THE ROLE OF CONTEXTS IN ASSESSMENT PROBLEMS IN MATHEMATICS

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Since the introduction of context problems in teaching and assessing number, there have been arguments about whether a context makes a problem easier or harder for students. This article [1] contributes to this dispute by discussing the three main roles that contexts may play in problems aimed at assessing students' mathematical understanding:

- enhancing the accessibility to problems
- contributing to the transparency and elasticity of problems, and
- suggesting solution strategies to students.

How helpful contexts are, both for students and for teachers, comes significantly to the fore when bare number problems (such as 26 + 10 =) are compared with context problems. To illustrate the role of contexts, examples are given from several primary school studies in the Netherlands and other countries

Different meanings of context

When the term 'context' is used in an educational setting, then several things can be meant by this term (see also Wedege, 1999):

- *the learning environment*: this includes both the different situations in which learning takes place (e g , Lave, 1988; Säljö and Wyndhamn, 1993; Nunes, Schliemann and Carraher, 1993) and the interpersonal dimension of learning (Bauersfeld, 1980; Cobb, Yackel and Wood, 1992; Voigt, 1994)

- *a characteristic of a task presented to the students*: referring either to the words and pictures that help the students to understand the task, or concerning the situation or event in which the task is situated. The description of a context, given by Borasi (1986), comes close to the interpretation of a context as a task characteristic

The distinction between 'learning-environment context' and 'task context' can also be applied when the term 'context' is used in relation to assessment. Similarly to an instructional situation, in an assessment situation 'context' can refer to the environment in which students are assessed, the tools and formats that are used for it, and the rules that apply to the assessment. Based on what Brousseau (1984) called the 'didactical contract', the latter is named the 'assessment contract' (see Elbers, 1991 and Van den Heuvel-Panhuizen, 1996). In addition to this 'environment' meaning of context, the term can also refer to the situated information in an assessment problem. This 'task context' is the focus of the rest of this article The idea is to rethink the influence of the context of the task on students' performance

The perspective taken here to discuss context is the Dutch domain-specific educational theory for mathematics education that is known as *Realistic Mathematics Education* (RME). [2]

As in most approaches to mathematics education, RME aims at enabling students to apply mathematics. In RME, this connection to reality is not only recognizable at the end of the learning process in the area of applying skills, but also reality is conceived of as a source for learning mathematics. Just as mathematics arose from the mathematization of reality, so learning mathematics has to originate in mathematizing reality. Even in the early years of RME, it was emphasized that if children learn mathematics in an isolated fashion, divorced from their experiences, it will quickly be forgotten and the children will not be able to apply it (Freudenthal, 1968). Rather than beginning with certain abstractions or definitions to be applied later, one must start with rich contexts demanding mathematical organization or, in other words, contexts that can be mathematized (*ibid.*) Thus, while working on context problems, the students can develop mathematical tools and understanding.

Although, in general, the term 'context' is often normatively employed as a requirement that the teaching and the problems used for it are authentic and reflect real-life situations (Wedege, 1999), this is not true for RME. Within this approach to mathematics education, 'realistic' means that the context of the problems is imaginable for the students However, it must be acknowledged that the name *Realistic Mathematics Education* is somewhat confusing in this respect. This all has to do with the Dutch verb *zich REALISE-ren* that means *to imagine*. This implies that it is not authenticity as such, but the emphasis on making something real in your mind that gave RME its name. For the problems presented to the students, this means that the context can be one from the real world, but this is not always necessary. The fantasy world of fairy tales and even the formal world of mathematics can provide suitable contexts for a problem, as long as they are real in the students' minds and they can experience them as real for themselves

Requirements for assessment problems in RME

Aside from the content of the assessment, which can set its own content-specific requirements for the problems, there are two general criteria which problems must fulfill in RME if they are to be suitable for assessment: they must be meaningful and informative (Van den Heuvel-Panhuizen, 1996)

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In the following section I will examine these criteria and discuss how they are linked to the characteristics of RME.

These requirements reflect another approach to assessment problems than is typical within the psychometric approach. More about this difference can be found in Van den Heuvel-Panhuizen and Becker (2003).

