Fork in the Road

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We met in Recife, Brazil at a Psychology of Mathematics Education (PME) conference. It was July 1995 and DR was about to take up a post as a mathematics teacher educator in Canada after completing a Ph.D., while LB was an experienced teacher of mathematics teachers in the UK, whose background was teaching secondary mathematics. The following two extracts illustrate how we started telling each other stories about our practice over email

Extracts from e-mail 1: Raising an issue – right/wrong

> LB: When interviewing people for the Post-Graduate Certificate in Education course (our one year, post-degree programme), I ask about how the interviewee's images of mathematics have changed through their own education and schooling. Often this reveals in a particular person an association of mathematics with the security of getting 'ticks' – "it's a subject where it's possible to know that you're right" – and this is seen as a good feeling. When they start university, however, everything goes so fast that the only response is to rote learn the material. They do not feel in control.
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> DR: On the first day of teaching the course, I invited one of my students to lead a sharing of reasons for wanting to become teachers of mathematics. One thing that struck me was the response of many of them, that they want to teach mathematics because they can't write. They were distressed, after reading the outline, that my course includes a fair bit of writing as that didn't fit their experience of what mathematics was about. The idea that a course in teaching mathematics should include 'creativity' as an evaluation component disturbed them as well.
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> I suspect that one element in this is their belief that in mathematics there is no room for creativity in answers. They chose mathematics as their field of study because the answers were always clearly right or wrong. Writing cannot be evaluated by a simple right or wrong. It's a more complex, multi-dimensional continuum of better <—> worse. This comes up again later in the course when we consider the subjectivity of marking. I suspect that they will come to that class with a belief that marking in mathematics is objective, because of this right/wrong character they believe mathematics has.

These sorts of exchanges led us to see that we were noticing examples of the dichotomy right/wrong in students' images of mathematics as they started to work at becoming teachers. Having ourselves gone through this transition from mathematician to mathematics teacher, we recognise that as mathematicians it was important for our world to be sharply divided into disjoint categories. Much of mathematics, for example proof by contradiction, explicitly requires that mathematical statements be clearly right or wrong.

Categorising into disjoint sets is such a part of us as mathematicians that learning not to do so in teaching contexts was an important step in our own transitions to becoming teachers. As teacher educators, we recognise that helping our students to find ways out of the dichotomies that worked well for them as mathematicians is an important part of their becoming teachers. Even in considering this transition, the mathematician's habit of mind comes into play, for we need to resist the apparent implication that 'mathematician' and 'mathematics teacher' are disjoint categories. We have encountered mathematicians who are not mathematics teachers, and mathematics teachers who are not mathematicians, but place ourselves in the intersection of the two sets

Copes (1982) has experienced students of mathematics in a similar position and reports finding the Perry Development Scheme useful in supporting his communications with them. He uses a condensed version of Perry's continuum of positions (world-views) which people may move through over time, citing four categories starting at one extreme with dualism:

> a person believes that every question has an answer, that there is a solution to every problem, and that the role of an authority is to know and deliver those answers (p. 38)

and the third of which is relativism:

> [a person has] come to see several reasons that not all opinions are equally good' and 'validity depends upon context'. (p. 38; see also Belenky, Clinchy, Goldberger and Tarule, 1986)

The dilemma for the teacher educator is that:

> We cannot communicate effectively with dualistic students about relativism [...] The only solution to this dilemma is to help students change their conception of the world to one that is sophisticated enough to deal with more complexity. (p. 39)

We do not have a view of mathematics as only producing right or wrong answers. If it is true that 'as you define your subject so you will teach it' (Davis, 1986, p 355), then it seems important for someone who is teaching mathematics to be able to extend their views so that they can broaden their range of teaching approaches to work successfully with the students they are teaching. For instance, Buerk (1982), in teaching bright women who were mathematics avoiders, identified that they needed to be taught relativistically not dualistically.

The raising of the right/wrong dichotomy through our e-mail conversations, trading stories of practice, led to a continuing interest in exploring dichotomies We recognised them in our reading and in our practice, noticing them seemingly everywhere, which had the effect of making us more aware of our own awarenesses and the related behaviours of our students. We started to ask what we might do as teacher educators to support ourselves and our student teachers in finding ways out of dichotomies.

