EXAMINING THE INTERACTIONS BETWEEN MATHEMATICAL CONTENT AND PEDAGOGICAL FORM: NOTES ON THE STRUCTURE OF THE LESSON

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Research conducted during TIMSS and later (Stigler *et al.*, 1999; Stigler and Hiebert, 1999) undertook a thorough analysis of lessons in the United States, Japan, and Germany. This article focuses on certain aspects of mathematics lessons in Russia. Specifically, the attempt is made to analyze classes as a complex system of interactions between mathematical content and pedagogical form. Just as good chess players, making their first move, are already thinking about how best to prepare for the middle and even the end of the game, good teachers construct their lessons as organic and well-thought-out entities. This article examines certain principles of lesson construction and looks at certain examples of it.

The well-formed story and the well-formed lesson

Stigler and Hiebert (1999) compare the lesson to a story, noting that:

> well-formed stories consist of a sequence of events that fit together to reach the final conclusion Ill-formed stories are scattered sets of events that don't seem to connect. (Stigler and Hiebert, 1999, p 61)

They conclude that:

> well-formed stories are like coherent lessons. They offer the students greater opportunities to make sense of what is going on (Stigler and Hiebert, 1999, p. 61)

It has been observed that students understand and retain the content of a lesson better if the lesson is coherent (Fernandez *et al.*, 1992; Yoshida *et al*, 1993), much like the listeners of a coherent narrative. The well-formedness of a story, however, is hardly reducible to mere coherence. To put it more precisely, the phrases "what is going on" and "well-formed" have a much deeper significance when used in reference to a literary narrative.

In his analysis of the short story *Gentle breath* by the great Russian writer Ivan Bunin, Vygotsky (1971) notes that the events in the narrative are not presented in chronological order. Inspired by the literary theories of the Russian Formalists (Shklovsky, 1990), Vygotsky distinguishes between *a plot* and *a subject* in the story. The *plot* refers to the factual and chronological sequence of events in the narrative, while the *subject* refers to the rearranged sequence of episodes that transforms the narrative into a work of art For Vygotsky, the key to the psychology of art appreciation consists precisely in the analysis of the interactions between the form and the content of the work of art. He remarks that the form and the content give rise to two distinct sets of emotions, which are in certain respects contradictory (as, for example, in a cold and defamiliarized narration about exciting events):

> The law of aesthetic response is the same for a fable as for a tragedy: *it comprises an affect that develops in two opposite directions but reaches annihilation at its point of termination*. (1971, p 214)

Laws discovered in the analysis of art cannot be carried over directly to the content of a lesson and it would be naive to attempt to do so Like the short story or the play, the lesson does presuppose a re-working and a deliberate arrangement of certain materials, not just their simple transmission. Still, despite the often repeated dictum that teaching is an art, putting a lesson about solving quadratic equations in the same category as a short story by Bunin or a play by Shakespeare can fairly be considered to be an exaggeration. A lesson often looks much more like a section in a reference book, where materials are also organized according to certain principles and not just thrown together in any old way, but where the organizing principles are much simpler and in no way oriented toward an aesthetic reaction or some other complex response

Nevertheless, the aim of a lesson (at least, a good lesson) cannot be reduced to making sure that the important formulas end up written in the middle of the board in large letters. Discussions of possible ways to motivate students invariably reach the conclusion that intrinsic motivators are necessary (and not just extrinsic ones, such as praise for success and punishment for failure (Posamentier and Stepelman, 2002). Such intrinsic motivators may include, for example, showing the students:

> how the topic to be presented will complete their knowledge about a particular part of mathematics. (Posamentier and Stepelman, 2002, p 56-57)

As the authors of a textbook for future teachers observe:

> The more dramatically you do this, the more effective the motivation. (Posamentier and Stepelman, 2002, p. 57)

In fact, the point at issue here concerns precisely those principles of lesson construction that are capable of eliciting an emotional response from the students. Once the existence of such a connection between lesson construction and student reaction is recognized, it becomes clear that it is an object that deserves thorough investigation

However, the principles of lesson construction and the patterns of students' responses to various lesson constructions

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remain insufficiently examined. Experience with similar investigations in other fields (including art) suggests the possible directions that such an analysis might take, as well as the potential misperceptions and pitfalls that it might encounter.

