On the Nature of Counting Activity: Perceptual Unit Items*

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As a part of a project concerned with the child's construction of the whole numbers, we have found it crucial to explain the nature of counting because its meaning and use vary greatly even among children of the same age [Richards, 1980; Steffe, Richards and Thompson, 1981]. In this paper we investigate the nature of counting for three six-year-old children who could count only perceptually accessible objects at the time they were interviewed. The larger project is a teaching experiment with six children being conducted at the University of Georgia.** Pre-experimental interviews conducted in October of 1980 are used to investigate three aspects of counting for Brenda, Shawn, and Tarus

1. The production of number word sequences, including, (1) the role of rhythm in these productions and (2) the awareness of number words they utter.
2. The construction of an object concept prior to counting and their use of it to produce countable items while counting
3. The coordination of their productions of a number word sequence and of countable items, including the role of rhythm in that coordination.

Number word sequences

The utterance of a number word sequence is part of children's counting activity. In fact, we characterize counting as the production of a number word sequence where each number word is accompanied by the production of a countable item. Several types of number word sequences — strings and chains, to name two of interest — have been identified [Fuson & Richards, 1980]. A string may be thought of as a number word sequence that can be accessed by the child at "one" only. A chain, on the other hand, can be accessed at any number word of the sequence. We turn immediately to Shawn's protocol to illuminate the behavior of a child in a number word interview whose number word sequence "1-2- -10" is basically a string. Abbreviations of protocols are made without sacrifice of meaning.

I: Shawn, can you count to ten for me? Out loud?
S: "Sure. 1-2-3-4- -10."
I: Start at five and count down to one.
S: "5-4-3-2-1."
I: Start at six and count down to one.
S: Sequentially extends fingers in synchrony with uttering "1-2-3-4-5." He then uttered "six-5-4-3-2-1."
I: Beautiful. Start at ten and count down to one.
S: Touches the flannel board in synchrony with uttering "1-2- -10", where the points of contact form a lineal pattern stretching away from his body. He starts backward, touching the flannel board in the reverse direction while uttering "10-9-8." He then stops and returns to the starting place of his forward counting action and touches the flannel board in synchrony with uttering "1-2-3-4-5-6-7." His points of contact again form a lineal pattern forward. He then reverses the direction of his pointing action and points twice while uttering "7-6." He then returns to the starting place of his original forward counting action and utters "1-2-3-4-5-6" while touching the flannel board forward. He then reverses the direction of his pointing actions and points in synchrony with uttering "6-5-4-3-2-1."
I: Okay. Beautiful! Here is another one. What number comes right after seven?
S: Touches the flannel board in synchrony with uttering "1-2- -8. Eight, eight, eight!"
I: What number comes right after three?
S: "Three, four."
I: What number comes right after five?
S: After some preliminaries, hits a cup in synchrony with uttering "1-2-3-4-5-6."
I: You got it. Very good.
I: What number comes right before two?
S: "Three."
I: What number comes right before three?
S: "Four."
I: What number comes right after six?
S: Selects poker chips in synchrony with uttering "1-2- -6." "Seven, seven."

The part of Shawn's number word sequence "1-2- -10" starting with "four" was a string because he started at "one" to produce the number words right after "five" and "six." His behavior in producing backward number word sequences and number words right after "five" and "six" is illuminating from three perspectives. First, he successively used a "dropping-back-to-one" strategy for producing number words right before a particular one. Second, he was aware of the number words he uttered. Third, he performed rhythmic coordinations in the "dropping-back-to-one" strategy. Each of these observations will be considered in turn.

Shawn dropped back to one three times, each time producing a partial backward number-word sequence. This indicates that he based the production of backward number word sequences on forward number-word sequences. As he produced the initial forward string "1-2- -10" he synchronously touched the flannel board, his points forming a

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lineal pattern. In each of his subsequent productions of number-word sequences, whether forward or backward, he also synchronously touched the flannel board. On all occasions, the pointing act corresponding to any number-word utterance was in the same approximate spatial location on the lineal path as its corresponding pointing act in the initial forward number word production (e.g. a pointing act corresponding to "7" was always nearer to his body than a point corresponding to "8" in the same sequence of utterances). In effect, he used the lineal paths to support the "externalization" of the number word sequences. He was thereby able to coordinate the production of forward and backward number-word sequences.

Shawn's use of the dropping-back-to-one strategy also indicates that he was aware of, and could remember, the number-words he uttered. For example, he produced "1-2- -7" immediately after uttering "10-9-8". To do so in the context of producing the number word sequence "10-9-8- -1" he must have remembered uttering "eight" for the duration of uttering "1-2- -7" (i.e. he had to remember not only this last utterance, but also the direction of the number-word sequence of which it was the last utterance)

Shawn frequently coordinated uttering number word sequences with other motor activity. To start at six and count down to one, he coordinated the utterances of number words "1-2-3-4-5" with sequential finger extensions; to produce the number word right after five, he rhythmically hit a cup in synchrony with uttering "1-2-3-4-5-6"; to produce the number word right after six, he rhythmically selected poker chips in synchrony with uttering "1-2- -6"; and to produce the number words before ten, eight, and six when using the dropping-back strategy, he rhythmically tapped his finger on the flannel board in synchrony with uttering number words, and his finger taps formed a lineal pattern. Thus, Shawn did not just recite number word sequences. Rather, these productions were supported by and coordinated with rhythmic motor action. (Fuson and Meirkiewicz [1980] also report rhythmic coordinations.)

