# TOWARDS THE NOTION OF COLLECTIVE MATHEMATICAL PROBLEMS

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In a fifth-grade classroom (10-11 years old), the following task is presented for solving [1]:

46, 81, 70, 106

Which numbers are divisible by 2?

As soon as the task is stated, several students raise their hands to respond. To begin, Louis asserts that 46 is divisible by 2 because the digits 4 and 6 are themselves divisible by 2. The teacher invites the rest of the class to share their thoughts. Marie counters by explaining that she divided 46 directly by 2, obtaining 23. The teacher reformulates the two strategies: focusing separately on the divisibility of the digits 4 and 6, or dividing 46 as a whole. Zack builds on this by clarifying that Louis's approach involves verifying whether each digit is even or odd. Damien, in turn, argues that the two strategies ultimately converge, as both involve dividing by 2. Julie then proposes that this reasoning also applies to the number 106, since dividing it by 2 yields 53 and 53. Mathis contributes by breaking 106 into 100 and 6, explaining that 100 can be seen as two 50s and 6 as two 3s. The teacher reiterates this decomposition strategy and asks whether other decompositions might also be possible. Several examples of decompositions of 106 are then provided, leading the class to question whether the divisibility of a number's parts implies the divisibility of the entire number. Various ideas emerge, and different decompositions are recorded on the board. An observation of these examples then leads the class to wonder in which cases an even number divided by two results in an odd number. The discussion unfolds until the bell signals the end of the session.

This session highlights the students' engagement with the task. They proposed diverse strategies, built on each other's ideas, and collectively advanced toward resolving the task. However, one might question whether they truly solved a mathematical problem—or, more specifically, a good mathematical problem. The issue of what constitutes a (good) mathematical problem has long been a focus of research in the tradition of problem solving studies. In this article, I aim to further these reflections by examining this question from a collective perspective.

The cognitive theory of enaction (e.g., Maturana & Varela, 1992) highlights the fundamental role of inter-actions between an individual and their environment in the development and enactment of knowledge. The emphasis on the terms 'inter' and 'action' serves to underscore that, from an enactivist perspective, knowledge is conceived as an action carried out by the knower, and it draws attention to the crucial role of the environment in the emergence of each action (Towers & Proulx, 2013). According to this view, the environment and the individual are always in inter-action, creating a mutual compatibility that enables their respective functioning. The environment acts as a trigger for change in the individual, just as the individual acts as a trigger for change in the environment. These inter-actions and changes unfold within a loop of mutual influence, wherein each continuously shapes the development of the other. This dynamic invites us to consider the classroom not as a mere collection of individuals acting side by side, but as a collectivity that moves forward through re-actions with one another, and where the actions of some open up new possibilities for action for others. From such a perspective, the teacher's actions in the classroom are seen as triggered by those of the students, just as students' actions are stimulated by those of the teacher and their peers (Towers & Proulx, 2013). The actions of each individual are thus understood as contingent upon and elicited by the actions of others. Constituted in an inter-active manner, these actions cannot be easily attributed to any single individual. Rather, they appear to emerge from the classroom as a whole, arising from inter-actions rather than from an isolated individual (McGarvey *et al.,* 2022). In this sense, it becomes possible to conceptualize the mathematics classroom—typically composed of the teacher and students—as an entity, as a collective that jointly engages in mathematical activity. In recent years, various researchers grounded in enaction have investigated the classroom as a collective entity, theorizing about the potential of collective actions to generate new possibilities within the mathematics classroom (see, *e.g.,* McGarvey *et al.,* 2022). By taking the classroom collective as the unit of analysis, this body of work shows that new phenomena can emerge—phenomena that could not be brought forth by an individual alone. Indeed, such collective phenomena do not necessarily belong to or reside within each individual; rather, they arise through and from the inter-actions enacted by the collective (McGarvey *et al.,* 2022). In attending to the mathematics classroom at this collective level, I propose in what follows to conceptualize the notion of *collective mathematical problems*—as phenomena that arise through and by way of the inter-actions enabled by the collective itself. In a previous article (Barabé, 2023), I introduced an initial conceptualization of this notion, illustrating various forms it can take in the classroom. In this article, I build on this conceptualization, extending it further to formalize it by proposing a definition that captures its essence. I also examine how these collective mathematical problems can emerge in the classroom while (re)positioning them in relation to the notion of

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'good' mathematical problem for the classroom. To establish a foundation for this discussion, I begin by reviewing the concept of 'mathematical problem'.

