IDENTITY AS A NEXUS OF AFFECT AND DISCOURSE IN MATHEMATICAL LEARNING

EINAT HEYD-METZUYANIM

> If [...] we adopt the principles of discursive psychology, and base our investigations of the hypothesis that the mind of any human being is constituted by the discourses that they are involved in, private and public, we need not despair of making an empirical study of personhood, of the nature of individual selfhood, just because the sense of identity is 'subjective'. We can turn to study how a person's sense of being uniquely located is discursively displayed. We can study how selfhood is produced discursively. (Harré & Gillette, 1994, p. 104).

Discourse and self, though figuring side by side in the above quote and in many other discursively oriented studies of recent decades, are still often treated in mathematics education research from within very different theoretical frameworks and methodologies. Self related concepts, such as beliefs, attitudes and emotions are mostly studied under the umbrella of 'affect', while discourse is usually studied within groups that focus on language. My personal experience of the interface between the two bodies of research has emerged from having been part of two groups working on literature reviews of seemingly disparate bodies of literature in mathematics education. One was a review of research of classroom discourse (Herbel-Eisenmann, Meaney, Bishop & Heyd-Metzuyanim, 2017), the other an overview of research on identity (Hannula *et al.,* 2016). This work, in addition to my own research that examines emotional and social aspects of mathematics learning, led me to see the overlaps as well as some underlying theoretical gaps between literature on affect, discourse and identity. In this article, I see the concept of identity as a nexus that enables examining affect (including beliefs, attitudes, emotions and motivation) and discourse at the same time. I will review how some exemplary studies in the field have attended to affect and to discourse in their definition of identity. I will then offer a bridge between a discursive definition of identity (Sfard & Prusak, 2005) and the idea of 'self' as it appears in the cognitive strands of social psychology, by building on Bamberg's conceptualization of identity formation (Bamberg, 2011; Bamberg & Georgakopoulou, 2008) as achieved within interaction and in relation to dominant cultural narratives. This will be followed by a short example from my previous work that will show the usefulness of such an interactive view of identity formation.

Background

Generally in the field of mathematics education, studies on affect have focused on inner mental constructs such as *beliefs* (including beliefs about self and about mathematics) *attitudes, emotions,* and *values*. These studies are mostly concerned with the subjective experience of the individual, or what has been termed in the opening quote the 'selfhood'. Studies focusing on discourse, on the other hand, have focused on conversational structures, linguistic capital, teaching-learning interactions and other features of communication and talk, mostly in the classroom (Herbel-Eisenmann *et al.,* 2017). These studies are usually focused on the *structure* of the activity, including its social and mathematical aspects. At first sight, not only do the two domains seem to be focusing on very different phenomena, the constructs used for investigation are inherently different. While those of affect related literature are inner mental constructs, those of discourse literature pertain to the public and accessible sphere. Yet if one follows the reasoning of discursive psychology (as exemplified in the opening quote), selfhood itself is discursively constructed. Beliefs do not just exist in one's mind, they are internalized versions of culturally produced narratives. Even emotions, which are often thought to be purely subjective pheomena, are in fact shaped by discourses and their expression is governed by cultural rules (Harré, 1986; Lutz & White, 1986). However, discursive psychologists do make a clear point of avoiding the use of inner mental constructs such as beliefs or emotions as *causal* factors for certain behaviors. This avoidance is rooted in discursive psychology's rejection of the metaphors of physical sciences (such as a billiard-ball mechanics) as a basis for social sciences. To put this claim simply, humans do not act as physical objects, and therefore the discourse of physical sciences should not be applied to them. I will take up this point of disparity between discursive and cognitive psychology in the concluding part of this article, as it presents some nontrivial challenges to those wishing to build on the two strands of literature in mathematics education. For now, however, I will turn to areas of middle ground between studies of affect and studies of discourse. The term 'identity' has been turning up more and more in these middle grounds. Some researchers using it, such as Nasir (2002), seem to intentionally blur the divide between the affective, subjective experience and the social structure: "I view identity as a fluid construct, one that both shapes and is shaped by the social context. Indeed, identity is not purely an individual's property, nor can it be completely attributed to social settings." (p. 219). Others explicitly include affective concepts in their definition. For instance Bishop (2012) defines: "An identity also encompasses ways of being and talking; narratives; and affective components such as feelings, attitudes, and beliefs" (p. 38).

