opposite sides. A few teachers and a few students relate rules they learned about the angles made by a transversal of two parallel lines. The typical classroom treatment of the concept of "parallel" at the middle school level is rational – what is learned makes sense. However, the range of what is to be understood is quite narrow. There is little opportunity for seeing the new and the wonderful, for making sense of parallel lines in the context of related mathematical ideas, and there is very little opportunity for experiencing mathematical surprise. We would assume that the voyeur's eye quickly bores and stops watching.

The digital mathematical performance of *Flatland* offers some opportunities for more interesting experiences for the voyeur's eye. The expectation is that parallel lines never meet. But then some doubts and some opportunities for surprise creep in. Do parallel lines have to be straight? What about train tracks that curve? Are they parallel? Do parallel lines never meet? Are lines of longitude parallel? The Equator (a transversal?) intersects lines of longitude at 90˚ – doesn't this mean that the lines of longitude are parallel at the Equator? But lines of longitude meet at the Poles. *Flatland* explores these ideas in video clips where lines are drawn on spherical models (balloons and oranges) to investigate "straightness" and "parallelness" on a sphere. Although we live our lives on a sphere (or an approximate sphere), very few students have opportunities to explore spherical geometry. For most middle school students and teachers, the sphere takes them "to a [mathematical] place that's like nothing they've seen before." It's mathematically new and wonderful, it's related to bigger mathematical ideas, its geometry makes sense (satisfying the rational expectations of the voyeur's eye), yet it offers many surprises. Straightness becomes complex, and although there are no extrinsic straight paths on a sphere, there do exist intrinsic straight paths (geodesics or great circles) (Henderson, 2001; Henderson & Taimina, 2006). The concept of parallel also increases in complexity in the context of spherical geometry. There are no parallel lines on a sphere from an extrinsic perspective, but there are lines that "feel" parallel, and are intrinsically parallel, such as lines of longitude, for example, which we often depict as straight, parallel lines on flat maps of the Earth. And if we form a "triangle" using three geodesics on a sphere, the sum of its angles is not 180˚ and in fact can vary within a range of values.

The above experiences have the potential to be occasioned in *Flatland* through the song and its performance as a music video, and through the video performances about parallel lines and spherical geometry. They offer students and teachers opportunities for a fresh, rational experience of geometry with multiple opportunities for mathematical surprise and insight. Henderson and Taimina (2006) suggest through such informal approaches to geometry "many levels of meaning in mathematics can be opened up in a way that most people can experience and find intellectually challenging and simulating" (p. 59).

However, *Flatland* does not provide the immersive experience of a film. Films tend to be complete experiences. We enter a dark room, we willingly push our everyday reality to the periphery of our attention, and we deeply experience the film. Films are also linear, in that the sequence of experiences is predetermined, and our attention guided and controlled. In contrast, *Flatland* is multi-linear. The videos may be experienced in an order chosen by the user. Our attention is scattered and not guided, at least not anywhere close to the level of control exerted by a film. The menus in *Flatland* offer an organisation, and an opportunity to experience the content in a semi-linear fashion, but this remains an option. Our attention may wander, we might even click on a link that takes us away from *Flatland*. In fact, on the web page where *Flatland* resides there are links inviting us to explore other mathematical performances. [3] Like watching television, we hold the remote and may switch channels at will. Unlike watching a film, where leaving will cause a disturbance and a loss of our investment in the price of admission, escape from *Flatland* is just a click away. In addition, the production quality of *Flatland* is spotty. The video annotations are quick first drafts rather than polished performances.

