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A Luta Continua! An ethnomathematical appreciation of Paulus Pierre Joseph Gerdes

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Shortly after the time we met in Berkeley, California, in 1980, at the fourth International Congress on Mathematics Education [1], until months before his untimely death, Paulus often signed correspondences, letters and later emails, to me, and I assume to others, with the closing expression: "A luta continua!" [The struggle continues!]. I imagine that he adopted this exclamation from the slogan of the *Frente de Libertação de Moçambique* (Front for the Liberation of Mozambique) (FELIMO) [2] because he was in solidarity with its *raison d'être,* the independence of an African people from European domination in the form of Portuguese colonial rule. His anticolonial and antiracist sentiments are, I contend, foundational to appreciating the motivating force of his scholarly activities, that resulted in a vast corpus of publications that contribute uniquely to both mathematics and mathematics education.

The magnitude of Paulus's contribution is impossible to portray adequately in a few words. He read and wrote in several languages, including Dutch, German, French, Portuguese, and English. In some of these languages, he published hundreds of articles and books. Here, I provide only brief biographical information and discuss two areas of his work in ethnomathematics that may interest FLM readers: foundations of the calculus and the mathematics and aesthetics of basketry.

Paulus, as an acutely and politically conscious participant observer of the global social and political revolutions of the late 1960s and early 1970s, successfully fashioned a means to combine his quest for global social justice with his love of mathematics and appreciation of diverse cultures. In 1976, he arrived in Mozambique from the Netherlands, the country of his birth. He was educated at Radbound University Nijmegen, where he earned a bachelor's degree with honors in mathematics and physics. Afterwards, he participated in a solidarity mission in Vietnam, and returned to Nijmegen to complete a baccalaureate in cultural anthropology in 1974. A year later, he earned a master's degree in mathematics. While still in the Netherlands, he became a professor at the Centro do Terceiro Mundo [Center of the Third World], with links to the liberation and anti-apartheid movements in southern Africa [3].

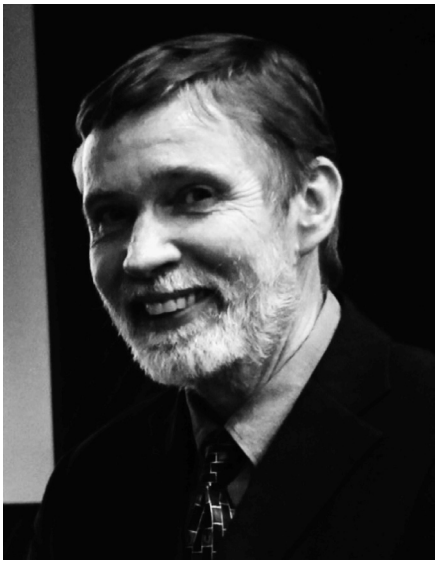

Figure 1. Paulus Pierre Joseph Gerdes (11 November 1952 – 11 November 2014).

In Mozambique, Paulus's mathematics teaching and research represented the incunabula of ethnomathematics. Several of his earliest publications in English appeared in this journal (Gerdes, 1985a, 1986, 1988). In 1986, he completed a doctorate at the University of Dresden, Germany, with a thesis on *O Despertar do Pensamento Geométrico* [The Origins of Geometric Thinking], later published (Gerdes, 2003). In 1996, he earned a second doctorate with a thesis on *Geometria Sona: reflexões sobre tradições de desenhar na areia entre os povos da África ao Sul do Equador* [Sona geometry: reflections on the tradition of sand drawings among the people of Africa south of the Equator], at the University of Wuppertal, Germany, an expanded version of an earlier publication (Gerdes, 1994).