Extracts from e-mail 2: Avoiding extremes

> LB: It's not only in their images of mathematics that I have this sense of students needing to adapt from a previously held position which is affecting their teaching. I'm forwarding an extract from an article which I wrote with a colleague John Hayter which will give a sense of how we identify students who are failing on the course Our image, developed with the teachers who act as mentors for our students in the school, is about becoming stuck in what could be a perfectly acceptable starting behaviour without adapting to become more flexible

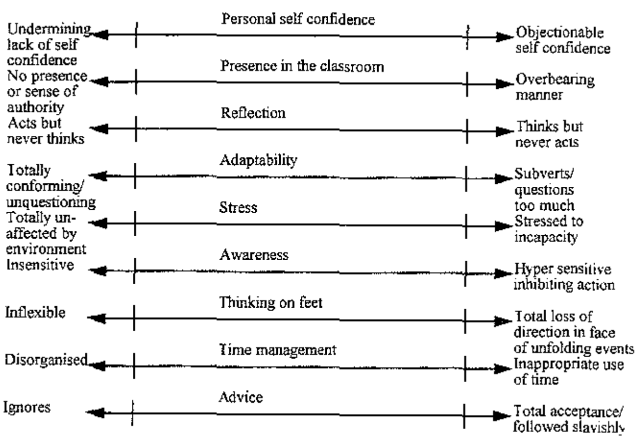

Figure 1

> Most student teachers start at one of the extremes for at least some of the continua and gradually move away with a large proportion of the central position being behaviour which is 'good enough for' [Reid, 1996] bringing forth a learning environment with the pupils they work with. No absolute best behaviour!
>
> > It was realised that an extreme behaviour along any one dimension would mean that the student was probably extreme on many others. The early mistakes which students make are capable of being turned into 'cartoons' – strong visual images which are funny to practising teachers because of an awareness of the problems which can follow from such behaviour An example would be:
> >
> > *Student teacher, with their back to the rest of the room, standing or sitting having an intense discussion with one pupil in the far corner of the classroom.*
> >
> > The cartoon would simply have the rest of the pupils engaged in various forms of outrageous behaviour with the teacher blissfully unaware of this Such cartoons seem to highlight the behaviours at extremes of the dimensions (Brown and Hayter, 1993, pp 14-15)
>
> Simply sharing such cartoon images with student teachers and getting them to describe their own such interpretations of the extremes (after they have had some experience) can be useful in raising their awarenesses in the classroom
>
> DR: Your continua certainly strike a chord Some of them seem to me more to be linked around the extremes of controlling vs adapting, e.g. 'Presence in the classroom' and 'Adaptability'. The masters students I am working with are experienced teachers, who would like to change their practice to include the teaching ideas that they are reading and talking about in their university courses, but 'control' is keeping them from 'adapting' in a couple of ways. One is their fear that the control they have as experienced teachers will go away when they try to do things in ways that aren't familiar to them. A teacher I am working with likened the experience to being a first-year teacher again, and it is easy to understand the hesitation to re-experience that feeling of uncertainty.
>
> The other control-related obstacle is the perception (quite accurate, I suspect, but perhaps exaggerated) that they don't have control of the systems in which they are embedded, control that they would need if they were to adapt their practices. The prescribed curriculum is one constraint which they often cite. The number of topics they are required to cover leaves them feeling that there is no time for 'experiments' with other ways of teaching. The need to be teaching their classes the same topic in the same amount of time at the same time as other classes at that level in the school is another constraint. If there is a common exam for all the grade 10 mathematics classes, then there is a reluctance to teach one class differently in case that leads to the students doing less well on the common examination

When talking about working with human beings who become teachers in one year, the image had been one of moving away from the extreme positions at the ends of a spectrum. We realised that both of us had in the past moved to a more complex reading of our worlds through becoming teachers and we started to talk about the common dichotomies which are around in the classrooms of our students and, more interestingly for us, about the ways in which we felt that we seemed to hold the complexity and, where appropriate, avoid the dichotomy.