Assessment problems must be meaningful

In RME, which is based on Freudenthal's idea of mathematics as a human activity, the primary educational goal is that the students learn to do mathematics as an activity. This implies that one should "teach mathematics so as to be useful" (Freudenthal, 1968, p. 3). The students must learn to analyze and organize problem situations and to apply mathematics flexibly in problem situations that are meaningful to them. From the point of view of the student, the problems must therefore be accessible, inviting, and worthwhile to solve. The problems must also be challenging (Treffers, 1987) and it must be obvious to the students why an answer to a given question is required (Gravemeijer, 1982). This meaningful aspect of the problems may also entail allowing the students to pose or think up questions themselves (see, for instance, Van den Heuvel-Panhuizen, Middleton and Streefland, 1995).

Another significant element is that the students can mould a given problem situation so that they, themselves, are in a certain sense master of the situation; that is to say, they are 'owner' of the problem. This is the case, for instance, in the percentage problems, where the students may decide whether someone has passed or failed a given examination (Streefland and Van den Heuvel-Panhuizen, 1992), or in problems in which the students may decide what to buy and so can control the degree of difficulty of a problem (Van den Heuvel-Panhuizen, 1996).

In order for problems to be meaningful with respect to subject matter, they need to reflect important goals. If something is not worthwhile learning, then neither is it worthwhile for assessing. Furthermore, the problems should not be restricted to goals that can be easily assessed, but, rather, should cover the entire breadth and depth of the mathematical area. As is indicated by the 'assessment pyramid' (De Lange, 1995), which means that an assessment should cover all topics of the subject matter and should include problems on each level: from basic skills to higher-order reasoning.

The emphasis on higher-order reasoning implies that the problem situations should be fairly unfamiliar to the students, as this will then offer them an opportunity for mathematization. In other words, problem solving in RME does not mean simply conducting a fixed procedure in set situations. Consequently, the problems can be solved in different ways. This aspect is present in the next requirement as well.

Assessment problems must be informative

Characteristic of the RME approach is that teaching as transmission of knowledge is replaced by creating rich learning environments for students as active learners. Education is designed to dovetail as closely as possible with the students' informal knowledge and help them to achieve a higher level of understanding through guided re-invention. In order to support this process of guided re-invention, the assessment problems must provide the teacher with a maximum of information on the students' knowledge, insight and skills, including their strategies. For this to succeed, the problems must again be accessible to the students, which now first of all means that the accompanying test instructions must be as clear as possible to the students.

Another point is that the students must have the opportunity to give their own answers in their own words. To state this more generally, if assessment is to offer insight into the students' mathematization activities, then these mathematization activities must be as visible as possible (see also Van den Heuvel-Panhuizen and Fosnot, 2001). This can best be realized with open-ended questions, in which the students work out a problem and formulate an answer on their own. So, there must be room for the students' own constructions – the problems must be such that they can be solved in different ways and on different levels. In this way, the problems make the learning process transparent – to both the teachers and the students.

The problems should also reflect 'positive testing', allowing the students to demonstrate what they know, rather than simply revealing what the students do not yet know. Again, this means that accessible problems must be used that can be solved in different ways and on different levels.

Such problems are also required if the assessment is meant to provide teachers with footholds for further instruction. This asks for assessment problems that reveal what is attainable for the child in the near future. In other words, giving insight into what Vygotsky called the 'zone of proximal development' requires 'advance testing' (Van den Heuvel-Panhuizen, 1996). A consequence of this view is that particular knowledge has to be assessed before it has been taught.

Another key requirement for problems to be informative is that they must provide an accurate picture of the student. This not only means that one must beware of certain inaccuracies that may arise as the result of inequitable differences in administration and analysis procedures, but also, importantly, justice must be done with respect to the students. In other words, the assessment should be fair to the students.

Contexts play an important role in fulfilling the requirement of having meaningful and informative assessment problems. Before dealing with how contexts can contribute to this requirement, a discussion follows on the different appearances contexts can have.

The different natures of context in assessment problems

As said before, in RME tasks contexts are viewed in a broad sense. They may refer to everyday-life and fantasy situations in which the problems are situated, but also to the mathematical context of, for instance, a bare number problem. What is important is that the task context is suitable for mathematization – the students are able to imagine the situation or event so that they can make use of their own experiences and knowledge. This might also be true for bare number problems, which can be meaningful outside any real-life context. Or, as Freudenthal (1991) stated, such problems fit or can be fitted into any context.