## The notion of mathematical problem

The notion of mathematical problem has been the subject of extensive research, particularly during the 1980s. Various definitions of this notion exist, but it is generally understood as comprising three key dimensions: emergence, uncertainty, and relativity. For instance, Brun (1990) defines the concept of a problem as follows:

> un problème est généralement défini comme une situation initiale avec un but à atteindre, demandant à un sujet d'élaborer une suite d'actions ou d'opérations pour atteindre ce but. Il n'y a problème que dans un rapport sujet/situation, où la solution n'est pas disponible d'emblée, mais possible à construire. C'est dire aussi que le problème pour un sujet donné peut ne pas être un problème pour un autre sujet, en fonction de leur niveau de développement intellectuel par exemple. (p. 2, bold removed) [2]

The dimension of *emergence* refers to the idea that for a task to become a problem for an individual, it must emerge as such for that person. A task becomes a problem if the individual accepts it as such (Agre, 1982). To be a problem, the individual must engage in the search for a solution: "A problem is not a Problem until one wants to solve it." (Schoenfeld, 1983, p. 41). Thus, when a person devises a series of actions or operations to achieve the goal set by an initial situation, as highlighted by Brun, they are engaging with a problem that has emerged for them. A problem emerges through an individual's inter-action with a task when they actively engage with that task. In this sense, many researchers propose distinguishing between a 'task' and a 'problem':

> Thus, the terminology of 'problem' is not very practical. Many researchers in the field use 'task' terminology, which is associated with the formulation of the task, and not in relation to a particular solver. A task can become a 'good problem' for some solvers in some conditions and a routine problem (or not a problem) for other solvers or in different conditions. (Hoshino, Polotskaia & Reid, 2016, p. 156)

Moreover, when a problem emerges for an individual, it must present a challenge; in other words, the person must not immediately know a solution, or the solution must not be readily available, to borrow Brun's terminology. This is the dimension of *uncertainty* in a problem. A task becomes a problem if it raises uncertainty for the individual attempting to solve it (Beghetto, 2017). Otherwise, it would not be considered a genuine problem but rather a routine problem, as described by Hoshino, Polotskaia and Reid. The distinction between a problem and a routine problem (or exercise) is often reiterated in the literature:

> A problem is a situation that differs from an exercise in that the problem solver does not have a procedure or algorithm which will certainly lead to a solution. (Kantowski, 1981, cited in Borasi, 1986, p. 132)

> First, a problem is only a *Problem* (as mathematicians use the term) if you don't know how to go about solving it. A problem that holds no 'surprises' in store, and that can be solved comfortably by routine or familiar procedures (no matter how difficult!) is an exercise. (Schoenfeld, 1983, p. 41)

For a task to qualify as a problem, it must challenge the person attempting to solve it. However, this challenge is also seen as relative to the individual and the context in which they find themselves. This is the *relativity* of a problem, the final dimension of this concept. A problem may be a challenge for one person but not for another, or it may be challenging one day but not the next, depending on the context and/or the person's evolving knowledge (Agre, 1982; Brun, 1990). These three dimensions—emergence, uncertainty, and relativity—form the foundation of the notion of a mathematical problem as it is typically defined in research.