In the rest of this article, we first discuss some ways out of dichotomies and then illustrate these examples through a collaborative teaching episode at St. George's School, Montréal when DR and LB visited Vicki Zack's (VZ) classroom for the first time and the three of us worked on mathematics together with VZ's pupils.

Ways out of dichotomies

We continued to share stories, but this time as teacher educators with a filter of dichotomies that we seem to be able to avoid, so we could begin to identify the ways we get around them. We started to find patterns which generated a list of types of our behaviours when faced with an either/or Here we discuss three of these, which we call: *both, fork-in-the-road* and *star*.

Both

One of our earliest conversations, begun in Brazil, came out of DR's perception that he was more a researcher and LB was more a teacher. As we explored the teacher/researcher dichotomy we became more and more aware of the ways in which teaching and researching are the same for us, so that in any situation we are *both* teacher *and* researcher. (Re-)Reading Hampden-Turner (1981) on the choice between "the ecology of 'both ... and' or the catastrophe of either/or" (p. 210) provided us with a way of speaking of this type of way around the seeming dichotomy.

> LB: Here's a story from my research where the teacher's attention is on tracking their practice and is working with me as a co-worker to help maintain that focus It illustrates how the 'noticing paradigm' John Mason [1994] was talking about at PME works in my research practice
>
>> I was travelling in a car with a teacher who had only been teaching for a year. He was depressed about what he articulated as the gap between his 'ideals' and the reality of his classroom He had been working on reflecting back over complete lessons looking for 'good' ones to take forward to the following year The only problem was that there weren't any.
>>
>> I asked him (it was nice that his attention was on the driving) to try to find moments or instances of times which were closest to the 'ideal' and articulate an account of such a time (rather than to account for it). This request generated two 'brief-but-vivid' accounts [see Brown and Coles, 1996, for details] and with energy he suddenly looked at me (not good news as he was driving) and said: "It's silence, isn't it!" This led to a new label or category which has been instrumental in him making progress on his teaching and he's really exploring the use of silence.
>
> I have no problems with this account as being *not* teacher, *not* researcher, but rather developing practice - but it also *is* teacher, *is* researcher for *both* of us. I call such a statement (e.g 'using silence') generated in discussions about practice a *purpose* This is because it is easily articulated - such base-level distinctions (categories) are:
>
>> the generally most useful distinctions to make in the world. (Rosch, quoted in Lakoff, 1990, p 49)
>
> and serves to provide a focus in the student teachers' planning for action which accrues a range of behaviours over time which can be used flexibly and automatically in the classroom There is a sense of being between detail (practice) and abstraction (theory, images of mathematics and mathematics teaching). Again I'm finding all this theory useful in describing my practices.
>
> DR: During my Ph D , we'd been observing two education students investigating several problem situations, and for our last session we interviewed them, but in fact they interviewed us too, and we as teachers discussed the 'research' experience we had all been taking part in, reflecting on our thinking and their thinking as it related to the teaching we do At that time, and since that time, the disjunctions researcher/subject, researcher/teacher broke down, and we go on as *both* researchers+subjects, researchers+ teachers.

Fork-in-the-road

When we come to a fork in the road, it seems we must choose one path or the other Unless we can somehow be in two places at once, *both* cannot resolve this dilemma In such a position, we could feel that we must necessarily lose one path if we want to gain the other This can make decision-making difficult or impossible. But provided we remember that roads are rarely connected in only one place, we leave the option open of later taking a secondary road from some point along one fork to a point along the other. We do lose the experience of the sections of each fork we do not travel, and the path via the secondary road may be longer, but still a destination we might have felt we were giving up on forever turns out to be accessible whichever fork we choose

A mathematical analogy is a decision tree in which each choice is a clear dichotomy, so that knowing the destination implies knowledge of the path taken. In teaching, as in life, the paths are not simply connected, so that many paths lead to every destination

> DR: I was reflecting on the choice I made to come here to Memorial University instead of going back to teaching in Montréal. At first, it was a very hard choice because I felt I was choosing between two paths which would take me to two very different places The life of an academic was one I always thought I wanted, but on the other hand I knew that teaching in schools was a joyful experience for me, and that Montréal was the one place that let me feel totally at home and comfortable.