An attempt to unite the individual and discursive aspects of identity has been made by Sfard and Prusak (2005) who defined identity as a "collection of stories [...] that are *reifying, significant* and *endorsable*" (p. 16). This definition has been attractive in its operationalizability and many of the subsequent works on identity in mathematics education have cited it. Most importantly, Sfard and Prusak's definition, theorizing both mathematics and identity building as discursive activities, offers a much-needed bridge between the individual and the mathematics with which she is engaged.

However, Sfard and Prusak's definition has not been received without objection. One of the blurry points seems to be the definition of narrative or stories. Though Sfard and Prusak qualify them as "reifying, endorsable and significant", what is actually a story and what may be simply stated as an attribute of a person remains unclear (Juzwik, 2006). Sfard (2006), in her response to Juzwik, explains that simple statements and attributes can be considered as narratives because they are reifications of "folded up" (p. 25) temporal sequences of actions. However, the inclusion of such a wide array of communicational actions (starting from attributes such as "he is smart" and reaching full life stories) may lead to more confusion than clarity. On the other hand, for the purposes of studying identity in relation to *learning,* the relatively static, fully articulated narratives such as those given in autobiographical accounts and considered by Juzwik as "stories" are less relevant. What is needed is theorizing of the process of *identity construction* within learning activities.

In my previous work I have used the term "subjectifying" to denote the activity of communicating about people, which is not necessarily reifying and significant, thus does not automatically fall under "identifying" (Heyd-Metzuyanim & Sfard, 2012). This distinction has aided me in differentiating ongoing classroom talk to mathematizing (talk about mathematical objects) and subjectifying (talk about people). Linked to Sfard's (2008) "commognitive" concepts for describing mathematical discourse, namely, words, narratives, routines and visual mediators, the addition of the "subjectifying vs. mathematizing" distinction was the first step towards articulating how students engage in mathematical discourse in ways that both enable them to develop it (that is, learn) and concomitantly construct their identities as mathematics learners. The refinement of this conceptual tool-set is the goal of the present work: I seek to offer conceptual tools for connecting between the individual, including their discourse about self or the "sense of self" as constructed in their discourse (Bamberg, 2011) with the discourse in which they are engaged. For this, I now turn to positioning analysis as a tool for extracting identity construction activity.

Positioning analysis as a tool for extracting identifying activity

Positions, according to Positioning Theory (Harré & van Langenhove, 1999), are sets of rights and duties, both restricted by the discourse in which the individual participates, and taken up by the individual as an agentive actor. *Positioning* or the act of taking up these sets of rights and duties, involves alignment with storylines, or master narratives. Positioning, and through it identity construction, is a reflexive process. This reflexive process is unearthed by Bamberg (2011) with the tool of "positioning analysis":

> Positioning analysis [...] studies how people as agentive actors position themselves—and in doing so become positioned. This model of positioning affords us the possibility of viewing identity constructions as two-fold: we are able to analyze the way the referential world is constructed, with characters (such as self and others) emerging in time (then) and space (there) [...] Simultaneously, we are able to show how the referential world (of what the story is about) is constructed as a function of the interactive engagement, where the way the referential world is put together points to how tellers "want to be understood"; or more appropriately, how tellers index a sense of self. (p. 10)

Thus, the reflexivity of positioning is unearthed by both looking at the story told by the person "as is" (its ideational aspect) and by looking at how the person places herself in the story, or what this story tells about her.

Though Bamberg and his colleagues (Bamberg, 2011; Bamberg & Georgakopoulou, 2008) use positioning analysis mainly to examine how children and adolescents position themselves in the social world (namely, the world populated with people), there is no reason not to appropriate this analytical form for the sake of examining how students locate themselves in relation to the mathematical discourse (as well as the social world). Before I elaborate on this point, I will briefly explain Bamberg and Georgakopoulou's (2008) three level analytical process.