The videos in *Flatland* sit and wait for us to click, to drag, to explore, but they do not pull us into an experience like a film does. However, unlike a film, which is designed to be a stand-alone experience, *Flatland* has been used in "classroom" settings where it offers starting points for voyeuristic experiences of geometry. For example, various digital productions of *Flatland* have been used in mathematics sessions with middle school students, in both online and face-to-face settings. *Flatland* has also been used in mathematics-for-teachers courses offered online. In such settings, *Flatland* is part of a classroom collective and exists in relationship with the teacher and with the students. It plays a supporting role by providing both a pedagogical model for the instructor and starting points for exploring geometry. A typical comment of teachers reflecting on the experience of *Flatland* and comparing this to their school mathematics experiences is that this "has shown me how much I was NOT taught in elementary school, and how MUCH I still need to know!" [2] We want to emphasise that such teacher comments are not about *Flatland*, or at least not just about *Flatland*. They are comments about the online mathematics experience that started with *Flatland*. The online experience also involved interventions by the instructor and by the students.

The vicarious eye

Boorstin states:

> [t]he vicarious eye puts our heart in the actor's body: we feel what the actor feels, but we judge it for ourselves.… The tension between the two impulses – the urge to be the character and to judge him simultaneously – gives the vicarious experience grit. (p. 67)

Whereas the voyeur's eye needs a wide-angle view, to experience context and relationship, the vicarious eye needs a close-up view of the actors. As Boorstin notes,

> We need to see the actor's face to trigger our vicarious response. … The voyeuristic experience may be grand or clever, but the vicarious experience can be profoundly moving.… [T]ime stands still. The rush of the

Figure 2. Audience Interviews

plot is suspended while we savor the empathic pleasures of the moment. (pp. 114, 67, 80)

Some scenes that are potentially moving did occur when *Flatland* was used in the context of a mathematics-for-teachers course. [2] Comments from teachers in this course were used to create the "Interviews" videos in *Flatland* (see Fig. 2), adding a vicarious feel. The Interviews dialogue performs the following (as well as other) comments from teachers reflecting on their experiences with spherical geometry:

> [Thinking back to my school mathematics experience] I feel like I was misled, misguided, told the half-truth about parallel lines. It is the first time that I have realised/felt that math isn't just BLACK & WHITE and can cause quite creative outcomes/discussions.
>
> I felt lost at first as I struggled to remember math concepts from childhood and adolescence. I felt confused. What did a poem have to do with math? I was perplexed. Was there not only one answer to a mathematical question? I felt apprehensive. How would I discuss a mathematical concept that I did not fully understand?

Figure 3. Students sharing what they learned and felt

Figure 4. "About the Math" Performances

> Then as I got into the swing of things, I felt more confident with my opinions, my answers and most importantly myself. I felt cheerful that I was experiencing math as a student and that I would hopefully be able to empathise with my future students. I felt happy that math instruction could be made to be engaging. Finally, I was giddy that I was thinking about math, actually thinking about math and not doing everything else to avoid it.

Nuggets of vicarious experiences are also contained in the Rehearsals videos, such as the "Learned & Felt" video (see Fig. 3) where students share what they learned and what they felt through the *Flatland* activities and performances and the "About the Math" performances (see Fig, 4). In fact, in one of the videos, *A Good Movie*, students identify moments in their "Parallel Lines" play that they consider to be vicarious. Is the vicarious experience important for mathematics education? Rodd's (2003) analysis of mathematics lectures through a performative lens appears to attend to the quality of vicarious experience, at least implicitly, when she discusses the role played by the personality of the lecturer. In many current mathematics education curricula (such as those in the Canadian province of Ontario, discussed earlier) there is an increased focus on communication, in being able to explain one's thinking to others and also in trying to understand the thinking of others – to let others into your mind and to understand the minds of others. However, the focus is typically on sense making rather than on feelings or emotions. Gadanidis and Namukasa (2005) have found that in mathematics-for-teachers sessions, having teachers share both what they have learned and what they felt, helped teachers realise the commonality of their experiences: their fears, their apprehensions as well as their joy of mathematical attention and insight. Mathematics is more than just a cognitive experience. It is also an emotional and an aesthetic experience (Sinclair, 2001; Sinclair, Pimm & Higginson, 2006), a deeply human experience (Higginson, 2004, 2006). The vicarious eye helps us see mathematics teaching and learning through the hearts of others, as human endeavours, offering opportunities for resonating with our own experiences and for broadening our perspective of what experiences are possible and what they may mean.