Tarus' number word sequence "1-2- -10" was slightly more advanced than Shawn's. He could utter number words right after given number words but did not appear to be aware of the links between adjacent number words. Because he could access his number word sequence at any point it constituted a chain rather than a string. His interview went as follows.

I: Can you count for me — out loud, 1-2- . like that?
T: (Very rapidly) "1-2-3-4-5- -17, 18, . . ."
I: Okay. That's fine. You're really good! Can you start at 3 and count down to 1? 3-2-1 like that? Do that
T: "3-2-1."
I: Can you start at 4 and do that?
T: Two-second pause, then counts slowly "1-2-3-4"
I: Okay. Can you go 4-3-2-1?
T: Sits silently for six seconds but is clearly mouthing something to himself
I: You did it, didn't you — 4-3-2-1. Okay, what comes right after 3?
T: "Four" after ten seconds
I: What comes right after five?
T: "Six" after two seconds.
I: What comes right after seven?
T: "Eight" after two seconds.
I: Eight! What comes right after ten?
T: "Eleven" after a five second delay. He silently utters "1-2- -10" mouthing the number words.
I: Yes, that's right. What comes right after 3?
T: "Four" after six second delay. Silently mouths "1-2-3".
I: Four! What comes right before 2?
T: "Three" after a five second delay. Silently mouths "1-2-3".
I: No, that's right after. What comes right before two?
T: "Five" after a three second delay
I: If you were counting, what number would you say right before you said two? Count and see.
T: Mouths number words silently with lips moving.
I: What number did you say right before two? (No answer) What number did you start with? (No answer) One, two. What number did I say right before 2?
T: "Three" after seven seconds
I: Three? Okay. Let's do this one. Start at 3 and count two numbers. Can you do that? Start at three and count two more numbers. "Three—"
T: "4-5-6."
I: Count just two more now. Start at three
T: "4-5-6-"
I: Start at two and count up to four. Start at two and count to four. 2—
T: "2-4-"
I: 2-3-4. Okay, start at three and count up to five.
T: "3-4-5- . ." (indistinct).
T: "3- (Delay of four seconds) -4-5."
I: 3-4-5. Right. Start at 4 and count up to 6.
T: "1-2-3-4-5-6-7."
I: Okay. Very good. Can you tell me the number that goes between 1 and 3?
T: "Two" after 10 seconds, mouthing number words.
I: Very good. Tell me the number that goes between 3 and 5.
T: "Six" after 2 seconds
I: Okay. How about the one between 2 and 4?
T: "Five" after 6 seconds.
I: Okay, very good.

Tarus clearly did not utter number word sequences backward. In the case of the number word sequence "4-3-2-1," he did imitate the interviewer, but that was all he could do in the case of backward number word sequences. He was able to utter number words right after 3, 5, and 7 without dropping-back-to-one and getting a running start. In the case of the number word after 10, he did drop-back-to-one and utter "1-2- . -10" in order to say "11," but his utterances were nonrhythmical in nature, in sharp contrast to Shawn's heavy reliance on rhythm. He never coordinated his production of number sequences with other motor activity as did Shawn. While this does not preclude the possibility that coordinations are crucial to the production of number word sequences in the case of strings, it makes it

unlikely that Tarus engaged in rhythmic number word productions. He just never displayed rhythmic bodily motion in producing number word sequences which were very close to being strings

That he could never utter a number word right before a given one is consistent with his inability to produce backward number word sequences. The possibility is great, however, that he did not know the meaning of "before," because he had a strategy to use (dropping-back) had he understood what was being asked. He did use the dropping-back-to-one strategy in another interview to produce "7" as being right before "8," but it was a spontaneously constructed problem — He wanted to utter "7" after uttering "8," but couldn't do it. So he dropped-back-to-one to get a running start.

Tarus did not produce two number words after 3; nor could he utter the number words between 3 and 5, and 2 and 4. Even though Tarus could utter immediate successors, he never seemed to be aware of the links between two adjacent number words, which indicates that when he uttered an immediate successor of a number word he seemed to "lose" that number word. This is dramatically displayed in another interview when he uttered "1-2- . . -7-8" as he counted drumbeats, but when he wanted to utter "7" after uttering "8", he had to start at one and utter the whole sequence over

Brenda uttered the forward number word sequence "1-2- . . -10" and the backward number word sequence "10- . . -1" with ease. She consistently interpreted "right after" as "right before", but these questions were asked immediately after she had uttered "10-9- . . -1." We believe that she interpreted "right after" with regard to that backward number word sequence because she was so consistent. After breaking this set, by using the phrase "If you were counting, what number would you say right after . . ," she did produce the number words right after "4," "7," and "9." She seemed also to be aware of the links between adjacent number words as indicated by being able to utter the number words between 1 and 3; between 4 and 6; between 4 and 2; and between 5 and 3. She could not utter the number words between 4 and 7, which indicates that her number word sequence "1-2- . . -10" was no more than a chain where she was only aware of the links between adjacent number words. Had she also been able to produce all the number words between any two nonadjacent number words of the number word sequence "1-2- . . -10", we would call her number word sequence a fabric. She was close to mastering "10-9-8- . . -1" as a chain, but it was still a string.