In the following, however, we will focus on the non-mathematical meaning of contexts and the different appearances they can have

Various kinds of contexts

Depending upon the opportunities they offer, various kinds of contexts can be distinguished. De Lange (1979), referring to the opportunities for mathematization, distinguishes three types of contexts. 'First-order' contexts only involve the translation of textually packaged mathematical problems, whilst 'second-order' and 'third-order' contexts actually offer the opportunity for mathematization. The difference between the latter two types of contexts is that third-order contexts are understood to be contexts that allow students to discover new mathematical concepts. Contexts also differ with respect to their degree of reality. Problems are often merely bare number problems that are 'dressed up' (*e.g.*, Bell, Burkhardt and Swan, 1992). The reality they encompass is largely cosmetic and they contrast with "relevant and essential contexts" (De Lange, 1995) in which the contexts make a relevant contribution to the problem. Although one's first thought here may be of a rich topic, presented in the form of an extensive task, even very simple problems may have a relevant and essential context.

This can even be true of multiple-choice problems. As an example, De Lange offers a multiple-choice problem in which students must estimate the width of the classroom. He also shows how artificial contexts, such as a story about a new disease in the 21st century, can be relevant. The disease in question was, in fact, AIDS, but was changed, for emotional reasons, to a science-fiction disease. This example clearly demonstrates that it is more important that the context stimulates and offers support for reflection than that the data and the situation be real.

Moreover, the degree of reality of a context is relative. De Lange wonders how close to the students' world the context must necessarily be. How suitable is a context that involves being an airplane pilot, for instance, if most of the students have never had such an experience. De Lange found that such contexts do indeed work, and with girls, too. The airplane context was used in a booklet on trigonometry and vectors, which was tested at an almost exclusively girls' school. This example also indicates the complexity of the contextual aspect. One single rule cannot always be found for choosing contexts, but we should at least try and create a balance between a good context and a good mathematical problem (De Lange, 1995).

In contrast to De Lange's experiences described above, which primarily involved extended assessment tasks, experiences with contexts in briefer tasks took place in the MORE [3] project (Gravemeijer and Van den Heuvel-Panhuizen *et al.*, 1993; Van den Heuvel-Panhuizen, 1996). In short-task problems, too, the contexts may be more or less real, may stand in various degrees of proximity to the students, and may offer more or less opportunity for mathematization. The single point of difference, however, is that the contexts in short-task problems are indicated by minimal means. Pictures have here a very special and multi-purpose function.

Pictures as bearers of contexts

Within RME, for short-task context problems, pictures are very often used to inform the students about the context. This means that not only the wording, like is the case in word problems, but also illustrations have the role of context bearer. The text, which explains what is asked in the problem, is often provided orally by the teacher. The advantage of having the text read out by the teacher is that the assessment does not become a test of reading ability. Moreover, to avoid the students having to remember the relevant numerical information that belongs to the problem, this information is placed on the test sheet.

The function of the illustrations is more important, however, than the typical obligatory picture accompanying a problem, whose usual purpose is to make the problem more attractive and to motivate the students.

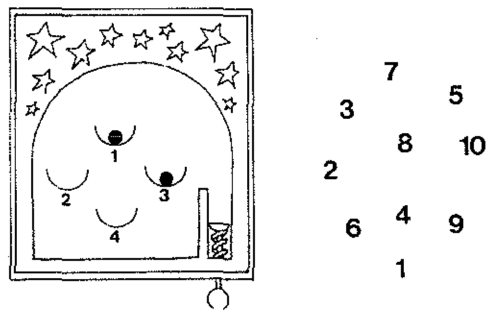

Figure 1: Pinball problem

Pictures have a number of functions, as follows:

- *motivator*, as already mentioned
- *situation describer*; one illustration can tell more than an entire story (see, *e.g.*, Figure 1, the *Pinball* problem)
- *information provider*; the necessary information can be derived from the illustration (see, *e.g.*, Figure 2, the *Shopping* problem)
- *action indicator*; a given action is elicited that has the potential of a strategy that leads to a solution (see, *e.g.*, Figure 3, the *Comparing height* problem)
- *model supplier*; the illustration contains certain structuring possibilities that can be used to solve the problem (see, *e.g.*, Figure 4, the *Candy* problem)
- *solution and solution-strategy communicator*; the solution and aspects of the applied strategies can be indicated in the drawing (see, *e.g.*, Figure 5, the *Whistle and watch* problem).