In the end, after much agonising, it was the simple recognition that my choice would not be irrevocable that got me unstuck. I can always go back to teaching and back to Montréal, so it was safe to go down the path that brought me here. And, in fact, once I got here I found that I could spend time teaching in schools, and also spend time in Montréal, so I go back and forth between the two paths.

LB: At school, before I took my examinations at 16, I could simply do mathematics. It was easy. I cannot remember enjoying the subject particularly but I knew that I was good at it from examination results, and, with many varied interests in my life, the lack of need to do homework was a strong motivating factor in my decision to get two easy A Levels (university entrance exams) – mathematics and further mathematics. In the UK at that time, it was usual to take three A Levels and the subject which I had loved at O level was English Literature. I was involved, motivated and inspired by a stimulating teacher.

Unfortunately, it was also usual for students to choose either arts or science A Levels and on the option grid for our choices further mathematics had always been against English and no-one could conceive of someone wanting to do both. After some soul-searching, I made a choice: I would always continue to read and be involved in literature, but if I were to give up the mathematics I knew that that would be that. A fork-in-the-road. So I did mathematics, further mathematics and physics at A Level. When I went on to study mathematics at university after A Levels, the choice I had made seemed bizarre since I was motivated to be Modern Literature librarian for the hall of residence, but not to invest the vast amount of time I would need to put in to engage with the hieroglyphics in the mathematics lectures.

Although I would in fact never go on to do A-Level English, nor take an English degree, I did end up editing the journal *Mathematics Teaching* and writing is part of my life as a mathematics educator. As my disappointed English teacher said: "At least if you're going to be a mathematician, be a literate one."

This story reminds me that, in travelling between schools where I visit my students on teaching practice around the Bristol area, I enjoy exploring new ways between schools so that I link up my mental map of the city in different ways. I have an awareness that all roads are connected.

DR: I was thinking about choosing paths as I was teaching, and it struck me that every time I ask a question and several hands go up I face one of those choices. I can't call on both Alex and Pat: I must choose. But I can come back to Pat after hearing Alex's response. It's not the same of course, because my question, and Pat's answer, have changed in the hearing of Alex's response.

It's the same thing too with choosing a method of teaching something. If I start off with integers as temperature and leave debts and credits for later it isn't the same as doing it the other way around, or doing both at once, or not doing either of those but them arising from the pupils I work with, but I do end up going down those paths, and switching back and forth whenever I need the features of one situation or the other.

Choices can seem impossible unless there is an awareness of interconnectedness and a unhooking from the quest for the 'right' choice or the 'best' order for doing things. Learners of mathematics and of teaching make their own connections anyway, and realising this can be freeing for future teachers. Clearly there are implications for children learning mathematics in classrooms where the teacher does not feel the responsibility to present their own linear mathematics and is willing to live with complexity.

Star

LB: I was reading the book on mental maps [Hampden-Turner, 1981] and came across this image which seems to me to be another 'way out' that we haven't got. Any stories for this one?

> Varela proposes that dualisms or dialectical 'contradictions' such as mind/body, whole/part, context/text, territory/map [...] should be conceived of as 'stars'. All stars consist of 'the it'/'the process of becoming it' where the slash or oblique stroke means "consider both sides of". Hence we must 'consider both it and the processes leading to it'. (p. 192)

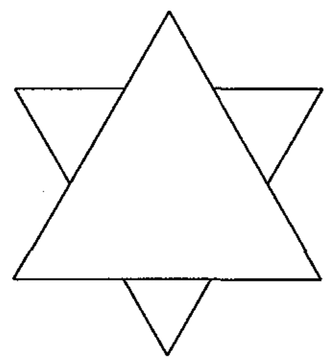

Figure 2

As the discussion progresses, I also like this bit:

> Hegel referred to his pairs as contradictions, a term applicable only to contraries of the same logical type, and their affirmation and negations were often, although not always, treated like win or lose clashes between symmetrical objectives, hence their revolutionary and violent implications. By using the image of two triangular shingles joined or imbricated in the shape of a star, with the one emerging from

beneath the other, conflicting ideas are presented as being at different levels of logical type – they become non-contradictory, mutually specifying and restrain each other in their complementarity. The network is unbalanced by the purposively striving tree, and the tree rebalanced by the constraints of the network. (p. 192)

DR: I really like the Star image as a way out of dichotomies. It works really well for something I've been mulling over for a while now: the interaction between individuals and the societies they are a part of. I am looking for ways to talk about such interactions which allows me to be thinking about both the triangles and the star at the same time, and so to be able to say something sensible about a part interacting with a whole. I kept getting tripped up with part/whole problems that I knew were avoidable but didn't have a nice image like 'star' to help me to avoid.