Constraints placed on counting activity by the quality of the number word sequence is clarified in the Marked Rings Interview, the Pendulum Interview, and the Drum Beats Interview that follow. These interviews were conducted to investigate (1) the construction of an object concept prior to counting and the use of it to produce countable items while counting, and (2) the coordination of the production of a number word sequence and of countable items.

Countable items

Children construct the object concept during the first two years of life [Piaget, 1937]. For this reason, the investigation of the countable items that six-year-old children are capable of constructing may seem to be misplaced. But Piaget [1937] was concerned with the child's construction of perceptual items *per se*. Our investigation focuses on children's use of the object concept to create countable items in the context of counting. We believe it is one of the most crucial investigations of children's number knowledge, because the countable items the children are capable of constructing constitute their basis for constructing other numerical concepts and operations.

von Glasersfeld [1981a, 1981b] has identified two distinct uses of the object concept. On the one hand, it serves in the representation of an object regardless of the absence or presence of actual perceptual signals. On the other hand, it serves in recognizing new sensory-motor experience as an instantiation of the object. In this latter case, the object concept serves as a template.

It is usually the case that, in the context of a counting task, the conceptual entity that serves as a template has been already established in the past experience of the child and is semantically linked to the sound image of a word. If, for example, a child was asked to "count the cookies" and the child had an established template semantically linked to the sound image of "cookie," this template would be applied during the subsequent count. In other cases, such a semantically-linked template may be modified in a particular counting context by restrictions or additions of particular sensory features (e.g. the rings with a black mark). In yet other cases, the "object" which is to serve as a template may have to be created immediately prior to counting. In any case, when perceptual signals constitute the material of application of a template in a counting task, we say that the child is counting perceptual unit items. In effect, the template serves as a "conceptual filter" through which perceptual items must pass if they are to be constituted as countable perceptual items. In all, we have identified five types of countable items — perceptual, figural, motor, verbal, and abstract [Steffe, von Glasersfeld, Richards, *et al*, 1981]. We are concerned here with countable items created from perceptual material and three children who are limited to constructing that type of countable item.

The object concept (in our terms, "template"), following Piaget [1937, p. 76], requires the establishment of experiential "objects" or "items" which then serve in the empirical abstraction crucial to form the object concept. We believe these experiential items are based on coordination of sensory material from more than one source. The coordination of sensory material could be a combination of perceptual signals from two modes of perception (visual, tactual, or auditory) or a combination of perceptual signals with kinesthetic (proprioceptive) signals. In the latter case, bodily movements would be involved. For that reason, we are interested in the supportive role of rhythm in the construction of objects which may not have been already constructed by the child — a particular movement of a figurine, or a drum-beat.

We investigate the three children's construction of countable items in the context of three interviews — the Pendulum Interview, the Drum Beats Interview, and the

Marked Rings Interview. In the Pendulum Interview, each child was asked to count the swings of a pendulum. The pendulum was a figurine (a toy golfer) balanced on two points on top of a cylindrical metal base. When set in motion by tilting the golfer away from the vertical position, the figurine swayed to-and-fro in near friction-free motion.

It is unlikely that any of the children would have already established an appropriate object in past experience, although this possibility exists. Assuming the child does not have an established object, it could nevertheless create experiential items (e.g., a to-and-fro motion) in the context of the swinging figurine. If the child becomes aware of a recurrent feature of the motion through empirical abstraction, the child would create an object which embodies these recurrent features. The child could then carve out instantiations (countable items) of this object from subsequent sensory-motor material deriving from inspection of further motion of the figurine. These countable items would be perceptual unit items and would be deliberately created. The term "item" is used to emphasize that, for the child, each motion would have a beginning and an end. The term "unit" emphasizes that the items would be equivalent for the child in the sense that they would be instantiations of the same object.

In the Drum Beats Interview, each child was asked to count rhythmic sequences of single drum beats and rhythmic sequences of dual drum beats that had been recorded on audio tape. Just as in the Pendulum Interview, it would be obligatory for the child to create the countable items sequentially in time. Again, we believe that the eventual creation of countable items would require some type of coordination of perceptual material deriving from two or more modes. In those cases where bodily movements are involved, rhythm may be crucial to the construction of the template.