It is obvious that each illustration does not always fulfil each function and that the function of an illustration cannot always easily be labelled. The above categorization is therefore not intended as a classification scheme for differentiating types of illustrations in context problems, but rather as a way of acquiring a sense of the potential for using illustrations as context bearers in assessment problems.

Word problems

Although word problems are often considered synonymous with context problems, there is big a difference between the

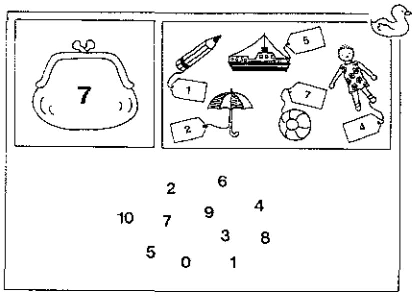

Figure 2: Shopping problem

Figure 3: Comparing height problem

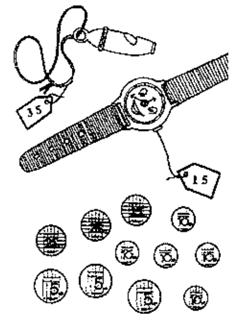

Figure 5: Whistle and watch problem

two. One characteristic for many word problems is that the context is not very essential - it can often be exchanged for another without substantially altering the problem (Treffers and Goffree, 1982) For instance, problems involving marbles, in which someone has 16 marbles and gains 10 more (see Figure 6), might just be changed to a problem involving pounds of ham (see Figure 7) (Freudenthal, 1980; Van den Heuvel-Panhuizen, 1996).

Moreover, in word problems the reality that is presented is often not in tune with the real situation of the actors in the problem. Think of the situation of the butcher's shop. Some of the hams in stock might have been sold when the new ham arrives In this word problem, the context reflects the world of textbooks. In this world, there is little space for reality with its unsolvable and multi-solvable problems. Therefore, Freudenthal (1991) has the suspicion that teaching word problems may cause an anti-mathematical attitude in children. From this perspective, it is not surprising that research has shown (*e.g.*, Verschaffel, Greer and De Corte, 2000) that word problems often do not foster in students a genuine disposition toward treating the word problem as a description of some real-world situation to be modelled mathematically. That the context does matter was recently confirmed by a study by Cooper and Harries (2002). They found that when students were given suitable realistic problems, many children were more willing and able to introduce realistic responses.

The role contexts play in assessment problems

Embedding assessment problems in everyday-life or fantasy situations that the students can imagine is a powerful means to getting meaningful and informative assessment problems

Contexts enhance the accessibility of problems

In the first place, contexts can contribute to the accessibility of assessment problems. By starting from easily imagined situations, presented visually, the students will quite quickly grasp the purpose of a given problem. The advantage of this direct, visual presentation - of which examples are shown in the section about pictures as bearers of contexts - is that the students need not wrestle through an enormous amount of text before they can deal with the problem. Moreover, in addition to making the situations recognizable and easily imaginable, a pleasant, inviting context can also increase the accessibility through its motivational element

Contexts contribute to the transparency and elasticity of problems

Compared with most bare number problems, context problems offer the students more opportunity for demonstrating their abilities. In bare number problems, such as long division, the answer is either right or wrong In a context problem, for instance, where one must figure out how many buses are needed to transport a large contingent of soccer fans, the students can also find the answer by applying an informal process of division, namely, repeated subtraction. Because the problem can be solved on different levels, its elasticity is increased. Not only can the quick students solve the problem, but the slower students can as well on a lower level, reducing the 'all-or-nothing' character of assessment

By giving the students more latitude in the way they approach the problems, the contexts further increase the transparency of the assessment. In bare number problems, the operation to be performed is generally fixed in the problem in question, to verify whether students are able to perform certain procedures that they had learned earlier. For this reason, bare problems are mostly unsuitable for advance testing. One cannot present a long division problem to a student who has never done one and expect to find footholds for further instruction.

An example of an assessment problem in which the context makes the problem transparent and elastic is the *Polar bear* problem (Figure 8).

This problem was given to third-grade students when the algorithm for long division had not yet been introduced. The

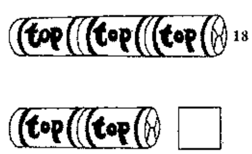

Figure 4: Candy problem

Jim has 16 marbles and wins 10 more

How many does he have now?