One possible misperception that might lead to inadequate attention to lesson construction and composition is the simple negation of their significance. In such cases, the explicit response (of the reader, audience member, student) is attributed exclusively to the qualities of the material, without taking into account the order in which the materials are presented. Meanwhile, just as the most successful line of verse would be less striking without the ones preceding it, as well as the ones following it, so too a problem given during a lesson turns out to be much less significant if it appears in another context. However, the common focus on, say, 'problems for motivation', although it does sometimes enable teachers to find wonderful problems, often completely overshadows any concern for the order of their presentation and their placement in the lesson.

To a large degree, lack of attention to the problem of lesson composition has been due to the fact that, for a long time, classroom procedure was thought of as being largely fixed and identical for all lessons. Traditionally, teachers conducted all lessons in virtually identical ways, beginning by checking students' homework, and following this with their own lecture. Today, teachers almost always begin with a 'do now' or a 'motivational activity', and then go on to 'seatwork' or 'teamwork' (see, for example, Brahier, 2000). This situation changes when teachers (at least the most active ones) recognize their new methodological freedom, which liberates them from these unofficial, but active, constraints and compels them to search for more effective forms of instruction.

In recent years, attention to lesson composition has grown with the increasing popularity of lesson studies (Stigler and Hiebert (1999). However, the analysis of lesson composition is hindered by the fact that the lesson schema is often seen as nothing more than a sequence of procedures without regard to their content (or conversely, as the presentation of content without regard for procedures). Which specific problems were given to 'do now', and why these problems were given when they were, are issues that often remain un-discussed. Meanwhile, lesson descriptions sometimes become as simplistic as, say: First the students factored quadratic trinomials, then they solved quadratic equations, and then they worked on word problems.

Forty years ago, these shortcomings of pedagogical research had already been addressed by the well-known Russian psychologist Zankov. Zankov (1962) quoted a passage from an article on the analysis of classroom practice:

> Two students were called up to the blackboard. Each was given a card with a problem, and the teacher explained what they were required to do. (Zankov, 1962, p. 67 [1])

And then he posed the question:

> What were the problems given to the students? Were these normal, standard problems, or did they reflect the original approach of the teacher? [. .] What kind of an explanation did the teacher give? The character of a student's activity depends on the character of the explanation. (Zankov, 1962, p. 68 [1])

The effectiveness of the lesson depends precisely on the interaction and interrelation of the form and content of the lesson, i.e. its procedures and its mathematical content. Bearing in mind all the differences between the two objects of investigation, it appears reasonable to apply Vygotsky's observations about art to the lesson i.e. it is precisely the interaction between its form and its content that determines its emotional impact on the students. New technology and the widespread use of video recordings have made it possible to isolate the connections between these two levels of the lesson, enabling a deeper analysis of the patterns of lesson construction. But technology by itself cannot replace analysis, which involves the identification of various principles of lesson construction and the examination of the reasons for their success or failure. Below, an attempt at such an analysis will be made using Russian materials.

Russian mathematics education: not comparing, but studying

During the Cold War, Russian (or more accurately, Soviet) mathematics education attracted a great deal of attention from Western educators (for example, McClintock, 1958; Swetz, 1978). With the end of the Cold War, this interest subsided. Naturally, the achievements of Russian education in the instruction of talented students, which were recognized at the time (Vogeli, 1968), still command much respect (Campbell, 1996). The new possibilities for Western mathematics educators to visit Russia, or for Russian ones to visit the West, have resulted in publications that identify various differences in the way mathematics is taught in the West and in Russia, as well as differences in the way children behave in the classroom (Toom, 1993; Watson, 1993). Nevertheless, it is impossible not to agree with Froumin (1996) that there is a lack of understanding of the Russian system and an excess of stereotypes about it among education specialists.

Explicitly or implicitly, analyses of the Russian system often end up being simple comparisons with, say, the American system (accompanied by the corresponding assessment of which of the two is better) or speculations about the possibility of transferring one or another element directly from one system to the other. Since the overwhelming majority of pedagogical traditions (no matter how they are evaluated) are deeply rooted in the culture and history of a country, it is clear from the start that simply transferring them from one context to the other will be impossible.

What is possible, however, is a careful examination of classroom practice that can lay bare various significant processes, patterns, or problems. Researchers who have been enriched by the experience of such an analysis will be able to see more in their own native system, if only because what they observe in it will no longer appear to them as the only possible theoretical approach.