The first level of analysis pertains to the nature of the characters and the way they are positioned in the story. For Bamberg, these are often stories about other people, which through their particularities reveal the authors' stance or moral position in relation to the people described in the story. In my work, I extend this analysis to the stories authored about mathematical objects. These have an important role in the position the student takes up in relation to the mathematics she is doing. For example, a student who has not mastered the discourse of fractions, may talk about the multiplication of fractions as a series of rigid calculations (*e.g.,* multiplying denominator times denominator *etc.*) thus positioning herself as a vigilant follower of prescribed routines. In contrast, a student who has fully mastered this discourse would talk about $\frac{2}{3}\times 9$ as 'being 6' because of how things *are in the world.* For example, she may explain that a third of 9 *is* 3 thus two thirds *are* 6. In this manner, she would detach herself from the mechanics of calculations, and position herself as reporter of the state of affairs in the world.

The second level offered by Bamberg deals with how the narrator positions herself or is positioned by others within the interactive situation. This is the "interactional work", the way the narrator wishes to be viewed by others. I have termed this in my previous works as "indirect identifying" (Heyd-Metzuyanim & Sfard, 2012): the activity of eliciting identifying narratives about oneself in others. The third level has to do with how the speaker positions herself in relation to the dominant discourse or master narratives. Within it, one can analyze the penetration of culture and widely held narratives into the individual's identity.

Applying this three-level lens to the world of students engaging with mathematical problem solving, I will use each of these levels to look at both the location of the speaker with relation to other subjects/people, which I term 'subjectifying' and the location of the speaker with relation to the objects, routines and mediators making up the mathematical discourse, which broadly falls under the 'mathematizing' activity. In what follows, I briefly exemplify an excerpt of talk and its analysis according to this analytical framework.

Positioning in a group of four seventh-graders

The excerpt below was taken from a lesson with a group of four seventh-grade boys studying with me on a weekly basis in an out of school setting (see Heyd-Metzuyanim, 2011, for an elaboration). The boys were all studying in an accelerated mathematics program [1] at their school, and arrived for additional mathematical 'enrichment' in my course. In my analysis, I will concentrate on Amir, who was particularly dominant in this group. Amir and the other boys were all good friends, and came to the five months long course because of their enjoyment of mathematics and of being with each other. The excerpt here was taken from the middle of the third lesson of the course, where the following problem was introduced, after discussing some other algebraic pattern-related problems:

> A car drove for 4 hours. During each hour the car covered 12 km more than in the hour before. During the third hour it covered 78 km. In total, how many km did it cover?

In the transcript {text} denotes overlapping speech. Inferred words are shown in parentheses (thus). The students and the teacher are sitting around a square table. Yoram and Amir are sitting on one side, facing Gil and Ram. The teacher sits at the head of the table, closest to Yoram.

1 *Teacher*	*[Reads the whole question, rather quickly. The students all read silently together with her]* In total, how many (kilometers) did it cover
2 *Amir*	Fifty four
	Yoram makes an unintelligible comment, the teacher laughs
5 *Amir*	In the fourth hour it covered ninety kilometers
6 *Yoram*	No
7 *Amir*	*[Writes in his worksheet]* so it's ninety plus seventy eight. Surely there's some formula [2] for it but {I don't feel like doing (that)} [3]. *[Writes]*
8 *Yoram*	{Yes, exactly} ah no! {I know}
8a *Amir*	{seventy eight}
9 *Teacher*	{All things} have a {formula}
10 *Yoram*	{I know}
10a *Teacher*	but first one has to find {the individual case}
10b *Yoram*	{it's plus eh *[pause]* twelve} *[pause]* times four *[pause]* {forty eight}
11 *Amir*	{Fourteen plus four} eight, {eighteen *[pause]* twenty three plus fifteen}
12 *Teacher*	{Amir and Yoram will you include us (in your conversation)? because we're not following you}
13 *Amir*	*[raises his voice]* two hundred eighty {eight kilometers}
14 *Yoram*	{It's a pity} you're not following
15 *Teacher*	*[Laughing]* really, that's not OK
16 *Amir*	That's it. I'm done! *[Straightens his back, wipes his mouth with his arm]*
17 *Teacher*	What? Wait. *[To Gil and Ram]* Did you follow him?
18 *Yoram*	No.
19 *Ram*	Uh-uh
21 *Yoram*	Why should we follow?
22 *Amir*	It was too simple *[smiling]*
23 *Yoram*	*[Looks at Ram, shrugs his shoulders]* It's without a formula. {It's not unique.}
24 *Amir*	{*[Sings]* sometimes I feel too } *[hums]*

In the following analysis, I will concentrate on Amir and the way he positions himself and is positioned by the other participants thereby constructing his identity as a "mathematically talented boy".