Figure 5. Music video image of "birds and fish together fly"

Figure 6. Students identifying a visceral experience.

The visceral eye

Boorstin states that through the visceral eye you seek

> not to feel what the character feels but to feel your own emotions, to have the experience yourself, directly … the passions aroused are not lofty, they're the gut reactions of the lizard brain – thrill of motion, joy of destruction, lust, blood lust, terror, disgust. Sensations, you might say, rather than emotions. (p. 110)

He adds:

> It's almost impossible to conceive of a visceral scene without music behind it.… Music in this context is basically a form of emotional sound effect. (p. 131)

Sensing beauty is a visceral experience and the potential for this exists in the context of sensing (before necessarily understanding) the "pattern, rhythm, symmetry, coherence, fit, balance" of spherical geometry. The music video in *Flatland* has the potential of providing some visceral mathematical experiences through the combination of music and the panning and zooming effects on images used to illustrate ideas in the song (see Fig. 5). In the video *A Good Movie*, students identify the popping of a balloon in their play as a visceral experience (see Fig. 6). In our performances for teachers and students, we have used balloons to illustrate what might happen to the Earth if lines of longitude were forced to be extrinsically parallel: as one of the poles is stretched to be, as the song says, "Equator-wide," the balloon pops with a loud bang.

Is the vicarious experience important for mathematics education? McKee (1997) suggests that there is a danger: "Flawed and false storytelling" might "substitute spectacle for substance, trickery for truth" (p. 13). However, our goal here is not to suggest the need for spectacle, but rather that a spectacular – a visceral – experience can be used to complement a good performance. Boorstin suggests that the mix between the voyeuristic, the vicarious and the visceral "can vary, but there must be a mix to make the movie work" (p. 137). Also, as noted above, the visceral experience does not have to be just about the thrill of change. It can also be about beauty: the depiction of beautiful mathematical ideas that give us pleasure in an instant, like depictions of the "worlds" of non-Euclidean geometries. In her poem "Math Lust," Molly Peacock notes

> But we want
> and we need
> our mathematical lust.
> It's our imagination
> our numerical trust!
> It's the raining of ideas
> till their five or nine or three.
> It's the juice of numbers
> in you, in me
> all unencumbered. [4]

Different media: Different categories? Different directions?

Boorstin's categories shed new light on mathematics education, especially in our case as we explore the new media affordances of the Web to generate digital mathematical performances for online and face-to-face classrooms. As we have argued above, these categories were also applied by Norman in other fields such as design, suggesting that they can be applied to different "objects" other than films. However, we also believe that it is possible that new categories may emerge as we continue our investigation of and our work with new media. Borba and Villarreal (2005) and Borba (2005) have argued, supported by several examples, that different media such as mathematics software or different Internet interfaces are likely to change mathematics in the way it is developed and the way it is learned. They propose that knowledge is produced by humans-with-media, as a means of emphasizing the active role of media such as orality, writing and information technology. We would like to see if the culture that is being created around the existence of the Internet will not create categories that go beyond voyeuristic, vicarious and visceral eyes.

It could very well be that other categories might be useful, such as "emergent," meaning the capability of a piece of digital performance to communicate with other pieces in new, unexpected ways. Or maybe, "multimodal" could be another one, meaning that the piece of art was able to take advantage of combining orality, writing, pictures, animation, etc. Even, "humour" could be one, as found in a recent digital mathematical piece we developed, where we perform a well known humorous mathematical scene that can be used to challenge children and adults to think mathematically. [5] We have no evidence that these kinds of categories will be useful to characterise "good mathematical performance," even though we hope to develop research to see if the conjecture that the Internet world can generate new categories is a reasonable one.