For instance, regarding the divisibility-by-2 task, one might argue that Louis and Marie (among others) approached the task in a certain way, engaging with it such that it became a problem for them. Others, however, might challenge this notion, claiming that it is unclear whether uncertainty or a genuine challenge was present for these students and suggesting instead that this was merely the resolution of an exercise—especially since such a task is relatively straightforward for students at this age level. When we examine the notion of a mathematical problem and, by extension, problem solving, we often focus on the individual: on what the task generates for that person in terms of a problem or problem solving activity. Alternatively, we may analyze the task itself to determine whether it qualifies, or not, as routine (see, *e.g.,* Woodwart *et al.,* 2018). However, the three dimensions central to the notion of a mathematical problem highlights its dynamic, emergent and evolving nature. A mathematical problem emerges when an individual encounters a situation that introduces uncertainty and actively engages in addressing it. While this situation is generally considered to be an initial task posed for resolution, uncertainty may not always be present at the outset but can emerge *during* the activity. This engagement with emerging uncertainty can thus occur as the activity unfolds. Consequently, these three dimensions underlying the notion of a mathematical problem do not necessarily refer to an initial task that immediately prompts a problem for an individual but can also be understood as an event or situation that arises and generates a problem for them during the course of resolution. Close connections can be drawn here with certain strands of research on problem posing, particularly those focusing on problem posing *during* problem solving (Silver, 1994). As discussed in Barabé and Proulx (2015), this line of work is situated within an implicit perspective, in which problem posing is seen as defining the very activity of problem solving itself, manifesting implicitly through it, without any (explicit) request to pose a problem being made. Thus, to determine whether a person has solved one or more mathematical problems, it is essential to examine the mathematical activity itself, considering the presence of uncertainties and the person's engagement in overcoming them. From this perspective, rather than viewing the notion of a mathematical problem as emerging *a priori* from a proposed mathematical task, it can also be understood as arising from the ongoing mathematical activity itself, assessed during or even *a posteriori* to the solving of the task. This leads to moving beyond the initial task and what it generates 'directly' for an individual, connecting instead to the activity that can transform the task into a genuine problem. These three dimensions of a mathematical problem bring into question the inter-action between an individual and the situation they encounter. This inter-action inevitably evolves, progresses, and shifts as the mathematical activity unfolds. Such a conceptualization of the notion of mathematical problem as a phenomenon that can take shape or emerge in the classroom shifts attention toward the activity taking place. Since the mathematics classroom is a space where teachers and students interact with one another, another way to analyze what occurred during this session in terms of the problems addressed or worked on is to consider what emerged at the collective level—that is, what arose from the mathematical activity carried out in the classroom.

## Collective mathematical problems and emergent mathematical activity

By focusing on what can emerge from a collective activity, it becomes possible to revisit the three dimensions that constitute the notion of mathematical problem in order to develop the concept of a *collective mathematical problem.*

> (1) Emergence: For a problem to exist, it must emerge within a collective. This emergence occurs as the collective interacts with a task it seeks to solve. A problem emerges for the collective if there is a desire to address a mathematical situation that arises, with the intention of resolving it. The collective then accepts to engage in solving this situation. A collective mathematical problem is thus brought to the forefront by a collective, in a given situation, as it commits to resolving it.
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> (2) Uncertainty: When a collective mathematical problem emerges, it presents a challenge for the collective, which does not immediately know how to resolve it. A mathematical situation can thus become a collective mathematical problem as soon as a mathematical uncertainty arises within the collective—one that it does not immediately know how to overcome, yet chooses to engage with in an effort to resolve.
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> (3) Relativity: The collective mathematical problem that emerges depends on the collective itself. Faced with the same initial task, a mathematical situation may emerge and generate a collective mathematical problem for one collective but not for another. Similarly, a collective may engage with such a problem at one moment but no longer perceive it as a problem the following day, depending on the progression of their mathematical understanding and skills, for example.