My 'research site' for this mulling is the mathematics class because it's familiar. As a teacher, I am used to interacting both with a number of individuals and also with a unified class. And I am aware that for each student the class is also an entity to interact with, one of which they (and I) are a part.

Montréal

In March 1997, we came together in Montréal to explore our common interests in the problems we use again and again in our teaching and research (Brown, Reid and Zack, 1998), the use of video as a research tool (Brown, Reid and Zack, 1997), and links between LB's work on intuition/analysis and DR's work on emotioning/reasoning. As a part of this process, we engaged a group of VZ's students from the previous year in a problem, the Arithmagon, and made a videotape of the subsequent whole-class discussion.

This was the first time that we had actually been present in the same classroom. VZ taught the first lesson, proposing the Arithmagon (see Figure 3) to a class of twelve-year-olds. She presented the problem silently (working with the 'using silence' purpose which we had been talking about the previous evening in planning), filling in the circles and then the squares according to the rule that the number in a square is the sum of the numbers in the two adjacent circles (see Figure 3a). As the children caught the pattern, she pointed to invite sums to go in the squares. (See Brown and Coles, 1996, for details of an analogous presentation.) Gradually, every child seemed ready to offer sums for the circles. VZ then offered the image in Figure 3b, and the children went off to work in small groups, exploring the problem they had set themselves from VZ's presentation. The three of us, all teachers, teacher educators and researchers (cf. Ainley, 1999), circulated, observing and questioning.

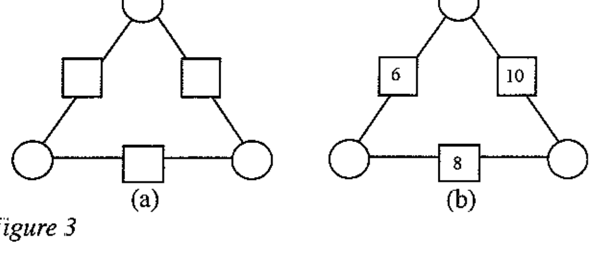

Figure 3

A week later, we returned to see what the problem had become for the children, what new ideas they had had and to see where they might go given some additional time. (The school roof had begun to leak the morning they arrived, so we ended up in the school gymnasium, with a bucket in the corner catching drips.) The initial discussion was in a group of 24, an unusual experience for a class that was used to doing mathematics as a half-class of 12. The children proposed conjectures, asked questions, rephrased, made connections. LB, acting as scribe, wrote on the board trying to re-present everything the children said.

Figure 4

As a part of our explorations of the way we use videotape, we engaged in a multiple viewing of the video VZ had made of the discussion. This viewing was 'multiple' in several senses. We were three people with different perspectives seeing different things. We were watching another instance of things we had watched before, DR having watched many videos of people investigating the Arithmagon, LB having watched many classrooms, and VZ having watched her students engaging with problems on many occasions. We also watched the video multiple times, both together, alone, in pairs and with others.

These multiple viewings are an important part of an enactivist research methodology (Reid, 1996), one inspired by the work on cognition of Maturana and Varela (1987; see also Varela, Thompson and Rosch, 1991). Enactivism foregrounds the role of the observer in any event, and acknowledges the complexity of interactions. For research methodology, this implies that we can never ignore the perspective or filter that each observer brings; it means that each observer will inevitably see different things. Some approaches acknowledge this, but see it as a problem to be solved, by attempting to modify the observers' perspectives in such a way that there is considerable overlap between what they see.

An enactivist approach instead attempts to preserve the diversity of interpretations, on the grounds that each interpretation says something significant about the observer–event. At the same time, each observer must be prepared to engage with the interpretations of other observers, without losing hold of their own perspective.