The Russian mathematics teacher usually works with the same group of students for at least two or three years. In addition, the Russian program is structured in such a way that the material is covered only once (a brief review at the end or the beginning of the year is possible, but in such cases students never end up, as it were, studying the subject from the beginning all over again). These characteristics presuppose a certain coherence in the entire teaching process and create a favorable situation for structuring each lesson as a unified whole. Russian lessons are therefore of particular interest for the analysis of the possibilities of lesson construction. Researchers (Wilson *et al.*, 2001) have noted that Russian lessons can perhaps be:

> characterized as a focused series of linked tasks, each of which comprises a blend of oral work, pupil demonstration, written recording and teacher questioning and explaining. (p. 41-42)

The remarks that follow attempt to analyze how this "blend" is achieved.

Several lessons

Below I offer short transcripts of several lessons that were taught in Russia. These lessons are limited to what are known in Russia as problem solving lessons, whose objective is indeed to teach students how to solve specific types of problems and to provide them with practice in such problem solving. Usually, such lessons are juxtaposed, to a certain degree, with lessons devoted to 'the introduction of new theoretical material' (although the lessons examined below also contain such material) or to lessons devoted to practice and repetition. It is useful to compare the methods of lesson construction employed in several different lessons, even if the following three examples display a certain degree of similarity. It should be kept in mind that all three lessons are devoted to subject matter that is quite typical for Russia. The lessons preceding them were devoted to related questions, and the students are therefore quite prepared, for example, at the beginning of the second lesson examined below, for solving equations with radicals.

Lesson 1. Solving equations that can be reduced to quadratic form: ninth grade (fourteen- to fifteen-year-old students)

1. The teacher says that the lesson will be devoted to solving quadratic equations and equations that can be reduced to quadratic form

2. The following equations are written out on the board:

   a) $(x - 1)(x + 2) = 0$

   b) $x^2 - 3x = 0$

   c) $x^2 - 4 = 0$

   d) $x^2 - 5x + 6 = 0.$

   The students solve them orally. Students called on by the teacher explain, from their seats, how they solved these problems.

3. The teacher explains, on the board, how to solve the following equation:

   $x^4 + 6x^2 - 7 = 0.$

   The teacher tells the students that the technique used here is called 'chunking'.

4. The following equations are written out on the board:

   a) $x^4 - 3x^2 = 0$

   b) $(x^2 + 1)^2 - 4 = 0$

   c) $({}^1/_x)^2 - {}^5/_x + 6 = 0.$

   The students solve these problems in their notebooks. Students, called on by the teacher, explain from their seats how they solved them

5. The teacher explains that the preceding problems all made use of the same technique – chunking. The teacher then gives the students the following set of equations to solve:

   a) $(x^2 + 3x)^2 - 2(x^2 + 3x) - 8 = 0$

   b) $(x^2 + 4x)^2 - 2x^2 = 8x + 15$

   c) $\dfrac{12}{x^4 - 8x^2 + 16} + \dfrac{1}{x^2 - 4} - 1 = 0$

   The students solve them in their notebooks

6. The solutions to these problems are worked out on the board.

7. The teacher asks the class to figure out a way to use chunking in the following cases:

   a) $\left(\dfrac{x^2 + x + 1}{x}\right)^2 - 3\left(x + \dfrac{1}{x}\right) - 3 = 0$

   b) $2x^4 + x^2(x^2 + 1) - (x^2 + 1)^2 = 0.$

8. The students solve the problems in their notebooks

9. Solutions are proposed and discussed. The teacher shows the class how equations can be reduced to quadratic form.

10. The teacher sums up the lesson

The content of this lesson, as well as its structure, is quite characteristic for a strong class in Russia. The technical difficulty of the material gradually increases throughout the course of the lesson.

At first, the students are given a set of oral problems Here, in effect, they repeat and go over what they have already learned. The first set of problems contains all the basic types of quadratic equations covered earlier. The oral format, with none of the students writing, and everyone listening to the person speaking, makes it possible to obtain the greatest degree of attention from the students, and to get them to focus on the problem-solving procedure. These techniques are the foundation for everything that follows.

The second key step in the lesson consists of a direct explanation by the teacher. The teacher introduces a new idea of chunking. This is immediately followed by a symbiosis - the new idea is combined with the old techniques rehearsed earlier. The reliance on the old techniques is underscored by the direct reference to the previously worked-out problems - the chunking operation reduces the new problems to the old ones.