Figure 6: Marbles problem

The butcher has 16 pounds of ham in his shop and orders 10 pounds more.
How much does he have now?

Figure 7: Ham problem

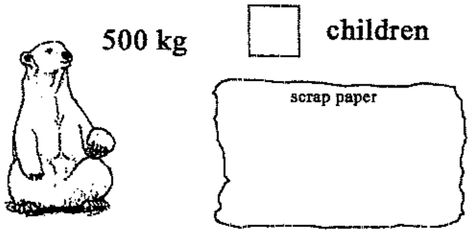

Figure 8: Polar bear problem

problem was particularly meant to support the teacher in looking forward in search of footholds for further instruction. Although at first view it is not very realistic, the children know this type of question from quizzes in television programs. So, the question is quite clear for them and they can imagine the situation. The latter and their natural curiosity help them to find, at least, a start of a solution. The advantage is that the problem does not give an indication about what kind of mathematical procedure must be carried out. A broad range of strategies is possible. Moreover, the students can influence the difficulty level of the calculation by choosing either a rounded off or a more precise weight to work with. So students have ample room for their own constructions.

As can be seen in the student work on the pieces of scrap paper (see Figure 9) this context problem reveals a lot about the students' knowledge and their thinking. The transparency and the flexibility of the problem brought in by the context provide the teacher with strong footholds for further instruction.

Contexts suggest strategies

A third important aspect of contexts in assessment (assuming they are chosen well) is that the contexts can suggest strategies. The latter is, for instance, the case in the *Bead* problem (Figure 10)

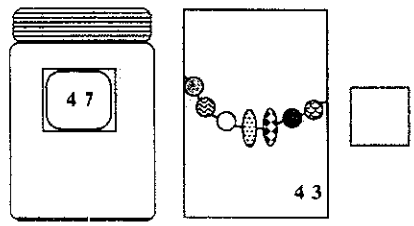

Figure 10: Bead problem

In this problem, the context prompts the students to find the number of beads left in the jar after they have used 43 beads for making a necklace; not by calculating 47 - 43, but by adding on from 43 until 47 is reached. As will be shown later in this article, this strategy-providing function of the context influences the students' achievements remarkably.

By imagining themselves in the situation to which the problem refers, the students can solve the problem in a way that was inspired, as it were, by the situation. Sometimes this will mean that the students use an accompanying drawing in a very direct way as a kind of model (see Figure 4), while, at other times, it is the action enclosed within a given context that elicits the strategy (see Figure 3). How close the students stick to the context with their solution depends upon the degree of insight and the repertoire of knowledge and skills they possess.

This role of strategy provider is not only important with respect to expanding the breadth of assessment problems and the potential this creates for advance testing, but it touches the core goal of RME as well: the ability to solve a problem using mathematical means and insights – an essential element of which is formed by the ability to make use of what the contexts have to offer.

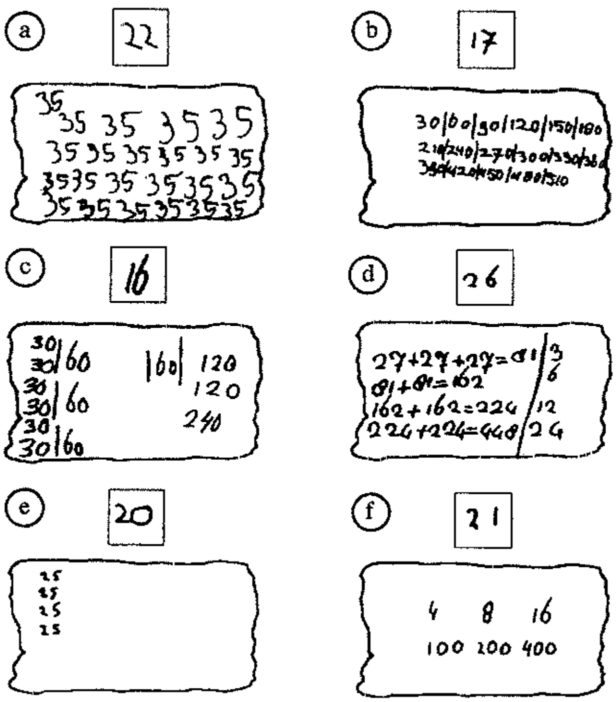

Figure 9: Students' work on the Polar bear problem

The context matters

Numerous research projects have confirmed how important it is that students make sense of the situation that is at hand in a problem, and how contexts can contribute to this sense making.