In the Pendulum and Drum Beats Interviews, it could be argued legitimately that, in some cases, the child need not create a template or countable items to perform as if it were counting. The basis of the performance could be coordination of the utterances of a number word sequence with various visually based features of the swinging figurine. Consequently, in order to investigate further the construction of a template in the context of an activity that could be called "counting", the marked ring interview was planned. Each child was presented with rings and asked to count the rings with marks on them. The marked rings were placed with marks down so that marked and unmarked rings could not be distinguished by immediate visual inspection alone. The marked ring interviews were audio-video taped (as were the others) and protocols and protocol analyses were developed from the audio-video tapes.

MARKED RING INTERVIEWS

Brenda and Shawn both understood what it was that they were to count (marked rings) prior to initiating their counting activity. That they searched for countable items is a strong indication that they had constructed a template prior to counting. A ring — *qua* object — became a countable item only when the perceptual feature of a mark became present to visual perception. The marked ring then became an instantiation of the template and was considered as a perceptual unit item or, for short, a countable item. The two children never made the error of counting an unmarked ring. The unmarked rings were not constituted as countable items.

It seems to be the "gap" between the children's template and the lack of visible marks that served as a propellant in their search for countable items. Their number word sequences were not a propellent for counting because they always uttered a number word after searching activity and after careful inspection of the marked rings. These conjectures seem to be substantiated in the next protocol of Tarus. Tarus' protocol also offers invaluable insight into the nature of the template for these three children — it is contextual and is formed as a modification of an already existing template.

I: (Places 11 rings in front of Tarus and explains the task.)
T: Inspects eight rings in all, but does not perform a single counting act. He always places inspected rings back with uninspected rings.
I: Starts again, placing 21 rings in front of Tarus, seven with marks on the bottom, and shows Tarus one of the marked rings, drawing his attention to the mark. Picks up three rings and also asks Tarus whether or not they are marked.
I: Explains the counting task.
T: Searches for a marked ring and finds one immediately. Places it in his hand.
I: Count out loud.
T: "One-two" (counting the marks).
I: Don't count the marks, count the ring!
T: Searches for 8 seconds, inspecting one ring before finding a marked ring. He places it in his hand while uttering "two."
T: Finds four more marked rings, six in all, after searches of 7, 16, 2, and 8 seconds. He uttered "three," "four," "five," and "six" while placing the respective marked ring in his hand.
T: Searches for 20 seconds without finding another marked ring.
I: Is that it? How many did you get?
T: "Six."

In the initial interview with 11 rings, Tarus' failure to perform a single counting act indicates that he did not construct countable items. One should not say that he was incapable of constructing countable items because he did so in the interview with 21 rings. Moreover, he seemed to be aware that he was not just to count rings, because he picked up and inspected eight unmarked rings without counting any of them. His inspection of these eight rings indicates that he was attempting to construct a meaning for the word "mark", but failed.

In the interview with 21 rings, Tarus finally made a semantic connection between the word "mark" and the marks on the rings. After making the connection, he thought he was to count the marks as indicated by his counting "one-two" when he found the first marked ring. At this point, he finally "understood" what he was to count — the marked rings! Our claim is that he made a modification in the template "ring", forming the template "marked ring".

He incorporated the visual feature of a mark into the template "ring", which he had already formed, abstracting that visual feature for the purpose of counting. He was now aware of the significance of the mark on the rings as indicated by his future searches for marked rings and by his constitution of the marked rings as countable items. He could now count, where the counting activity involved the search for countable items, a search made possible by a conceptual understanding of what he was to count — the marked rings.

Rhythm seemed to be unimportant in the construction of countable items in the marked ring interviews. This should be expected because the children had already created a template semantically linked with the sound image "ring". In the Pendulum Interview, however, it was very unlikely that the children had created, prior to observing the figurine, any particular movement of the figurine as an object that they were going to count. For this reason, careful examination of the children's behavior is crucial in the attempt to observe any indication of the construction of experiential items as well as a template and countable items.

PENDULUM INTERVIEWS

I: Places figurine in front of Brenda. What does this look like to you?
B: "I don't know."
I: That's an old man chopping wood. Count how many times he chops. Starts the figurine and utters "one" as it completes a forward swing.
B: "T-w-o, t-h-r-e-e-, f-o-u-r, . . , t-e-n, eleven, . . , twenty-five".

Brenda uttered each number word as the figurine moved through its forward swing and seemed to be aware of the forward swings. But there was no overt bodily motion, other than possible eye movements, as she uttered number words, and thus, no motor rhythm. She then uttered the number words "26" through "30" during six rather than five swings. In doing so, she rhythmically coordinated her utterance of the "tens" part of the number words with one change of direction of the figurine and the "units" part with the other change of direction in a sing-song fashion. Brenda's change from uttering a number word during a forward motion to coordinating her utterances with the changes of direction indicates that her activity from "25" on could be construed as a coordination activity without the production of countable items.

There were two aspects of Shawn's behavior that indicates that he created countable items and actually engaged in counting activity. The first was that Shawn coordinated rhythmic bodily movement with to-and-fro swings of the figurine. The second was that he did not change the aspect of the swinging figurine he attempted to coordinate with utterances of number words when his utterances lagged behind. Instead, he increased the tempo of the utterances enough to restore the coordination he had established.