The students accomplish the proposed symbiosis on their own, but the time they spend working by themselves is short, and their results are immediately discussed. The teacher also plays an important role, generalizing the students' results theoretically, and repeating that a general technique has been employed.

The next block of problems is the same in essence, but more difficult. Therefore, more time is devoted to it - the students work under less pressure, individually and not in one big group. The key point, however, is that this block of problems will also be checked more thoroughly, on the board.

The last problems again shift us from written work by individual students to oral work by the whole class. The technical difficulty of the solution has in large part been overcome, but a new kind of difficulty appears on the new technical level - the newly introduced variable may look quite complicated. Now the students are asked to 'grasp' this new variable.

To a certain extent, they are helped by the fact that the last problem in the previous block already involved a fraction as a new variable, and the other problems also required them to perform certain (although obvious) operations in order to see how they could be reduced to quadratic equations. In an oral discussion of the problems, the class is introduced to the general idea that putting an equation into quadratic form can presuppose certain preliminary transformations. This fact is emphasized, one more time, by the teacher.

Lesson 2. Using equivalent equations to solve equations with radicals: tenth grade (fifteen- to sixteen-year-old students)

1. The teacher tells the students that the lesson will be devoted to solving equations with radicals.

2. The following equations are written out on the board:

   a) $\sqrt{2x-3} = 1$

   b) $\sqrt{3x+1} = -2$

   c) $\sqrt{2x+6} = \sqrt{x-1}$

   d) $\sqrt{2x-6} = \sqrt{x+1}$

   e) $\sqrt{2x^2+2x-4} = x-1$

   f) $\sqrt{2x^2-6x+4} = 2-x$

Three pairs of students are called up to the board, one after the other, to solve problems a and b, c and d, e and f, respectively. In the meantime, the other students solve these problems in their notebooks. The solution of the problems involves finding the roots of the equations and checking them (for example, equation (e) is solved as follows:

$$\sqrt{2x^2+2x-4} = x-1,$$
$$2x^2+2x-4 = (x-1)^2,$$
$$2x^2+2x-4 = x^2-2x+1,$$
$$x^2+4x-5 = 0,$$

Therefore, $x = 1$ and $x = -5$

Plugging these values into the original equation shows that only $x = 1$ fits, and that $x = -5$ is not a root of the original equation). The students discover that certain solutions have extraneous roots that do not fit the original equation and must be rejected.

3. The teacher asks why there were extraneous roots in certain cases but not in others.

4. Students, from their seats, offer various responses (focusing on differences between various types of expressions). These responses are discussed and rejected.

5. The teacher asks the students to break up into pairs and to figure out (by plugging in the obtained values at every step of the solution) at what point the extraneous roots first appear. The students discover that this happens when the expressions are squared.

6. The teacher asks again why there were extraneous roots in certain cases but not in others.

7. The students break up into pairs and discuss this question.

8. The results are discussed by the whole class.

9. The teacher sums up the results, observing that if equations of the form:

$$\sqrt{f(x)} = g(x)$$

are replaced by the system:

$$\begin{cases} f(x) = (g(x))^2 \\ g(x) \geq 0 \end{cases}$$

and if equations of the form:

$$\sqrt{f(x)} = \sqrt{g(x)}$$

are replaced by the system:

$$\begin{cases} f(x) = g(x) \\ g(x) \geq 0 \end{cases}$$

then no extraneous roots will appear In some cases above, the roots of the obtained equations satisfy the inequalities, and in other cases they do not.

10. Two more equations are given:

$$\sqrt{x^2 - 5x + 6} = \sqrt{3 - x}$$

and:

$$\sqrt{2x - 3} = x - 3.$$

The students are asked to solve them in their notebooks by constructing a system like the ones above

11. The results are analyzed on the board.

12. The teacher sums up the lesson

This lesson begins as a routine exercise in perfecting a technical skill Students are given problems that are perfectly analogous to problems examined earlier The method of working with such problems is likewise sufficiently traditional - it is assumed that most students will do their work more quickly in their notebooks than those solving the problems on the board In addition, there are many boards and the solved problems are not erased, so the students who stay in their seats can work at a pace that is comfortable for them, with enough time to check their own solutions against the ones on the boards