A collective mathematical problem can thus be defined as a mathematical situation that arises contingently from a collective activity and that presents a mathematical uncertainty for the collective—an uncertainty that the group does not immediately know how to overcome, yet chooses to engage with in an effort to resolve. A collective mathematical problem thus occurs when a mathematical uncertainty, experienced *in situ,* emerges from a collective activity, and the collective engages with it. It seems important to note that this mathematical uncertainty is considered at the level of the collective—that is, at the level of the group—and could therefore be experienced differently by each individual [3]. However, the conceptualization of collective mathematical problems proposed here aim to focus on what results from the inter-actions within the collective, and not on what is produced or experienced by each individual taken in isolation, although this could certainly be of interest. Furthermore, these uncertainties, as elaborated in Barabé (2023), can take various forms, leading to a distinction between two types of collective mathematical problems: *content-related* collective mathematical problems and *meta-mathematical* collective problems. The former refers to uncertainties related to mathematical concepts, techniques, or methods, while the latter pertains to meta-mathematical dimensions, such as engaging in mathematics with others (*e.g.,* having to explain or justify ideas to ensure mutual understanding). For instance, when a mathematical error emerges in the classroom, the collective may struggle to make sense of it. The error may create doubt about its plausibility and/or lead to difficulties in collectively understanding, explaining, and justifying what does not work and what could work. Addressing the error, with all the explanations, justifications, and validations it may entail, can quickly introduce additional challenges for the collective, prompting it to resolve the collective mathematical problem related to the error. Similarly, the sharing of a strategy or mathematical idea—whether correct or not—can create a need for further explanations or justifications that are not readily available, prompting the collective to engage with an uncertainty tied to the development of mathematical explanations and justifications to satisfy the collective. Errors to overcome, strategies requiring clarification, justification, and validation, and conjectures to investigate are all examples of collective mathematical problems, whether content-related and/or meta-mathematical (both of which can occur simultaneously), that can emerge from classroom inter-actions. Their emergence can lead the collective into authentic problem solving, particularly with problems contingent on ongoing activity, where the pathway to a solution remains unknown to the collective. The argument put forward in this article is that solving these collective mathematical problems constitutes a legitimate form of problem solving, one that takes into account ongoing mathematical activity and thus goes beyond the 'direct' emergence of a problem from an initial task. These collective mathematical problems are embedded in the mathematical activity initiated by the collective, illustrating the various challenges it experiences through its engagement in the activity.

I now return to the session briefly described earlier to examine more closely what can occur when we focus on the collective and what can emerge from it. Over the 50-minute session, various uncertainties emerged from classroom inter-actions, and the collective engaged with them. Several collective mathematical problems surfaced:

1. Can we verify whether each digit of a number is divisible by 2 to determine whether the entire number is divisible by 2?

2. How can we do this with 106? How can we decompose 106? What is the value of the 10 in 106?

3. If the decompositions are divisible, is the entire number also divisible?

4. In what cases does dividing an even number by 2 results in an odd number?

The *first collective mathematical problem* arose from an initial claim that 46 is divisible by 2 because both 4 and 6 are divisible by 2. In the classroom, this claim prompted a request for validation—not just of the answer but of the strategy itself. The strategy became the new problem to solve, as the collective debated its validity in all cases. This constitutes a collective mathematical problem, as an uncertainty emerged contingently from the mathematical activity occurring in class, and the collective engaged in resolving it. In doing so, the collective proposed various explanations, examples, and counterexamples to make sense of the strategy. This led them to assert that the strategy works and could also work with the number 106, which was part of the initial task data. This proposition quickly triggered mathematical argumentation, as the collective was divided on the validity of the strategy for this particular number. The example of 106 in the context of dividing each digit by 2 led to a *second collective mathematical problem:* determining whether the strategy works for 106. This problem remained more localized, focusing specifically on 106 while introducing additional challenges related to a three-digit number, one of which is zero. To resolve this problem, the collective presented various explanations and justifications, explored multiple decompositions of 106 (as illustrated in Figure 1), and reflected on the value of the 10 in the number.

These decompositions were then examined by the collective, focusing on their parity and leading to the formulation of a mathematical question: "If the decompositions are divisible, is the entire number necessarily divisible as well?" This mathematical question became a *third collective mathematical problem* to resolve, as the collective faced an uncertainty it sought to overcome. Justifications and examples were proposed to address this question. The investigation of these examples led to the formulation of a conjecture that an even number divided by 2 always results in an odd number. This conjecture gave rise to a *fourth collective mathematical problem:* "In what cases does dividing an even number by 2 result in an odd number?" Faced with this uncertainty, the collective explored new examples, some yielding odd numbers (as shown on the left side of Figure 2) and others even numbers (as shown on the right side of Figure 2). The collective then proposed explanations, validated and invalidated different elements, and sought generalizations, among other activities.

Once this fourth collective mathematical problem was resolved, the bell rang, marking the end of the session. Further strategies or ideas could have been shared and discussed, potentially leading to the emergence of new collective mathematical problems to solve. A remaining question is to understand how such collective mathematical problems can form in the classroom.