This combination of perspectives has then three important features:

- there are multiple perspectives in combination;
- each perspective says something different (one might say they contradict);
- each perspective can be made sense of by other observers (they are consensual)

We use the phrase 'multiple contradictory consensual perspectives' (Reid, 1996) as a shorthand to describe this approach.

In viewing the video after the event, given our conversations, we looked at the video with a common filter of these ways out of dichotomies to see whether there was evidence for any of the behaviours being present in our teaching and researching behaviours.

DR is sitting on the floor, leaning against a wall, on which there is a chalkboard. The children are facing him, also sitting on the floor. LB is sitting at the other end of the chalkboard. VZ is at the side, running the video camera. DR starts:

DR: *We'd like to talk some more about the Arithmagon problem – we worked on last week – and the first thing we need to know is how much you remember about what you were doing so we can get back up to speed [...]*

Both

DR: *So there's that conclusion I wonder if we want to do something like write that down. Up on the [...] do we have chalk? OK* [LB is getting up to write as this is said.]

Using the board was not something we had planned to do, but in the moment it was a natural move, and LB was already getting up to write as DR wondered out loud whether to keep a record One might ask whether the writing came out of a teaching intention or whether it was something we did in order to have a record for our research. In fact, both! The writing on the board could be seen as unplanned, but at the same time there is planning implicit in who we are as teachers+researchers

Josh	[mumbling] *the two circles have to equal the square.*
DR:	*Equal how?*
Josh:	[gives an example]
Jason:	[clearly] *The square is the sum of the two circles on either side*
DR:	*Is that a good way to say it? Or does anyone have a further refinement of that?*

One reading of this is as asking leading questions to further elicit the students' thinking It is again a teacher+researcher behaviour.

Matthew:	*Because of the first one, if the sum of the squares is odd, it is impossible*
DR:	*The sum of the squares is odd, because of the first statement. OK* [20 second pause. LB writes.] *Does that mean if the sum of the squares is even, then it is possible?*

After the children had left, VZ, LB and DR discussed what had just happened. DR wondered why he had effectively asked 'Does that mean it always works for even numbers?' He speculated that he might have been inspired by the sight out of the corner of his eye of LB drawing a double-headed arrow. Later, having watched the videotape, he wondered if he could have seen what LB was doing, and he noted that the question was the sort of thing he often asks in all sorts of contexts. Speculating about converses is part of his being a mathematician. There are many more stories that could be told about why DR asked that question Some are false in the sense that no one would believe them. Many are true in that some people would believe them. Even if one story contradicts another, they are both true in this sense.

Fork-in-the-road

Matthew:	*Because of the first one, if the sum of the squares is odd, it is impossible.*

[Two minutes later]

Becky:	*I don't understand what* [...] *Matthew said about* [...] *if the sum of the squares is odd* [ ] *it being impossible.*

The chalkboard (see Figure 4) offered connections between ideas, so that after the class had chosen one path, Becky could ask it to return to another path left unexplored. Of course, the discussion that ensued was not the one that would have happened had she asked for clarification immediately, but it continued in a similar direction.

Richard:	*We cannot do a problem with using negative numbers.*
Josh, Ezra and others:	*You can!*
DR:	*So maybe we have two theories, one that you can and one that you can't.*
Ezra:	*Yeah.*
DR:	*Maybe we can come back to that then. Mark that with a question mark.*

The chalkboard allowed DR to make choices, to choose forks, in terms of what was considered next, since the option to take a connecting road was always there: "Maybe we can come back to that then" A suspended choice, like a choice made tentatively, accepts one or both parts of a seeming dichotomy without denying either. Like exploring down one fork in a road before deciding which path to follow, the dichotomy is held unresolved until we know more.