Pioneering work in this area was conducted by Donaldson, who, in collaboration with McGarrigle, demonstrated how context-determined the results of Piaget's conservation experiments were (see Donaldson, 1978). Through a slight alteration of the context (a mischievous teddybear shifted the blocks instead of the researcher), the number of conservers suddenly increased in comparison to the original version of the experiment.

A similar confirmation of the influence of context was provided by Carraher *et al.* (1985). These authors showed that many children who were unable to perform certain calculations at school had no trouble doing so when selling candies at the market.

Research by Foxman *et al.* (1985) showed that some students who did poorly in certain addition problems were even able to multiply and divide when asked to organize a party.

Joffe (1990) points out that, when the same calculations are presented in different situations, the calculations are not interpreted as being similar ones.

Carpenter and Moser (1984) further discovered that young children could solve word problems through informal strategies before they had learned to do bare arithmetic problems.

The same results were found in research conducted by Hughes (1986), in which a group of 60 children, varying in age from three to five, was presented with a number of addition problems - some in a game situation and others in a formal presentation. Depending on the conditions, the differences in achievement were enormous. When an actual box was used, where blocks were put in and taken out, or when this box or a shopping situation was referred to, the achievements were much higher than in the formal situation, where no reference was made to reality.

Context problems versus bare problems on written tests

That the difficulty level of a problem is inflenced by the way in which the problem is presented to children is, in this section, further elaborated by contrasting context problems with bare problems that both deal with the same numbers and operations.

Clements' finding

The discovery made by Clements (1980), with regard to different results in assessment problems, was quite revealing. He compared two text items that were part of the same written test. One item consisted of a bare arithmetic problem and the other of a context problem, but both held the same mathematical content (see Figure 11).

Question 5:	Write in the answer 1 – ¼ = _______ (Answer)
Question 18:	A cake is cut into four equal parts and Bill takes one of the parts. What fraction of the cake is left?

Figure 11: Two parallel problems from Clements (1980)

Of the 126 sixth-grade students, 45% answered the bare arithmetic problem correctly, while 78% found the correct answer to the context problem. The interviews conducted afterwards with the students revealed that they had had support from the imagery in the cake problem. Evidently, it did not matter that the context problem involved more reading, comprehension and transformation.

More examples of revealing alternate presentations

Many years later, before being aware of Clements' intriguing finding, I found similar results when problems were presented both as a context problem and as a bare number problem. Large discrepancies in the achievement scores emerged. Take, for instance, the *Bead* problem (see Figure 10). When this problem was given to second-grade students in November, 60% of the students answered it correctly, while the corresponding bare number problem (47 - 43 =) that was presented at the same time, was only answered correctly by 38%. However spectacular these discovered discrepancies may be, it is what lies behind these differences that is important, rather than the differences themselves. Evidently, the context elicited a different strategy than did the bare number problem. The strength of the context presentation lies in the fact that it provides students with the opportunity to solve problems by using informal strategies that are linked to contexts. As a result, instruction that has not yet been given can be anticipated. It is the opportunity to 'test in advance' that makes the alternate presentation so appropriate for providing indications for further instruction.

Another striking example is the *Comparing height* problem (Figure 3) in which two boys compared their height. This problem was administered to a second-grade class (students aged 7-8 years) in April/May. Although, at the time this problem was administered, problems such as 145 - 138 involving bridging ten above one-hundred had not yet been handled, around 50% of the students were nonetheless able to calculate this difference. In contrast to the *Bead* problem, no corresponding bare number problem was simultaneously administered in this case. We did not want to frustrate the students with a type of problem for which they did not yet know the procedure. Moreover, such a problem would have revealed more about what the students were not able to do than what they were able to do.

In the context problem at hand, by contrast, it did become clear what the students could do. Compared with the bare subtraction problems involving bridging ten under one-hundred (33 - 25 = and 94 - 26 =) that were included on the test because the students had already dealt with them, the score for the *Comparing height* problem (that involved higher numbers) was respectively 12% and 15% higher. Instead of using the (often laborious) subtraction procedures that are usually taught for bare number problems, the students apparently also used the informal strategy of adding-on, which was elicited by the context.