Suddenly, the routine flow of the work is interrupted The class shifts from what is essentially individual work to a collective discussion. The discussion is oral and, in this instance, superficial - its purpose is to remove possible initial assumptions and to spur the students on to a more thoughtful search for an answer

This is what the students begin doing as they discuss their results with their neighbours (note that in Russia students usually sit two to a desk). This work, however, is structured by the teacher in such a way that it is interrupted by a collective discussion, which sums up the results obtained so far and poses a new question As a consequence, the students find the answer to the given question independently, formulating it in ordinary, colloquial language The teacher expresses the same thought in mathematical language, making it somewhat stronger at the same time

Following this, collective and group discussions give way to individual work Working individually, the students apply the new algorithm Their results are then examined and checked on the board, with the whole class once again working collectively.

Lesson 3. Comparing numbers and proving inequalities: eighth grade (thirteen- to fourteen-year-old students)

1 The teacher tells the class that the lesson will be devoted to inequalities and to solving problems involving inequalities

2. The following problems have been written out on the board beforehand:

a) Compare the numbers $x - 2$ and $x + 3$; $x^2 - x$ and $-1 - x$; $x^2 - 4x$ and -4; and $x/2$ and $x/3$, given that x is positive

b) One person walked a certain distance at a speed of 4 km/hr, while another person walked the same distance at a speed of 3 km/hr, which of them covered the distance more quickly?

The students are asked to solve these problems orally The teacher calls on several students in a row and they explain (from their seats, without writing anything) how these problems are solved.

3. The students are given the following problem:

> Two cars travel from one city to another on the same road The first car drives at a speed of 40 km/hr, while the second car drives at a speed of 20 km/hr for the first half of the way, and at a speed of 80 km/hr for the second half of the way. Which car will be the first to reach the destination?

The students solve this problem in their notebooks

4 A student is called up to the board and writes down a brief solution

5 The teacher asks the students to try out the same problem with different numbers. He himself suggests 60, 30, and 120, respectively, and asks the students to explain why he chose these numbers

6. In a discussion, the students:

a) explain that these numbers, like the previous ones, are such that the second is twice as small as the first, while the third is twice as large

b) solve the new problem

c) suggest other variations on the problem such as 30, 15, and 60

7. The teacher asks the students to think about how they might state and solve this problem in a general form.

8 The students work individually and in pairs

9 One pair of students explains their solution, which involves looking at the same problem for the numbers x, $x/2$, and $2x$.

10 A student asks what will happen if one takes numbers that are, respectively, three times smaller and three times greater than the first The teacher asks the students to develop the problem even further by answering this question as well

11. The students work in pairs

12. One pair of students writes down the formulation of the problem for the numbers x, x/k and kx. Another pair explains the solution (the comparison may be reduced to a comparison between the numbers 2 and $k + 1/k$).

13. The teacher sums up the results, commenting that this problem is a generalization of the original problem The teacher asks the students to solve the same problem for the numbers 40, 20 and 60 for homework, and try to generalize it

This lesson, too, begins with oral work, whose purpose is to repeat what the students have already learned and to prepare them for what they are about to learn. For instance, there is a recurring shift throughout the lesson from a given distance and the speed with which it was covered, to the time it took to cover it; this shift appears for the first time in this oral portion of the lesson.

The first problem which is then examined is somewhat harder than the oral one, but is in principle not very different from it. Once again, there is a gradual increase in the level of difficulty - the new problem is solved by the students individually, but it is preceded by a preliminary oral problem that is worked out by the whole class, and it is examined and checked orally, once again by the whole class. As in the lesson analyzed above, the teacher's question interrupts this flow The collective discussion now plays a different role - in effect, it sets the stage for a new problem, which is a generalization of the previous one.

The students' creative work on this problem is done individually, while their results are discussed collectively. In the course of this discussion, a new question is posed. And if previously it was the teacher who posed the question that enabled a transition from a specific problem to a generalization, then now it is a student who poses such a question, in effect following the teacher's example Now the steps examined above are repeated: individual work is once again followed by a collective discussion (note that the idea behind the proof of the inequality, which is connected to the completion of the square, was already mentioned in the oral exercises). Finally, in the conclusion of the lesson, the teacher asks the students to continue their work on the generalization at home, now completely independently.