I: Shows Shawn the figurine and explains the task. She then sets the figurine in motion and utters "one" as it swings forward.
S: "One" (Imitating I). "Two, three, four, five, six, seven, eight, nine, ten, . . ." As he utters "six, . . , ten" he swings his body to and fro in time with the figurine and utters a number word for each forward swing of the figurine.
S: ". . . 11, 12, 13, 14, 15, 16, 17, 18, . . ." He reduces his bodily motions to slight head nods during these utterances and starts to get behind the forward swing of the figurine at "17".
S: ". . . 19, 21, 22, 23, 24" I stops him at "24". He increases his tempo at "19" and restores the utterance of the number word to correspond to the forward swing at "21".

When Shawn uttered "one, two, three, four, five" there was no indication that he did anything other than coordinate the number words with the forward swings of the figurine. We believe he was imitating the interviewer's initial coordination of a number word with a forward swing of the pendulum. But when he emphatically moved to and fro rhythmically while uttering "six, . . , ten" we believe he created an experiential item based on his bodily movements and his visual perception and then empirically abstracted a template from these experiential items. After establishing what he was to count, he became less dependent on rhythmic bodily movement because he was now aware of what he was counting — forward swings of the figurine. This claim is corroborated by his restoration of the utterances of the number words with the forward swings after his utterances lagged behind.

There was no indication that Tarus created a template during his apparent coordination of utterances of number words and perception of swings of the figurine. Tarus simply uttered number words, apparently in coordination with each swing, running out of number words at "14".

I: Here is a little man with an axe. He's going to take some chops. I want you to count the chops. Count them. (Sets figurine in motion.)
T: Stares at the moving figurine and says nothing.
I: Stops the figurine. "Count them." Sets the figurine in motion.
T: "One, two, . . , 14, 15, 14, 15" Tarus seems to utter a number word for each forward as well as backward motion. When he repeats "14" and "15" there is no coordination of the utterances with swings of the figurine.
I: Okay.

There is a distinct possibility that Tarus just uttered a number word sequence, with fortuitous synchrony with the swings of the figurine. This possibility becomes more plausible when his behavior in the drum beats experiment is investigated. There, when he heard the word "count", he uttered a number word sequence very rapidly and without regard to the drum beats. In any case, although he "saw" the swinging figurine, there was no decisive bodily motion that would indicate that he had formed an abstraction and was counting instantiations of a template.

DRUM-BEATS INTERVIEWS

All three children established a template in the Marked

Rings Interview, albeit Tarus with some difficulty However, it is claimed that Tarus never created a template in the context of the Pendulum Interview. Nevertheless we feel that Tarus may be able to create a template and countable items in the context of performing sequential coordination activity. In the Pendulum Interview, the experiential items had to be "cut out of" the experiential background constituted by visual perception of the swinging figurine. The perceptual signals in the Drum-Beats Interview were much less complex than in the Pendulum Interview. So, it should be more likely that Tarus would create a template that we call "drum-beat" and count subsequent countable items. We now turn to the interviews. The directives were recorded on audio-tape as well as the drum beats. We start with Brenda ("Ta" is used in place of "Tape")

Ta: Count carefully now (. .) (Each beat is represented by a period. There was approximately one second between beats)
B: "One, two, three". Brenda started two beats behind.
I: You didn't start soon enough.
Ta: Count carefully now (. .).
B: "Two, three, four," immediately after each appropriate beat.
I: Four! You have to start right off. When the tape says "count carefully now," you have to listen and count
Ta: Count carefully now (. . . .).
B: "One, . . , seven," immediately after each appropriate beat.
I: Very good! How many did you get?
B: "Seven."
Ta: Count carefully now (. . . . . . .)
B: "One, . . , 13," immediately after each beat
I: You can do it, can't you!
Ta: Count carefully now (. . .) (There is approximately one second between each beat of a pair and 1 5 seconds between each pair.)
B: "One, two — three, four — five." Brenda misses the fifth beat and utters "five" after the sixth beat. "Four" is uttered after the fourth beat, so she is still coordinating number words with appropriate beats at this point But the fifth beat occurred immediately after the utterance "Four" which caused her to not hear that beat.

Brenda did not execute any bodily motion as she coordinated number words with the single beats, beyond slight head nods as she uttered number words. The most convincing indication that she created a template "drum-beat" is her behavior in the case of the dual beats. There, we believe, she missed counting the fifth beat simply because she did not hear it — not because of any asynchrony in the coordination of utterances and beats. She intended to count the drum beats as indicated by her modification of the number word utterances to correspond to the rhythm of the beats. Moreover, she always uttered number words after each beat except in the first case of three beats, where she waited two beats before she started uttering number words While she certainly coordinated beats and number words, the fact that her utterances always followed a beat corroborates our belief that she constructed beats as countable items.

Shawn never coordinated utterances of number words with drum beats. He seemed to have no idea of what the interviewer expected him to do The only time he engaged in counting activity was after the interviewer asked him how many beats he heard (seven beats). Only then did he utter number words — "one, two, . . , five" in synchrony with touching each finger of his left hand with the index finger of his right hand.