Level 1 positioning—the objects referred to and described through the narration

With regard to the human subjects involved, and specifically to Amir, there is not much text in this excerpt. Amir does not directly talk about himself, except when claiming "I'm done!" [turn 16]. The other participants do not talk about Amir either, except the Teacher who asks "Amir and Yoram" to "please include us" [turn 12] and some deictic markers referring to "you" (directed at Gil and Ram) and "him" (Amir) [turn 17].

The mathematical objects talked about in this excerpt are mostly numbers ("ninety", "two hundred eighty eight"). These serve as both standalone objects (as in "twenty-three plus fifteen") and as qualifiers relevant to the task ("two hundred eighty eight kilometers"). Also figuring in the excerpt are routines of calculation such as "four times twelve" and "ninety plus seventy eight". These are talked about as a naturally given, almost automatic derivation of the story about the car, as can be seen by Amir's "so it's … " [turn 7], the "it" referring to *the appropriate routine to solve this problem*. This "it's" and the absence of any verbs in Amir's talk, imply that there is no human agency involved in the choice of the routine or in its performance and that Amir is simply 'revealing' the mathematics underlying the task. Such alienated talk has been termed by Sfard (2008) as 'objectified' talk and is often the hallmark of skilled mathematical discourse. Moreover, the routines used are qualified as "too simple" [turn 22] by Amir and as "not unique" by Yoram [turn 23] implying that there are other routines (unique and not simple) that would be worthier of discussion. Another object figuring in Amir's talk is the elusive "formula" [turn 7] that "surely" can be connected to the problem, but that Amir does not "feel like" [turn 7] finding. Thus its existence is hinted at but not examined further. The important point to notice here is that level 1 positioning analysis shows that Amir is positioning himself in a very particular way with respect to the mathematics at hand, despite its seemingly neutral, 'cognitive' appearance. He is not just telling a story about the mathematical objects, he is also telling a story about himself. This story could be summarized as *reporter of the state of affairs* in the world.

Level 2 positioning—how the subjects position themselves in relation to others and to the mathematics

Through his hasty calculations and refusal to let others into his calculational talk, seen in his ignoring of the teacher's request to "include us" [turn 12] and in his disregard of Yoram's alternative claims [turns 8, 10], Amir positions himself as competing in a race with the other boys. Interestingly, the other boys do not challenge this position, neither do they try to join in the race. Even Yoram, who at the beginning attempts to offer an alternative routine (multiplying 12 by 4 and thereby perhaps "finding a formula"), contents himself from turn 13 onwards with following Amir's calculations silently, affirming them (as true but "not unique" [turn 23]) and aligning with Amir by half humorously commenting that "it's a pity" the teacher, Ram and Gil are not "following" [turn 13]. Amir's positioning of himself as winning the race is strengthened by his declaration "I'm done!"[turn 16], complemented by a decisive wipe of his mouth. Importantly, this is done with a mischievous smile and while keeping eye contact with his friends across the table. These non-verbal signals mitigate the potential negative effects of his "racing" and superiority, which may explain the silent approval of his friends.

The teacher, on her part, collaborates in positioning Amir as the "winner" of the race. Initially, she groups Amir with Yoram as solving together and shutting everyone else out ("Amir and Yoram, will you include us?" [turn 12]). Once Amir declares "I'm done!" [turn 16], the teacher moves to position him as the sole winner, by asking the other two boys if they "followed him" [turn 17] (and not "them"). Her humorous tone of voice indicates an appreciation of Amir's quickness, which while highlighting the identity of Amir as successful, simultaneously positions the others as slower and perhaps less smart. This positioning of Amir as superior to others is established even further by the teacher expression of her own confusion ("what? wait" [turn 17]), which aligns her with the other, supposedly confused participants and positions Amir as the sole solver of the problem.

Amir's positioning of himself as superior to others is amplified by the position he takes up (together with Yoram) in relation to the mathematical routines he is employing. These are described by both boys as "too simple" [turn 22] and inferior to "a formula" [turn 23] which would have been in Amir's reach had he "felt like" finding it [turn 7].