Discussion

The obvious distinguishing characteristic of all of these lessons is the fact that the students spend comparatively little time on individual work, and that the role of the teacher is quite great This difference between Russian and many Western lessons, which has also been observed in elementary school classes (Wilson *et al.*, 2001), can be explained by the greater popularity of Piaget's views in the West, and by Vygotsky's influence (in however simplified a form) in shaping pedagogical practice in Russia

What should be emphasized, however, is that the prominence of the teacher's role does not imply that the teacher treats the class as a lecture and in general does all the talking. In all three lessons, the teacher talks relatively little. Rather, the teacher's contribution consists in organizing the lesson from a fairly rigidly calculated sequence of segments, in the course of which the students can construct their own knowledge.

Students in all classes construct their mathematical knowledge in a special classroom world, which is different from the real world. Continuing the comparison between the lesson and the short story made above, it can be said that, just as readers experience events that might take place in the real world with an increased intensity and concentration when reading a story, students encounter questions that might appear in the real world in a more condensed state in the world of the classroom. It is fair to say that this condensation, this *artificiality*, is quite intense in the lessons examined above. They are constructed not according to some simple principle of composition applied to mathematical materials (such as 'from the simple to the complex' or 'all about equations of a given type'), but according to their own special laws (where the *plot* and the *subject* do not necessarily coincide).

What are these laws? The key factors in the effectiveness of the analyzed lessons are the *rhythm* and *tempo* of the action. They hold the students' attention and maintain a certain degree of emotional intensity They are achieved in large measure through *the variety of actions and procedures* in which the students are involved, and also through *the thought-out progression* of these actions and procedures For example, it is clear that the students cannot work orally for a long time, since this requires a great deal of attention and concentration. It is necessary to alternate oral activities with 'low pressure' exercises, such as calm, individual work In an analogous way, the structure of the lesson makes use of *the alternation of blocks of new and old materials.* Routine materials flow more easily, but become boring when used in excess; the quantity of new materials that can be introduced during a lesson, on the other hand, is, of course, limited

The specific ways in which various procedural and topical blocks alternate, and the specific ways in which the substantive aspect of the lesson is connected to its formal character, is what determines its construction and its impact on the students (including its emotional impact). These formal and substantive aspects of the lesson are in their turn determined by the goal of the lesson. In this respect, the role of formal procedures can vary quite widely The three lessons examined above may be described in terms of the following schemas:

Lesson 1: Collective oral work - explanation by the teacher - individual written work - collective oral work - individual written work - collective work (on the board) - individual written work - collective oral work

Lesson 2: Parallel written work individually and on the board - collective oral work - individual-group written work - collective oral work - individual/group written work - collective oral work - summary by the teacher - individual written work - collective work (on the board)

Lesson 3: Collective oral work - written work - collective work (on the board) - collective oral work - individual written work - collective work (on the board and oral) - individual-group work - collective work (on the board) - collective oral work - individual-group work - collective work (on the board) - teacher's remarks.

However, the substantive role of, say, oral work, can vary greatly. In these lessons, it is always connected with collective work by the entire class, but its purpose can range from going over standard and well-known materials, as in the beginning of Lesson 1, to defining newly observed patterns, as in Lesson 2. It can be used to discourage students from looking for a simple and obvious answer, as in Lesson 2, or it can be used to encourage them to search for new questions, as in Lesson 3.

Collective work in these lessons is quite frequently employed for the introduction of new material, leaving individual work for exercises to practice what has been discovered. This is by no means the only approach, however. For example, the 'narrative hook' in Lesson 2 is connected precisely with the students' sudden discovery that a routine exercise contains unexpected content: the accumulation of material leads to an independent discovery.

In Lesson 3, the teacher, for a moment, breaks out of the traditional role of the teacher as the source of the problems in the lesson. He invites the students to take his place, as it were - the place of the person asking the questions (which is, to a certain degree, what actually happens). In this way, the procedure again turns out to be different from what the students expected, and this lends a certain dramatic tension to the lesson

The goals of the teachers in Lessons 2 and 3 are markedly different from the goals of the teacher in Lesson 1 Lesson 1 is relatively uninterrupted - its content consists of problems gradually increasing in level of difficulty, given by the teacher However, in Lessons 2 and 3 the teachers must spur on the students to perform independent and non-standard conceptual operations. As a result, the procedures employed in these lessons are also different in character.