## Collective mathematical problems and mathematical practices

A close examination of the mathematical actions foregrounded by the collective can contribute to a better understanding of how collective mathematical problems emerge in the classroom. Building on research related to inquiry-based approaches to mathematics education (*e.g.,* Cobb, Perlwitz & Underwood, 1994), mathematical actions that are inter-actively constituted and grounded in the social conventions of the classroom are referred to as *mathematical practices*. These practices encompass, for example, forms of activity such as explaining, justifying, validating, arguing, exemplifying, formulating mathematical conjectures or questions, overcoming errors or uncertainties, using symbols and representations, or drawing on a body of established mathematical knowledge. This session on the divisibility by 2 illustrates that the deployment of such mathematical practices in the classroom can not only advance the resolution of the initial task but also contribute to the emergence of new collective mathematical problems to be explored. Indeed, it

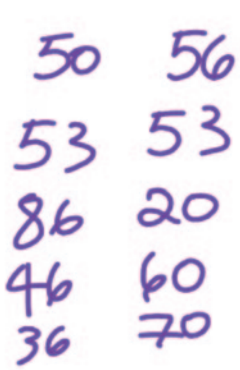


*Figure 1. Different decompositions of 106 proposed during the session.*

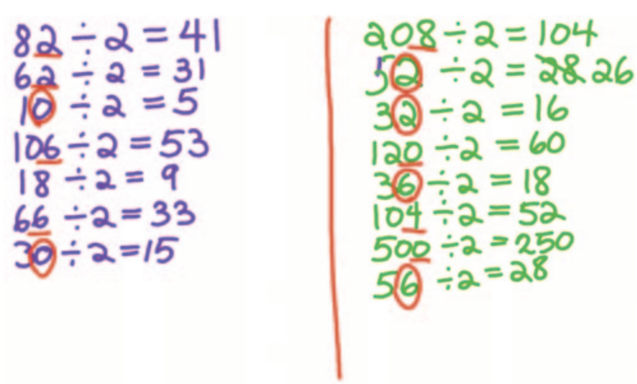


*Figure 2. Investigation of the collective mathematical problem arising from the conjecture of an even number divided by 2 resulting in an odd number.*

was the *validation* of the strategy that 46 is divisible by 2 because both 4 and 6 are divisible by 2 that gave rise to the first collective mathematical problem. To solve it, *explanations*, *examples*, and *counterexamples* were provided. An *argumentation* regarding the application of this strategy to the number 106 emerged, leading to the second collective mathematical problem. Various *explanations* and *justifications* were proposed to address it, during which different decompositions of 106 were suggested. This process prompted the *formulation of a mathematical question*, which became the third collective mathematical problem. *Examples* and *justifications* were presented to solve this problem, eventually leading to the *formulation of a conjecture*. The conjecture then became the focus of the fourth collective mathematical problem. In his work, Schoenfeld (2020) emphasizes that the importance of problem solving lies in the deployment of mathematical practices and mathematical ways of thinking. However, it appears that the deployment of mathematical practices also generates new mathematical problems to solve—collective mathematical problems that inherently emerge from such activity. A kind of 'snowball effect' seems to occur. Mathematical practices are put forth by the collective in response to an initial task. These practices, in turn, stimulate the use of additional mathematical practices by the collective. Their deployment can generate uncertainties for the collective, prompting it to resolve them. For instance, the validity of a strategy might be questioned and investigated; an explanation perceived as imprecise might generate further inquiries; an error might occur, prompting the collective to make sense of it; a conjecture might be formulated, prompting exploration of its domain of validity; or examples and counterexamples might be proposed, resulting in mathematical argumentation. The mathematical practices enacted in the classroom can thus generate moments of uncertainty, and when the collective engages in addressing these uncertainties, collective mathematical problems emerge. In turn, the resolution of these collective mathematical problems calls for the enactment of new mathematical practices. And so the cycle continues… This suggests a mutual influence loop between the deployment of mathematical practices and the emergence of collective mathematical problems, unfolding through and driven by the inter-actions that take place in the classroom as the collective works to solve an initial mathematical task. This recalls the work of Grenier and Payan (2002), who discuss the never-ending nature of exploiting a mathematical task, as new mathematical problems can always be formulated. However, while such problem formulation is often done either before the solving of the task, by modifying various didactic variables (*e.g.,* Zhang *et al.,* 2022), or after its resolution, as in Polya's (1945) well known *Looking Back* strategy, the example presented here highlights that mathematical problems can also arise contingently, that is, *during* the collective mathematical activity occurring in the classroom—emerging naturally and inherently from the activity itself. The concept of collective mathematical problems aims to shift our focus toward the mathematical activity as it unfolds in the classroom, drawing attention to the mathematical problems *experienced* by the collective.