Tarus, like Shawn, did not initially coordinate the utterances of number words with drum beats. But, unlike Shawn, Tarus did utter number word sequences. So, the interviewer varied the interview under the hypothesis that Tarus could coordinate number word utterances with sequential drum beats.

I: You did not count them! Like this — count carefully now (strikes the table five times)
T: Utters "one, two, . . . , five" silently but coordinated with the beats. Tarus looked at the beating motion
I: That is what you are supposed to do. Count the drum beats
Ta: Count carefully now (. .) Each beat is represented by a period.
I&T: "One, two, three, four" immediately after each beat.
I: You want to do that now? You do it by yourself
Ta: Count carefully now (. .).
T: Whispers "one, two, three, four" keeping in time with the drum beats.
I: How many did you get?
T: "Four."
Ta: Count carefully now (. . . .)
T: Whispers "one, . . , seven, eight" immediately ahead of the drum beats. He stops because there was no drum beat after "eight."
I: What did you get?
T: No reply, but is clearly mouthing words.
I: Did you get seven?
T: Shakes his head "no"
I: Eight?
T: Ignores the question and whispers "one, two, . . , seven" to himself, taking 14 seconds "Seven!"
Ta: Count carefully now (. . . . .)
T: Misses the first one because he was still answering the above question and utters "one, . . , twelve" in synchrony with the beats He utters number words at about the same time as each beat.
Ta: Count carefully now (. . . .).
T: "One-two, three-four, five-six," where the first number word of each pair is uttered after the appropriate beat
Ta: Count carefully now (. . . . .).
T: "One-two-three, four-five, six-seven, eight-nine. He gets behind and utters "one-two-three" for the first four beats and continues as above.

We believe that Tarus constructed a template semantically linked to "beat" in the context of watching and listening to the interviewer striking the table. While visual perception supported the construction of this template, the material of application of the template was constituted solely by audi-

tory signals. He could now conceive of the possibility of counting auditory items. There are two reasons for our belief. First, he was aware that there was no drum beat corresponding to "eight" in the case of seven single beats. This shows that he anticipated the drum beats before they actually occurred, which is tantamount to claiming that he constructed the drum beats as things to count — as instantiations of a template. Moreover, he counted three dual beats and five dual beats, admittedly starting erratically in the case of the five dual beats. Second, he displayed no bodily action as he uttered number words in synchrony with drum beats. This lack of motor activity can be considered as further corroboration that all he needed to construct as a thing to count was some auditory signal. He apparently did not need proprioceptive signals. In fact, we believe that he constructed the template when the interviewer struck the table five times. What seemed to be salient were the perceptual and auditory signals because he did not engage in any gross bodily movements as he whispered "one, two, three, four, five", other than moving his mouth to whisper the number words. In short, once he had constructed a template by empirically abstracting from visual and auditory material, he restricted the template in the context of auditory items as countable items.

Coordination of number word sequences and countable items

Counting involves coordination of a number word sequence and countable items. An act of counting is a coordination of an utterance of a number word with a countable item. In this section, we are interested in possible mechanisms of this coordination. In the pendulum experiment, Shawn's rhythmic body movements were intimately involved in the act of creating and counting countable items. While there is no evidence that motor rhythm played a role in either Brenda's or Tarus' counting activity, Shawn's behavior indicates that it can serve as a coordinating mechanism.

We return now to the marked rings experiment in order to examine the children's behavior for evidence of other possible mechanisms. In addition, performance in this experiment will supplement the evidence of the children's awareness of their number words gleaned from the number-word interviews. In this experiment, of course, the context is counting rather than producing number-word sequences.

RETURN TO THE MARKED RING INTERVIEWS

Brenda had found and counted two marked rings.

B: Searches for 16 seconds, inspecting two rings before finding a ring with a mark on it. She sees the mark and then utters "t-h-r-e-e" while placing the marked ring to her right, proximal to the first two.

B: Searches for 7 seconds, inspecting one ring before finding a ring with a mark on it. Again, she sees the mark and then utters "f-o-u-r" while placing the marked ring to he right, proximal to the first three.

B: Searches for 19 seconds, inspecting three rings before finding a ring with a mark on it. This marked ring is the third that she had previously counted. After she sees the mark, she utters "f-i-v-e" while placing the marked ring to her right, proximal to the other counted rings.

B: Searches for seven seconds, inspecting one ring before finding a marked ring. After she sees the mark, she utters "s-i-x" while placing the marked ring to her right, proximal to the other marked rings.

B: Searches for 120 seconds, inspecting 11 rings before finding a marked ring. During her search, she inspects two counted marked rings but does not recount them. After she finally finds a marked ring, she utters "s-e-v-e-n" while placing the marked ring to her right proximal to the other marked rings.

B: Searches for 20 more seconds, inspecting two rings

I: Are you done?

B: Nods her head "yes."

I: How many did you find?

B: "Seven."

Brenda's utterances of number words were totally nonrhythmic for the simple reason that she uttered number words only after finding a marked ring. The intervals, in seconds, between the utterances were 20, 16, 7, 19, 7, 120. So, it is not plausible that rhythm was the basic mechanism that Brenda used to "coordinate" the utterance of a number word sequence with the production of countable items. Nor did she use rhythm in coordination of a particular number word with a countable item.