One variant of ethnomathematics as a research program theorizes the emergence of cognitive practices that attend discursively to objects, relations among objects, and relations among relations. These cognitive practices can be shaped by cultural activities such as labor or intellectual stances. Paulus recognized cultural groups defined philosophically as well as geographically. Extending Struik's (1948, 1997) report and analysis, Paulus (1985b) published a book, *Marx demystifies calculus* [4], which he later revised and retitled as *The philosophic-mathematical manuscripts of Karl Marx on differential calculus: an introduction* (Gerdes, 2014). In these texts, Paulus synthesizes references from four different languages to provide an ethnomathematical account of Marx's critique of three theoretical formulations of the differential calculus and his presentation of an alternative perspective that attempts to account for the dialectics of motion and change to which a mathematical function is subjected in the process of differentiation (Marx, 1983). Understanding that adherents of the philosophy of historical and dialectical materialism constitute a distinct cultural group whose intellectual perspective varies from other Western philosophical and social worldviews, Paulus analyzed

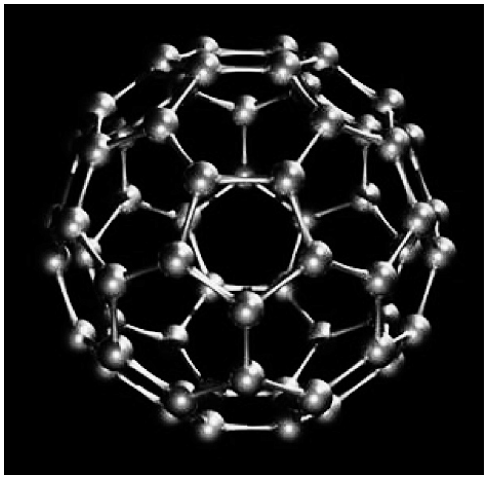

Figure 2. A model of the structure of a fullerene, C_{60}, carbon molecule, which also represents an African hexagonal basket weave.

the consequent critique of the foundational theorization of the calculus for its non-semiotic and theoretical representation of movement in how functions are differentiated.

From a geographical perspective, Paulus manifested particular interest in understanding geometrical ideas extractable from the material cultures of African peoples south of the Sahara. This research program comprised the preponderance of his work. He not only found occasions to relate traditional weaving and sand-drawing practices to known mathematical ideas such as the Pythagorean Theorem, Pappus's Theorem, and Arithmetic modulo *n,* as well as ideas in graph theory, combinatorics, and group theory (Gerdes, 1988, 1998b, 2005a, 2005b, 2007a, 2007b, 2010; Gerdes & Bulafo, 1994), but also, inspired by African cultural practices, to beget new mathematical constructs such as *Lunda*-designs and *Liki*-designs and theorems associated with these constructs (Gerdes, 1997b, 2000, 2005a, 2005b). He also connected the geometry of a particular African technique of hexagonal basket weaving to the fullerene [5] molecule of carbon, C_{60} (see Figure 2) (Gerdes, 1999a, see pp. 110-125; 1999b), a structure first discovered in 1985 by Curl, Koto, and Smalley, the 1996 Nobel laureates of chemistry.

Paulus's ethnomathematical investigations were guided methodologically by respect and beauty. He found seamless connections between beauty and geometric exploration and understood it to be a cultural value common throughout Africa. He would ask why the studied material products possess their specific form. He found that the forms were not arbitrary, but rather presented practical advantages and often were optimal solutions to production problems based on accumulated experience and wisdom. He found that the forms constituted not only knowledge of the materials' biological and physical properties, but also knowledge of "the properties and relations of circles, angles, rectangles, squares, regular pentagons and hexagons, cones, pyramids, cylinders and so forth" (Gerdes, 1997a, pp. 227-228). And he insisted that to uncover the mathematical knowledge of a culture requires respectful and attentive focus on encoded ideas of its material culture (Gerdes, 1998a).

Paulus was a great teacher and mentor, a valiant figure, a prolific researcher and writer, and a caring and generous human being. He respected African culture and knowledge. The volume of his production in ethnomathematics is unparalleled. His untimely death is a tremendous loss for the mathematics and mathematics education communities, as well as for all who struggle for a better, more just world. Memories of him, his interactions, his words, and his deeds will warm all of our hearts and inspire our souls.