Moreover, since we will develop research based on the qualitative tradition, is very likely that categories will emerge, in the inductive ways pointed out by Lincoln and Guba (1985) and Borba and Araújo (2004). Although we started from the framework constructed by others who studied arts, we can put them in "suspension" as we analyze data. In this type of research categories emerge as we, in this case, interact with students and the digital mathematical performance we are in the process of creating.

In the same line of thought, we need to investigate whether mathematics itself is being transformed by the performance affordances of new media and the Internet combined. If we think of art and of the Internet as being different actors, in the same anthropomorphic eyes that Borba and Villarreal (2005) use to see new media, it is reasonable to expect that they will interact with and influence and change mathematics. This type of investigation could give a new dimension to what we have been calling digital mathematical performance. In the meantime, we will pursue using the three categories we have presented in this paper as way of trying to see new paths for mathematics education.

Final considerations

In the opening keynote address to the 2005 Fields Symposium on Online Mathematical Investigation as a Narrative Experience, Richard Noss used the phrase *rare events* to describe the wonderful mathematics experiences he has witnessed in classrooms using technology. We have also experienced and witnessed similar rare events in mathematics classrooms, with and without the use of technology. These rare events act for us as mathematical and pedagogical models – as thinking tools – that guide our teaching practice. However, these remain for the most part personal, internal models. Performances are typically shared experiences, and mathematical performance challenges us to find ways of sharing with one another the rare events of mathematics education, to make them accessible for reflection, critique and dialogue, and as models on which to base the mathematics experiences we stage in our classrooms.

Peter Taylor, who is a professor of mathematics at Queen's University (Canada), tells the story of many years ago visiting the poetry classroom of his late friend Bill Barnes. Taylor noticed that Barnes brought to his class poems that he cared about, poems that he was passionate about. Taylor also reflected that the mathematics he brought to his own class was not interesting to him as a mathematician. It was not mathematics he cared about or was passionate about. So he wondered, what are the poems of mathematics? We suggest that the poems of mathematics – or the rare events, to use Noss's phrase – must make for good mathematical performances.

We believe that the performance of *Flatland* (perhaps a better produced *Flatland*) has the potential to be a rare event or a poem of mathematics. In fact, we have witnessed this happen in regular and in online classroom settings. What is the advantage of calling *Flatland* a performance? Why not call it a lesson or a set of starting points for mathematical investigation? We have seen in the discussion of Boorstin's three ways of looking at films that the language of performance allows us to see mathematics education in new light and to draw out attention to voyeuristic pleasure of mathematical surprise, to the vicarious pleasure of witnessing the deeply human aspects of feeling mathematical moments, and the visceral pleasure of mathematical change and beauty. As well, we have suggested that calling ourselves performance mathematicians creates a shift of identity that impacts on mathematics education, by helping us view mathematics not as confined to classroom activity or to the work of professional mathematicians, but as something that is shared with the wider world.

Acknowledgement

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Notes

[1] *Flatland* is available at http://www.edu.uwo.ca/mathsense/T/flatland.html.

[2] See Gadanidis, G., Namukasa, I. and Moghaddam, A. (forthcoming) 'Mathematics-for-teachers online: facilitating conceptual shifts in elementary teachers' views of mathematics', *Bolema*.

[3] Other mathematical performances can be accessed at http://www.edu.uwo.ca/mathscene.

[4] The complete poem is available at http://www.edu.uwo.ca/dmp/peacock.

[5] See http://www.edu.uwo.ca/dmp/waiter.html.

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Mathematics education is now a well established discipline drawing some of its challenges, its ideas, its orientations not only from psychology, but also from sociology, anthropology, philosophy and history and focusing its interests not only on formal learning in schools, but across all ages, outside as well as inside formal settings. In the early days, the content of what was taught was not seen as problematic. Nor was the pedagogical setting within which the teaching and learning took place. The focus was on the relationship between the learners and why they are required to learn. We have come a very long way since then…

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