Level 3 positioning—dominant discourses or master narratives

The level 1 and level 2 analysis prepare the ground for examining how the students position themselves with relation to stable and long-lasting narratives in their culture and society. In our case, these pertain to two aspects: narratives about what mathematics is as a discipline, and narratives about what it takes to succeed in it. With relation to the mathematics, Amir, assisted by Yoram, positions himself as engaged with easy or "too simple" arithmetic routines, implying that "real" mathematics (or worthwhile mathematics) is concerned with more general narratives, here referred to as "formulas" but most likely pertaining more generally to algebra. This hierarchy of arithmetic being less valued than algebra reflects the dominant discourse of school mathematics where arithmetic is learned at lower grades than algebra, and the stage at which the boys were located (7th grade, accelerated) where algebra was the top domain of the curriculum.

With relation to narratives about what makes one a successful mathematician, Amir aligns himself with the story that mathematical success (or talent) is about solving problems correctly, quickly and alone. As documented by many before (*e.g.,* Schoenfeld, 1989), these narratives are commonplace in students' declared beliefs about mathematical success, indicating Amir's position is aligned with the dominant discourse of mathematics success and talent.

Discussion

As the wider analysis of Amir's case revealed (Heyd-Metzuyanim, 2011), Amir was identified by his parents and by other significant narrators (psychological authorities and educational institutions) as a 'gifted' boy. He also identified himself as very successful in mathematics, despite having some minor and local difficulties with certain aspects of mathematical learning (especially writing and doing homework).

Yet such identity stories about Amir, as interesting as they may be, do not enable us to see how this identity was constructed and reconstructed time and again in his interactions both within the mathematical discourse and with the human participants around him (including the teacher and his fellow students). It thus leaves in the dark the *process* by which such an identity interacts with mathematical activity. The above three-leveled analysis of Amir's positioning within the people-populated and the mathematical-objects world shows us that within this activity of identity construction, both subjectifying actions (such as implying one is quicker and smarter than others) *and* mathematizing actions (such as constructing mathematical objects as existing "simply" of themselves) are important.

The possibility of looking simultaneously at the positioning of the student (that is, the story he is telling about himself) with relation to the mathematical objects, together with his/her positioning with relation to other participants, offers the link sought in this article between the selfhood, often talked about in terms of beliefs, attitudes and emotions, and the discourse within which these affects are produced. To make this link clearer, Table 1 summarizes the three levels of positioning both in relation to the analytical method and in relation to common methods for the study of discourse and affect in mathematics education.

As Table 1 shows, identity, as resulting from the ongoing accretion of positioning acts, can be related both to the commognitive method (Sfard, 2008) for analyzing mathematical talk and to psychological methods designed to elicit explicit narratives about the self. As such, it brings into full relief the ongoing processes by which not just subjectifying but also mathematizing activity continuously construct mathematical identities. The table's analytical power lies not only in the strength of partitioning activity into the various categories, but also in the potential to connect the table's rubrics with commonly used tools in the fields of discourse and affect in mathematical learning, such as classroom discourse analysis, interviews and self-report questionnaires (see the bottom part of each table cell). Therefore, the construction of identity, as conceived from Table 1, can serve as a nexus for *researchers*' discourse about the selfhood (or affect) and about discourse in the mathematics classroom.

Summary and some meta-level notes

My aim in this article was twofold: at the theoretical-empirical level, I was offering identity as a conceptual tool for studying affect and discourse interconnectedly, showing how using the tool of positioning analysis may aid in such an attempt. At this level, my work extends recent attempts by socio-culturally and socio-historically oriented scholars (*e.g.,* Radford, 2015; Skott, 2015) who have shown that viewing affect-related phenomena from a purely cognitivist, individualistic orientation limits our understanding of these phenomena. In particular, I used the lens of discursive psychology to show that phenomena which traditionally would have been analyzed using socio-cognitive psychological concepts, can be shown to be discursively constructed. For example, Amir could have been labeled as having 'high self-esteem' in mathematics, as holding certain beliefs about mathematics or as enjoying problem solving. Yet as shown in the positioning analysis, these beliefs and emotional expressions were constructed in a certain social context, in relation to wider cultural narratives. Therefore, identity—as an accretion of positioning actions resulting in a collection

Table 1. Three levels of positioning and their relation to tools tapping affect and discourse.