In order to show that these discrepancies between context problems and bare number problems do not only come up in the curriculum domain of the lower grades in primary school, I will conclude this section with an example related to estimation with decimal numbers in which fifth-grade students (aged 10-11 years) are asked to give an estimate of 1.49 x 0.740. The question was asked in two ways: as a bare number problem and as a context problem (see Figure 12).

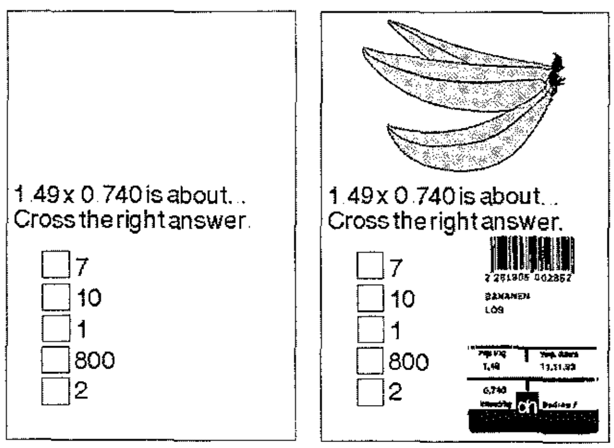

Figure 12: Two versions of the 1.49 x 0.740 problem

I designed this problem for a study that I carried out in 1997.

Again, the results were remarkable. The bare version was answered correctly by only 4% out of the 26 students, whereas 46% of the students came up with the correct answer to the context version of the problem. Again, it was found that the context apparently evoked a different way of reasoning and calculation to find an answer.

Many context issues still to be resolved

The role that contexts play in assessment problems in mathematics is an intriguing topic that again and again has new surprises in store for us and many unanswered questions.

Contexts can hinder finding an answer

Using contexts familiar to the students does not always provide support. Van der Veer and Valsiner (1991) have shown that certain limitations may also ensue from calculating within a given context. When an anthropologist attempted to ascertain how far an ethnic group was able to count and suggested that they count pigs, they counted no further than 60, as having so many pigs was inconceivable to them. Another example, taken from the research of Wood (1988), involves a problem in which candies had to be distributed in such a way that one student would get four more candies than the other. The children to whom this problem was presented refused to do it, however, on the grounds that it was not fair.

Other situations in which the context hindered students working on problems were reported on by Mack (1993) and Gravemeijer (1994). In Mack's study, a student's comment on fraction problems was that she does not like pizza and therefore never eats pizza, but that instead she likes ice cream and if the teacher would change the problem into an ice cream problem it would be easier for her. Gravemeijer describes a similar experience involving a problem in which 18 bottles of coca-cola must be shared fairly by 24 students at a school party. These students refused to interpret the problem as it was intended because, they said, some students did not like coca-cola and, moreover, not everyone drank the same amount.

Students' unwillingness to take into account the context

As is shown by many studies, students often ignore the context entirely. Balloons to be shared fairly are then neatly cut in half (Davis, 1989). Greer (1993), for instance, discovered great differences between achievement on straightforward items, where no limitations are presented in the situation referred to by the problem, and more complex items which had such limitations. The fewest errors were made on items of the first type. The complex problems, by contrast, were often answered using stereotypical procedures that assumed a 'clean' modelling of the situation in question. The students' expectations of how problems should be solved resulted in a lack of openness to potential limitations. They simply took the problem out of its context and solved it as a bare problem. In order to discourage this, according to Greer, students must be confronted with various types of problems and must learn to regard each problem according to its own merits.

Verschaffel, De Corte, and Lasure (1994), who repeated Greer's research along broad lines, also had to acknowledge a strong tendency by the students to exclude real-world knowledge and realistic considerations when solving word-problems in the classroom. It is worth noting, however, that in both these research projects, two problems stood out in a positive sense: the *Balloon* problem and the *Bus* problem. As Verschaffel, De Corte and Lasure remarked, these were the only problems in which the result of the calculation had to be connected to the context of the answer, so that the students would immediately notice an unrealistic answer (for instance, 13.5 buses or 3.5 balloons). Nevertheless, the authors sought the explanation for the low level of realistic considerations in the stereotypical and straightforward nature of the word problems used in the instruction and the way these problems were instructed, rather than in the mathematical structure of the problems.

