Letter to the Editor

From Dr. Caleb Gattegno, 80 Fifth Avenue, New York, NY 10011

I started reading the first issue of *For the Learning of Mathematics* when I had much to do, but instead of postponing that reading for more leisurely times, I postponed other jobs and kept on reading and reflecting on what your selection of articles was bringing to your readers.

May I first congratulate you on your first issue, which in appearance and content has raised the level of my hopes that, ultimately, a conscious and responsible approach to the challenges involved in teaching mathematics across the spectrum will become a reality. Your writers are given room to be able to develop their ideas to the point where they think they have said all they want to say, as they want to say it. The topics will appear basic to anyone who is concerned with research in teaching and learning, and varied enough to expose the extent of the challenge to the collectivity of people involved in math education and its improvement.

The following notes may find a place in a new column of letters to the editor or comments on matters raised by writers in previous issues. I very much hope so since your journal may generate that community of concerned people needed to make a difference in the field.

1. Radatz's article on errors, with its impressive bibliography, remains an academic survey of what people have thought about the topic. There is room for a more fundamental attack on the subject.

A first observation, open to all, is that there is no learning without the accompaniment of a string of errors which simply tell at which phase of the process we find ourselves. I got the impression that errors are seen by all these investigators as something to avoid, that they represent a blemish on the learners and their teachers. However much errors are used to guide suggestions to teachers, they are not welcomed as I think they should be. In fact, errors are the only guide we have to knowing how to place our students in front of the challenges they are facing. From their mistakes we can arrive at what we should not do and what we should do as teachers to lead our students towards what we consider reachable — mastery of the matter worked on. Mathematics is not knowledge to be memorized and retained, it is a way of working of the mind, a sensitivity to an inner reality where perception of relationships and their dynamics has become second nature. As soon as we know how to shift from asking people to remember something to considering how they act, we can supply criteria whose function permits errors to be caught and corrected spontaneously.

What contemplation of errors can do is wake us up to our prolonged ignoring of the students who are the ones involved in the process of learning. And 100% involved at that; while we need to be reminded that teaching means just letting the learners do the learning, learning that imposes its own inner demands. Errors do not vanish because of better presentations and the use of audio-visual or other devices in instruction, nor because we reach a deeper understanding of psychoanalysis or the psychology of thinking, but because people know what they are doing and how this connects to what they have to do and how they can relate properly to the challenge in front of them.

A study of errors is a study of learning, at least of the first phase of learning when their occurrence is natural — and welcome.

2. Easley's article has stirred more thoughts than I will permit myself to write in this letter to the editor. The most important point is that I would have thought that, with his awareness that all he was doing was not leading where he wanted to go, he could have asked other sets of questions of himself. Maybe his keen intellect, the considerable facilities he has available with his position at Urbana, and students available to do spade work, would serve him better and satisfy him more if he took the plunge and asked himself, "What is it all about?" My reading of the efforts *he* feels have not produced what he and others expected from them, is that they are not centered on what *can* yield what he hopes for. His team could have gathered results which would astonish them, meet their expectations and more, had they started with different equipment from that which they preferred to adopt. With success in teaching measured only by learning success, only if teaching is subordinated to learning can there be any chance of getting what Easley, and probably all teacher educators, wish to see happen in the classrooms of the world.

Easley seems ripe for a radical move, and I humbly suggest to him that, if his move is taken in the form of looking at learning and its effect on teaching, it will not be regretted. My feeling for him, as he comes across from his personal writing contained in his article, is that all he did this far has prepared him precisely for this shift.

3. To Joseph Agassi, may I say that I see more options open to us for the future of education, for mathematics education and teacher education than his article suggests. Perhaps it is the practitioner in me who, in more than 50 years of teaching and reflection on the same matters, has allowed components he has left out to take their place. Perhaps it is the empirical foundation of my ideas and my great ignorance of the vast contributions of writers known to Agassi which keeps me in contact with what I find comforting and which makes me optimistic about the future of education.

The other writers, too, made me think. To all I want to say thank you.

May you receive the necessary support to keep for yourself the function of serving all of us by channelling our experiences and our meditations to each other, wherever we are, through your welcome and beautiful journal on learning mathematics.