Brenda's number word sequence "1-2- . . .-7" constituted a chain. Being aware of the linkages between number words allowed her to "stretch" those linkages over time intervals of up to 120 seconds without breaking them (recollection of the number word uttered last was sufficient for her to utter the next number word). It is this chain property that permitted her to utter a number word after finding a marked ring. This does not, however, explain why she uttered a number word. It only explains the mechanism which permitted her to do it.

Of particular interest is the fact that Brenda picked up but did not recount two previously counted marked rings while searching for the seventh marked ring. She surely saw that the two rings were marked, but did not re-count them. In other words, the mere presence of a countable item did not trigger counting activity. This leads to the contention that she had separated the act of creating a countable item from the act of counting it. When performing the latter act, she uttered a number word as she placed the marked ring on the board in front of her. This motor act probably served the role of coordinating the countable item with the utterance of a number word. That is, the motor act was an essential part of the counting act itself; she uttered the appropriate number-word as she performed the placing act after she had created a countable item. The separation of the act of creating and the act of counting a countable item allowed her to decide whether or not to count a particular item. In short, this separation elevated the act of counting an item from the status of a counting reflex to the status of an intentional act.

Shawn had already found and counted four marked rings.

S: Searches for 13 seconds, inspecting seven rings before finding a marked ring. After he sees the mark, he places the marked ring on one of the stacks.

I: How many have you got now?

S: Points to two rings while uttering "1-2."

I: (Interrupts) Don't count. How many have you got?

S: Raises three fingers.
I: Count them!
S: Counts the marked rings by placing his right index finger on each one and abruptly moving it in synchrony with uttering number words "1-2-3-4-5." He then stacks them back up
S: Inspects four marked rings. He missed seeing a mark and is directed to that ring by I. He sees the mark and then picks the ring up and places it with the others already counted
I: How many have you got now?
S: Hides his eyes in his hands for eleven seconds and then raises six fingers
I: Six. Okay
S: Counts the marked rings as before "1-2-3-4-5-6."
S: Searches for 30 seconds, inspecting 12 rings.
I: Find them all? How many did you find?
S: Grabs the stack of six counted rings and utters "1-2-3-4-5-6" in a sing song fashion without any accompanying motor activity but holding the marked rings in his hand

Shawn's act of counting the sixth marked ring actually involved two separate parts; one of placing the marked ring with the counted marked rings and the other uttering the number words "1-2-...-6" and extending six fingers. These two parts of the act of counting were necessary because Shawn could not utter the next number word after "five," given only "five". Any link that may have been constructed by Shawn between "five" and "six" prior to finding a new marked ring decayed during the search for the sixth marked ring. That Shawn was able to extend six fingers shows that he was aware of "five" as a number word that he uttered last when uttering "1-2-3-4-5." The claim that Shawn could recollect uttering number words was also made in the number word interview when Shawn was asked to start at ten and count down to one. There, he recollected uttering "eight" for the duration of uttering "1-2-...-7". The decay of a link between number words constrained his counting activity and he resorted to dropping-back-to-one to find the appropriate number word. Shawn's behavior also strongly indicates that the completion of a counting act requires the utterance of a number word, because he dropped-back-to-one after the interviewer had said "six".

After Shawn completed his final search, inspecting 12 rings during the search, the interviewer asked him how many rings he found. He grabbed the stack of six counted rings and uttered "1-2-...-6" in a sing song fashion without accompanying counting activity. This behavior provides important indication of the nature of what Shawn produced through counting. We believe it was a "lot" in the sense of von Glasersfeld [1981] — an experientially-bounded plurality, where the items of the plurality need only be discrete units. Shawn's sense of wholeness is indicated by his grabbing the counted rings with one hand and uttering the number word sequence "1-2-...-6" over his hand. The hand containing the rings constituted an experiential boundary for the counted rings, which were surely perceptual unit items.

Shawn also provided indication at other points of his counting activity that he possessed a sense of wholeness concerning the counted items. He stacked the counted items on more than one occasion as well as when counting them to find how many he had found. More generally, we believe that when a counter with perceptual unit items counts a collection and re-utters the last number word of the number word sequence uttered during the counting activity to indicate "how many" were counted (as Brenda did after she found the last marked ring), that number word refers to a "lot", as we use the term. We do not consider the lot established through counting to be numerical. That must await abstract, numerical units which our three children were not yet capable of constructing.

Tarus' complete protocol for the marked-ring experiment is contained in the section *Marked Rings Interviews*. After Tarus finally constructed the template "Marked Ring," the intervals of search were 8 seconds, 7 seconds, 16 seconds, 20 seconds, and 8 seconds. His recollection of uttering number words over these intervals was sufficient for him to utter the next number words, which is corroboration of the claim that his forward number word sequence was a chain. The links between number words permitted him to coordinate number words with countable items without reconstructive activity. But they were not sufficient, for he moved a marked ring to one hand with his other hand as he uttered number words. This motor act was a fundamental mechanism of coordination for Tarus as well as for Brenda and Shawn. But in Shawn's case, when the links connecting number words decayed over the duration of search, the motor act was not sufficient for the coordination.