	Subjectifying	**Mathematizing**
Level 1 positioning	A factual description of people's actions and feelings. "I failed the test", "I know the answer", "this is boring", "I hate long computations". Self-efficacy questionnaires may tap this when asking specific questions about past actions (*e.g.,* "I usually succeed in solving word problems"), though the meaning of "success" is sociocultural, thus pertaining to level 3.	What the mathematical objects are, which routines are used, what are the expressed mathematical narratives. "it's 54", "it's 4 times 12" Commognitive analysis (Sfard, 2008) captures this relation through the "alienated" vs. "process oriented" talk about mathematical objects and routines
Level 2 positioning	How the participants are positioning themselves with relation to each other. mostly achieved through implicit identifying. The concept of "Engagement structures" (Goldin, Epstein, Schorr, & Warner, 2011) is highly overlapping with this level.	How the subjects position themselves with relation to other people who preform mathematical routines (*e.g.,* quicker, more sophisticated, accurate). Commognitive analysis (*e.g.,* Heyd-Metzuyanim & Sfard, 2012) captures this through the analysis of subjectifying and mathematizing actions.
Level 3 positioning	Who the subject is in relation to common narratives about doing mathematics. *e.g.,* "Clever people solve math problems fast" Self-reports (such as attitude questionnaires) often tap this rubric, for instance with items that pertain to success or failure, since those are defined by the culture.	What is the mathematics that the student is engaged with? How is it delineated (*e.g.,* "fractions", "decimals")? What is considered by the student as worthy, difficult or beautiful? Self-reports and questionnaires on beliefs about mathematics often tap this level.

of narratives about the self—serves best to capture both the selfhood and the mathematical interactions by which it is constructed.

Yet there is another aim to this article—a ‘meta-goal’ it should perhaps be called—and that is to turn the community of researchers’ attention to identity as being a possible nexus—or ‘meeting point’—for scholars studying affect and scholars studying discourse.

This latter goal brings up some epistemological and ontological obstacles lurking behind attempts to link studies of the self and studies of discourse. The main obstacle pertains to the unit of analysis, and its ontological and epistemological status. In other words, what we are looking at when we study the self in the activity of mathematical learning, and how we may come to know it. Is it inner mental constructs, inaccessible to our senses and only theorized as existing by psychological research, or is it public, communicational activity, to which one may ‘point’ clearly? And if those are mental constructs (or selves) that we look for, how are those related to the publicly accessible communication that these selves produce? As the quote at the beginning of this article reminds us, “we need not despair of making an empirical study of personhood”. Yet the study of this personhood has been historically conducted by communities studying affect and discourse (which can be largely, at the risk of over-simplification, mapped to the disciplines of psychology and sociology), using very different methodological tools as well as underlying theoretical assumptions. From my personal experience, concepts such as ‘attitudes’ and ‘beliefs’ do not map neatly onto concepts such as ‘narratives’ or ‘discourse’. Neither is it simple to interpret results of studies conducted via quantifiable measures of self-report surveys, within socio-cultural and situated theories of learning. The question of *(in)commensurability* is eventually inescapable when attempting to bridge between research stemming for cognitivist and discursively oriented research. Are these gaps impassable? For example, can research on attitudes towards mathematics inform research on how such attitudes are socially constructed? And vice-versa—can research on the discursive formulation of identity narratives inform research that collects statistical measures of beliefs and attitudes? I pose this as a challenge to our field, to start inquiring into these possibilities.

Notes

[1] In Israel, this was called at the time of study a “Mofet” classroom and was a class where students were chosen according to entrance exams and previous grades in elementary school.

[2] Hebrew: *Nuskhah*, literally ‘formula’ or ‘equation’. Here the students mean by it an algebraic expression of some sort.

[3] Hebrew: *Ein li ko’ach*, literally ‘I have no energy’. A colloquial way of saying ‘I don’t feel like it’, or ‘it makes me tired’.

References

Bamberg, M. (2011) Who am I? Narration and its contribution to self and identity. *Theory & Psychology* **21**(1), 3–24.

Bamberg, M. & Georgakopoulou, A. (2008) Small stories as a new perspective in narrative and identity analysis. *Text & Talk* **28**(3), 377–396.