## Final reflection on 'good' mathematical problems

While the notion of a mathematical problem often serves as a point of departure, several studies in problem solving research also focus on what makes a problem a 'good' problem. Liljedahl (2020) argues that good mathematical problems are those in which students initially get stuck, but which lead them to think, to experiment, to try things out, and ultimately to mobilize their knowledge in original ways in order to break through the impasse. For Schoenfeld (2020), a good problem fosters mathematical investigation—that is, it allows for the formulation of conjectures, the establishment of connections, abstraction, generalization, and the generation of new problems to be solved. While research in this field often focuses on the initial task proposed to students as a means of triggering rich and authentic mathematical activity, one question that arises is whether collective mathematical problems—emerging from *in situ* mathematical activity—may also be considered good mathematical problems. Good problems may certainly exhibit various characteristics, depending on their formulation and design, but it seems that such characteristics may also arise within the classroom over the course of activity. In this context, future research could seek to better understand whether the collective resolution of routine problems (*i.e.,* exercises), which gives rise to such emergent collective mathematical problems, can lead to mathematical activity similar to that which is typically elicited by initially formulated good problems.

Such research could help challenge the widely held dichotomy between routine problems and good mathematical problems, by illustrating the richness of mathematical activity that may emerge when collective mathematical problems are recognized and investigated within the classroom. Given that most of the teaching material used by teachers consists largely of routine problems (Beghetto, 2017), such research may help to bridge the gap that sometimes separates the scientific and practical communities. However, further research exploring the conditions under which collective mathematical problems emerge and are investigated—through an analysis of teaching practices that foster, support, or make use of them, as well as the resulting mathematical activity among students—would be valuable for better understanding the scope and limitations of this notion of collective mathematical problems in classroom teaching. Moreover, because collective mathematical problems are inherently tied to the ongoing activity, it is reasonable to consider that they may carry a certain significance for the collective that seeks to resolve them. This raises a central question: For whom are these problems good mathematical problems? To address this question, it may be helpful to consider different perspectives: for instance, that of the researcher, who seeks to observe students engaging in mathematical activity akin to that of mathematicians; that of the teacher, who aims to support students in learning specific content; or that of the students themselves, as they encounter and formulate problems through the mathematical activity unfolding in the classroom. In this sense, this article highlights the complexity of the notion of (good) mathematical problems and proposes to broaden it by paying closer attention to what unfolds and emerges through classroom activity—emphasizing the emergent and contingent nature of problems and the situated, socially constructed nature of mathematical activity in the classroom (Cobb, Perlwitz & Underwood, 1994), while also inviting consideration of the multiple perspectives involved in defining what constitutes a good problem for the classroom.


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## Notes

[1] This session comes from a *Teaching Experiment* conducted as part of a research project on problem solving (see Proulx, 2018, for more details).

[2] In English: A problem is generally defined as an initial situation with a goal to achieve, requiring the subject to devise a series of actions or operations to reach that goal. A problem only exists within a subject/situation relationship, where the solution is not immediately available but is possible to construct. This also means that what constitutes a problem for one subject may not be a problem for another, depending on their level of intellectual development, for example. (free translation)

[3] Obviously, the teacher could, quite often, prevent these uncertainties from unfolding in the classroom by immediately offering a way to resolve them each time they begin to emerge. The teaching context thus plays an important role in the emergence of collective mathematical problems in the classroom. This role of the teacher, which would warrant more in-depth investigation, nevertheless lies beyond the scope of the present article.

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