When Tarus placed each counted marked ring in his hand, we believe that he possessed a sense of wholeness as did Shawn. Grasping the counted marked rings provided an experiential boundary for the marked rings he had counted. So, when the interviewer finally asked him "How many did you get?" and he answered "six" 20 seconds after he had counted the sixth one, "six" clearly referred to the marked rings that he had grasped in his hand.

Summary

There are various findings of the study which contribute to understanding the nature of counting activity for children who are limited to counting perceptual items. These findings are listed by the topical sections of the paper.

Number word sequences

1. There is no relationship between these children's type of number-word sequence and their ability to create a template or countable items. For example, Shawn created a template in the Pendulum Interview, but his number word sequence "4-5-...-10" was a string. On the other hand, Tarus' number word sequence "1-2-...-10" was a chain, but he never created a template in this Interview.

2. Tarus and Brenda could both access the number word sequence "1-2-...-10" at any number word, but Tarus was unaware of the linkages between adjacent number words in the sense that he "lost" a number word as soon as he uttered its immediate successor.

3. All three children remembered the last number word they uttered while counting over intervals as long as 120 seconds even with intervening counting activity.

4. The last number word these children uttered while

counting referred to an experientially-bounded plurality where the items of the plurality need only be discrete units.

5. Rhythmic motor activity can play a crucial supportive role in children's number word sequence productions. This is demonstrated by Shawn's performance in the number word experiment. However, Tarus' performance in the same experiment suggests that other children develop number word sequences without ever employing rhythm.

6. Children can use the dropping-back-to-one strategy to produce a backward number word sequence based on the forward number word sequence even in the case of forward number word sequences that are strings.

Countable items

7. The establishment of a template is contextual. For example, Shawn constructed countable items in the Pendulum Interview but not in the Drum-beats Interview. The child's previous experience seems to facilitate the construction of new templates. But a new template can be constructed as well as restricted in a short time-frame, as was indicated when Tarus established a template for "beat" by observing the interviewer's striking motions and then restricting that template to establish "drum beat".

8. Rhythmic motor acts can provide support in the construction of a template as well as countable items. It was claimed that Shawn created a template in the Pendulum Interview by swinging his body to and fro in time with the swings of the pendulum.

9. A template which has been established in past experience may be modified to fit a context by a restriction or an addition. In the marked ring experiment, all three children modified a template "ring" to form a template "marked ring."

10. In the Marked Rings Interview, the initial collection of rings did not constitute countable items, because the children searched for marked rings and counted only marked rings. A countable item, then, is viewed as an instantiation of a template. The creation of a countable item was intentional, as indicated by the search activity of the children. There were numerous unmarked rings inspected, but not counted, before a marked ring was found.

11. Children apparently are not limited to empirical abstraction from perceptual material in incorporating new material into a template. Both Brenda and Shawn counted marked rings immediately upon hearing "marked ring" even though they did not see marks. Tarus, however, was forced to form a connection between "mark" and the perceptual material which constituted the marks. His behavior dramatically corroborates the claim that an object concept and, thus, a plurality, is a precondition for counting. Initially, he "saw" the rings and actually picked rings up and inspected them, but did not count. He knew he was not to count just rings, but had no idea of what it was he was supposed to count. After viewing the marks and hearing "mark" spoken in conjunction with viewing the mark, he then thought he was supposed to count marks. A corrective was crucial in his final construction of a "marked ring," after which he proceeded to behave exactly like Brenda and Shawn, initiating appropriate search activity to locate countable items.

Coordination of number word sequences and countable items

12. Counters with perceptual unit items can separate the act of creating countable items from the act of counting them.

13. Rhythmic motor acts can support the coordination between the production of a number word sequence and countable items. This is demonstrated by Shawn's performance in both the pendulum experiment and the second portion of the marked rings experiment, when he recounted the previously identified marked rings.

14. Non-rhythmic motor acts can be used to coordinate a number word utterance and a countable item. In the case of Brenda and Tarus, a motor act was intimately involved in each of their individual counting acts in the marked ring experiment. For both children, a marked ring was not counted until the motor act had been performed.

15. The utterance of a number word is essential to complete a counting act. This is demonstrated by Shawn's performance in the second portion of the marked rings experiment. Even after he had extended six fingers when attempting to count the sixth marked ring he had found and the experimenter had said "six," he still recounted all the marked rings.

16. Number-word linkages play an essential role in the coordination of number words and countable items. Shawn, whose number word sequence "1-2- . . -10" was basically a string, had to drop-back-to-one and recount all the marked rings he had found in order to count the sixth. Brenda and Tarus, both of whose number word sequences "1-2- . . -10" were chains, could pick-up counting acts where they had left off before a search. This suggests that while the lack of construction of a number word chain does not necessarily affect the child's ability to create countable items, it constrains the child's coordination of the number word sequence and countable items.

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