Bishop, J. (2012) “She’s always been the smart one. I’ve always been the dumb one”: identities in the mathematics classroom. *Journal for Research in Mathematics Education* **43**(1), 34–74.

Hannula, M.S., DiMartino, P., Pantziara, M., Zhang, Q., Morselli, F., Heyd-Metzuyanim, E., Lutovac, S., Raimo Kaasila, R., Middleton, J.A., Jansen, A. & Goldin, G.A. (2016). Attitudes, beliefs, motivation, and identity in mathematics education. In Hannula, M.S., DiMartino, P., Pantziara, M., Zhang, Q., Morselli, F., Heyd-Metzuyanim, E., Lutovac, S., Raimo Kaasila, R., Middleton, J.A., Jansen, A. & Goldin, G.A. (Eds.) *Attitudes, Beliefs, Motivation and Identity in Mathematics Education: An Overview of the Field and Future Directions,* pp. 1–35. London: SpringerOpen. https://doi.org/10.1007/978-3-319-32811-9_1

Harré, R. (Ed.) (1986) *The social construction of emotions.* Oxford: Basil Blackwell.

Harré, R. & Gillett, G. (1994) *The Discursive Mind.* Thousand Oaks, CA: Sage Publications.

Harré, R. & Van Langenhove, L. (Eds.) (1999) *Positioning Theory: Moral Contexts of International Action.* Oxford: Blackwell.

Herbel-Eisenmann, B., Meaney, T., Bishop, J.P. & Heyd-Metzuyanim, E. (2017) Highlighting heritages and building tasks: a critical analysis of literature on mathematics classroom discourse. In J. Cai (Ed.) *Compendium for Research in Mathematics Education,* pp. 142-185. Reston, VA: NCTM.

Heyd-Metzuyanim, E. (2011). *Emotional aspects of learning mathematics - how the interaction between identifying and mathematizing influences the effectiveness of learning* (Unpublished Dissertation). Haifa: University of Haifa.

Heyd-Metzuyanim, E. & Sfard, A. (2012). Identity struggles in the mathematics classroom: on learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research* **51–52,** 128–145.

Juzwik, M. (2006) Situating narrative-minded research: a commentary on Anna Sfard and Anna Prusak’s “Telling Identities.” *Educational Researcher* **35**(9), 13–21.

Lutz, C. & White, G.M. (1986) The Anthropology of Emotions. *Annual Review of Anthropology* **15**(1), 405–436.

Nasir, N.S. (2002) Identity, goals, and learning: mathematics in cultural practice. *Mathematical Thinking and Learning* **4**(2), 213–247.

Radford, L. (2015) Of love, frustration, and mathematics: a cultural-historical approach to emotions in mathematics teaching and learning. In Pepin, B. & Roesken-Winter, B. (Eds.) *From Beliefs to Dynamic Affect Systems in Mathematics Education,* pp. 25–50. Berlin: Springer.

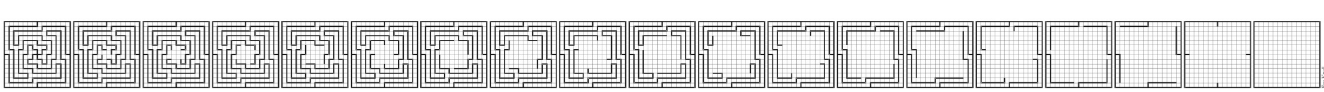
Schoenfeld, A.H. (1989) Explorations of students’ mathematical beliefs and behavior. *Journal for Research in Mathematics Education* **20**(4), 338–355.

Sfard, A. (2006) Telling ideas by the company they keep: a response to the critique by Mary Juzwik. *Educational Researcher* **35**(9), 22–27.

Sfard, A. (2008) *Thinking as communicating.* Cambridge UK: Cambridge University Press.

Sfard, A. & Prusak, A. (2005) Telling identities: in search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher* **34**(4), 14–22.

Skott, J. (2015). Towards a participatory approach to “Beliefs” in mathematics education. In Pepin, B. & Roesken-Winter, B. (Eds.) *From Beliefs to Dynamic Affect Systems in Mathematics Education,* pp. 3–24. Berlin: Springer.



An algorithm for producing mazes with 4-fold symmetry.