Context problems that do not allow one to take the context into account

Although there is increasing emphasis on assessment in which the students can demonstrate the application of mathematics with realistic considerations, students who do exhibit this approach are not always at an advantage in terms of getting a high score. Paradoxically, context problems often have to be 'undressed'. Cooper (1992), using a number of test problems from the English national assessment items, showed that the students who were able to marginalize their knowledge of reality had the best chance of answering the problems correctly. One of the examples, the *Lift* problem (Figure 13) involves a lift that has to take 269 people to the upper floors during the morning rush hour. A sign states that no more than 14 people may take the lift at one time. The question put to the students is, "How many times must the lift go up and down?"

The accompanying marking scheme indicates that the only answer that is considered correct is 20, obtained by dividing 269 by 14. Neither 19 nor 19.2 is acceptable. So, if students take into consideration that the lift is not always full, that some persons will decide to take the stairs, and that, for instance, a wheelchair needs to use the elevator, then their answers would be considered incorrect. The difficulty is that there is nothing in the question that indicates that the students may not use their knowledge of reality. Actually, this information is only in the marking scheme! Noteworthy, in Cooper's discussion of this problem, is that he firmly suggested that teachers caution their students to be aware of the practical, real and everyday world, while he does not address the issue of the context-unfriendly marking scheme. After all, if other answers to the problem were also consid-

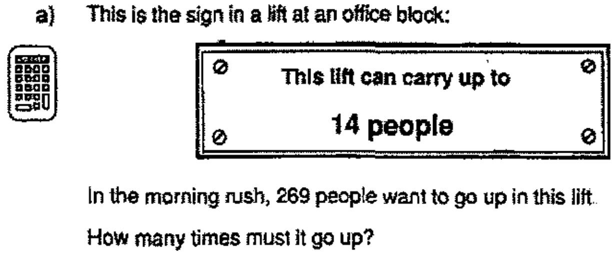

Figure 13: Lift problem (SEAC, 1992)

ered correct, then the students would really have the opportunity to take the context into account. That students truly do this was revealed when Cooper and Harries (2002) did further research into this problem. After the problem was rewritten to encourage a more realistic pattern of responses (by asking the students to give reasons for why particular answers were given by other students), it became clear that the students were very able to apply a more realistic approach to this problem. However, again the marking scheme was not brought into the debate by the researchers.

Taking the context into account is not distributed evenly among students

Another issue that needs attention is that the knowledge of situations appealed to in context problems is not distributed evenly among students from different social backgrounds (*e.g.*, MSEB, 1993) and between female and male students (Meyer, 1992; Boaler, 1994)

Concluding remarks

> The real purpose of emphasizing realistic problems [is] to encourage students to construct and investigate powerful and useful mathematical ideas [...] based on extensions, adaptations, or refinements of their own personal knowledge and experience (Lesh and Lamon, 1992, p 39)

Although Lesh and Lamon made this statement more than ten years ago, it has not lost its relevance The same is true for Boaler's (1993) comment that

> the degree to which the context of a task affects students' performance is widely underestimated (p. 13)

At the same time she is warning us not only to focus on the tasks and their contexts:

> Students' learning is multidimensional and the context is just one of the many facets of the learning environment with which their learning is engaged (*ibid.*)

Being aware of this, it is also true that nowadays a great many issues remain to be resolved about the nature of realistic problems, realistic solutions and realistic solution processes. Based on the findings discussed in this article, my recommendation would be to do more research into the effects of alterations in presentations and comparing context problems with bare problems in order to get a better understanding of these issues However, staying within the subject matter domain of mathematics education will not bring us all the in-depth knowledge we need for this. The context issue cannot be restricted to one single subject. Instead, it should be investigated beyond the boundaries of the content area of mathematics, as is suggested by Kastberg, D'Ambrosio, McDermott and Saad (2005)

Notes

[1] An earlier version of this article was a paper in a symposium, *Assessing mathematical reasoning by embedding tasks in contexts* during the Research Pre-session of the National Council of Teachers of Mathematics, Philadelphia, PA, April 2004

[2] A concise overview of the philosophy and principles of RME can be found in Van den Heuvel-Panhuizen (2001).

[3] 'MORE' is an acronym for *Methoden Onderzoek REkenen-Wiskunde*.

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[The rest of the references can be found on page 23 (ed)]