Notes

[1] We exchanged ideas about the need to increase awareness of Africa's contribution to mathematics and embarked on a thirty-four year friendship and collaboration. It was a chance meeting since three years earlier, I completed a master's degree in mathematics and two years after that left a mathematics doctoral program to work for Caleb Gattegno. I only attended ICME-4 because of the encouragement of Sandy Dawson and Marty Hoffman, two mathematics educators who had close links to FLM's founder, David Wheeler.

[2] FELIMO, founded in 1962, fought for the independence of the Portuguese Overseas Province of Mozambique, achieved its goal in June 1975, one year after the Carnation Revolution in Lisbon.

[3] For more details, see Ubiratan D'Ambrosio's appreciation of Paulus at professorubiratandambrosio.blogspot.com/2014/11/paulus-gerdes-in-memoriam.html - .VKdiRcaRPfY

[4] Published to commemorate the centennial of Karl Marx's death, the book is an extended version of a 1983 presentation Paulus gave at the Eduardo Mondlane University, Maputo, Mozambique, at a seminar on the then significance of Karl Marx's work.

[5] This refers to buckminsterfullerene, named after R. Buckminster Fuller (1895-1983), the designer of geodesic domes. Prior to its discovery, only graphite and diamond forms of crystalline carbon were known.

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From the archives

Editor's note: *The following remarks are extracted (and slightly edited) from an article by Paulus Gerdes (1994), published in FLM14(2).*

The ethnomathematical movement

The scholars who are engaged in ethnomathematical research are normally socially actively engaged. In this sense I should like to speak of an *ethnomathematical movement* that might be characterized by the following features, among others:

- *Ethnomathematicians* use broad conceptions of mathematics, including, in particular, counting, locating, measuring, designing, playing, explaining [1];
- *Ethnomathematicians* emphasize and analyse the influences of socio-cultural factors on the teaching, learning, and development of mathematics;
- *Ethnomathematicians* draw attention to the fact that mathematics (its techniques and truths) is a *cultural product.* They stress that every people—every culture and every subculture—develops its own particular mathematics. Mathematics is considered to be a *universal, pan-human activity.* As a cultural product mathematics has a history Under certain economic, social, and cultural conditions, it emerged and developed in certain directions; under other conditions, it emerged and developed in other directions. In other words, the development of mathematics is *not unilinear* [2];
- *Ethnomathematicians* emphasize that the school mathematics of the transplanted, imported curriculum is *apparently* alien to the cultural traditions of Africa, Asia, and South America. Apparently this mathematics comes from outside the Third World. *In reality,* however, a substantial part of the contents of this school mathematics is of African and Asian origin [3];
- *Ethnomathematicians* try to contribute to knowledge of the mathematical realizations of the formerly colonized peoples. They look for cultural elements that survived colonialism and that reveal mathematical and other scientific thinking. They try to *reconstruct these* mathematical thoughts;
- *Ethnomathematicians* in Third World countries look for mathematical traditions that survived colonization and for mathematical activities in people's daily life and analyse ways to incorporate them into the curriculum;
- In the educational context *ethnomathematicians* generally favor a critical mathematics education that enables the students to reflect about the reality they live in and empowers them to develop and use mathematics in an emancipatory way. The influence of the well-known Third World pedagogue Paulo Freire is visible [4].

Notes

[1] See the chapter "Environmental activities and mathematical culture" in Bishop (1988).
[2] Compare Ascher & Ascher (1986, pp. 139-140).
[3] Compare *e.g.,* Bishop (1989): "One of the greatest ironies different cultures and societies contributed to the development of [the so-called] Western Mathematics—the Egyptians, the Chinese, the Indians, the Moslems, the Greeks, as well as the Western Europeans. Yet when Western cultural imperialism imposed its version of Mathematics on the colonized societies, it was scarcely recognisable as anything to which these societies might have contributed"; and Bishop (1990).
[4] Compare *e.g.,* Frankenstein (1983), Frankenstein & Powell (1994), Ferreira (1992).

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