Star

The chalkboard made individuals' statements into objects. They remained visible to all after the sound of the words had faded

Matthew: *I know that odd numbers can be solved with decimals Because if you have one, one and one, you can add that up to three, divide by two and you get one point five So you put point five into the circles*

Jason: *I don't think "that's allowed" is right. Because you can't break the rules. It's like you can't break the rules of physics. If it's possible to do it with zero or decimals or negatives, then it's allowed*

Josh: *You have to read what the problem says* [...] *If there's nothing about positive, you can use any numbers you want*

After the children left, at the end of the lesson, DR, LB and VZ talked about what was written on the board. They looked at the whole on another level. They saw the problem of 'breaking out of natural numbers' (see Figure 5) in the writing on the board. The children would not have seen this problem as they did not see the board as a record and presentation of their thinking No individual, child or adult, in the room 'understood' everything on the board in the way the three adults did talking after the event But the recording produced a record of the complex set of ideas, parts of which become accessible to individuals at different times The society of we three teachers had another reading of the history and, as it was recorded on the board, we could talk immediately about what went on, as researchers, teachers, etc.

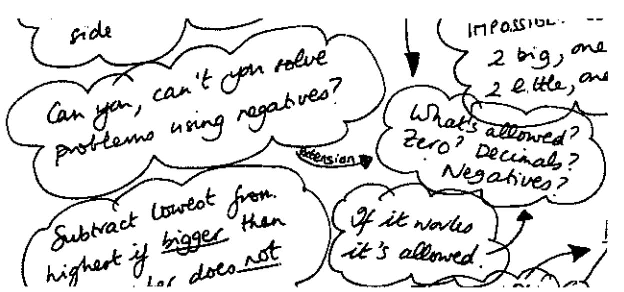

Figure 5

In this discussion, we slipped back and forth between speaking of the class as one entity and referring to the individuals who constituted the class. The board formed a visible memory of the class as a whole In the mind of the class there existed a problem, the text of which never appeared, but which was nevertheless present, so that Josh could refer to "what the problem says".

The dichotomy individual/group is often raised. We can see it in this lesson, when we speak of individuals' actions, as well as the understanding of the group represented on the board. But we can also recall that the class as a group emerges from the actions of individual children, and so is of a different and incomparable logical type In considering the interests of individual children and the class (which are sometimes seen as opposed), we can instead see them in this context as the same, so that the dichotomy of individual interests/group interest disappears The triangle of individual supports the triangle of group, and we work with the star

Looking ahead

In working together in a classroom in this way, we were interpreting the actions as teachers and researchers which have shown us that our e-mail conversations are indeed illustrative of who we are as people+teachers+researchers. There are ways out of dichotomies and this 'consensual co-ordination of action' (Maturana and Varela, 1987) has allowed us to work in awareness with our student teachers as they grapple with the complexity of the whole of teaching whilst focusing on the parts and moving away from extremes of behaviour in the classroom Teaching is essentially an uncertain activity. They never will meet the same children in the same situation facing the same difficulties or asking the same questions again and so working at ways out of a need to be certain seems an important part of what we do with them.

It also became apparent that it was possible to work with children so that they, too, saw mathematics as a 'domain of explanations' (Maturana, 1988, pp. 33-38) VZ's children already seemed comfortable with working with this ethos and were prepared to come to know, exploring and changing their minds in a messy space where they were forming complex connections and no artificial simplicity was being offered

The challenge remains of how to work with student teachers so that they learn to be comfortable in these messy spaces and trust that their actions will complexify over time. Showing them examples of our own teaching seems too like showing pupils how to do the mathematics; it does not allow them to enter a domain of explanations, but more to try to imitate what we do One problem with working with student teachers is that often they cannot see what an experienced teacher is doing - it all looks so easy - or they try out a 'model' lesson from which they cannot then distill out the elements to support their own later planning and action.

We will continue working in this area and hope to share, in a later article, some strategies for working with student teachers of mathematics so that they can develop strategies for working with their children getting out of the dichotomies For one of those student teachers with a right/wrong image of mathematics, the fork-in-the-road image seems important to allow them to let go of the agonised question: 'But how do I know what's the *right* thing to do?' They need to develop an awareness that they will simply act (This is a case of a fourth way out of dichotomies: MU - unask the question. The name is due to Hofstadter, 1979.) They will come to know through that action a little more about that particular class and those individuals in it and about their relations to mathematics. There really is no right way, just many connected paths, and, as Varela (1987) indicates, the path is not laid down in advance - it is created through the walking

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