Naturally, it would not be difficult to invent a way of presenting the same mathematical content without breaking any established canons of classroom procedure. But it is unlikely that such a lesson would be equally engaging to the students Just as works of literature often depart from the chronological sequence of events (the plot) and narrate them in a different order (the subject) that allows readers to see and feel their inner significance more vividly, the *subject* of a lesson may differ from its *plot* i e. its mathematical content as it would be laid in a manual, as a simple logical progression.

In the students' perception, the procedures of the lesson are in many ways connected to a specific content that seems 'natural' for these procedures. By breaking this connection, by playing on the contrast between what is subconsciously expected and what is actually presented, the teacher creates an additional emotional tension (recalling Vygotsky) and in this respect the teacher does, to a certain degree, resemble the creator of a work of art.

Finally, on the practical aspects of lesson composition: even more than a literary composition, a lesson can be analyzed to determine to what extent it is a product of the author's individual creativity, and to what extent it reflects the influence of certain current models and accepted ideas Teachers usually compose their lessons on their own (or more precisely, in co-authorship with their students). However, like writers of literary compositions, teachers possess various kinds of templates.

In the Russian system of mathematics education, which has been quite centralized up to now, teachers are expected to make use of recommended model lessons. These model lessons are published in the teachers' manuals that accompany textbooks and are also presented orally by expert teachers in special lectures on the subject. Without going into a detailed analysis of these lessons and comparing them, for example, to the lessons described above, it should be pointed out that these lessons are typically characterized by many of the same traits that were mentioned earlier, such as a high intensity and variety of forms of classwork. Working teachers are in large part educated through these examples

At the same time, the structure of the lessons that are offered by these published manuals is usually not very complex. Using the terminology introduced earlier, it can be said that in most cases the *plot* and the *subject* of a lesson coincide. More precisely, the authors of the manuals typically present just the lesson's *plot* Their decision to do so is probably a reasonable one: a 'dramatization' of the lesson hardly seems feasible in the absence of a real audience Thus, it falls to teachers themselves to invent such a 'dramatization', by taking into account the specifics of their classes and creating a unique combination of mathematical content and classroom procedures

And just like authors of literary compositions who have critics and admirers, such teachers can at least sometimes expect to have adult observers and experts in their audience. In Russia, the practice of so-called open lessons is widespread, and teachers (from the school, the school district, and sometimes even the whole city) are often invited to attend the classes of their colleagues. Such visits, and the discussions that follow them, inevitably exert an influence on teachers, broadening their conception of how lessons can be structured.

Conclusion

The preceding remarks make clear that the interaction between the procedures and the content of a lesson deserve careful study. Clearly, the significance and drama of this interaction become weaker if the special *condensed* world of the lesson is ruptured (Stigler and Hiebert (1999) describe the astonishment of Japanese mathematics educators at the ease with which lessons in the United States are interrupted without particular necessity). Clearly, it would also be interesting to analyze lessons in other countries, and for students of other ages, and with a different mathematical content (we should note parenthetically that much of what is traditionally studied in Russian schools can seem to be outdated and unnecessary). Teachers are often compared to actors, but they are actually actors and directors and playwrights The works they create call for analytic descriptions.

Note

[1] Translated from the original Russian by the author

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Returning to the *Oxford English Dictionary*, we learn that a *principle* is

> A fundamental truth or proposition, on which many others depend; a primary truth comprehending, or forming the basis of, various subordinate truths . .

[. .] I take the first point as reminding us that to call something a law is not a neutral act: you usually use the label *law* only for assertions that you feel have some intrinsic structural priority over their consequences. I take the second point as emphasizing (requiring, even) the *economy* of thought inherent in something we call *law*: a single principle can cover much territory. (pp 74-75)

Stars, for example, are everywhere in poems. Nevertheless, the emotional tone of the phrase "the stars looked down," taken in isolation, is as yet indeterminate. Those stars could present a benevolence blanketing us with their warmth, they could be comforting witnesses that all's right with the cosmos, or else they could be glaring down, affecting a sinister penetration, a cold perception of our unimportant place in the cosmos. The poet, using such a phrase, has the liberty of carving an exquisite, meticulously specific, emotional context for the phrase, and to achieve this, can call up the full resonance of the metaphoric tradition regarding the image of stars. (pp. 88-89)

(Mazur, 2003)
