Audacity and Reason: French Research in Mathematics Education*

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I—Two hundred years later

Just under two hundred years ago in France, the National Convention decided that "the revolution that had taken place in the social and politic system" should "also take effect in the theory of the sciences and the arts" (introduction aux Séances des Ecoles Normales, An IV, Imprimerie du Cercle Social) and that one of its aims was to "give the French people a system of instruction worthy of their new destinies" (decree of 24 Nivôse, Year III of the French Republic, one and indivisible) We know that the most prestigious scientists then applied themselves to the development of ideas about the best teaching methods to initiate the greatest number of pupils in the sciences.

I would like to put across the idea that a little of this enthusiastic and audacious spirit blows through our desire to develop the field of research which *didactique des mathématiques* constitutes in France and that our approach is not exempt from this revolutionary tradition Is it not contrary to generally accepted ideas to attempt to describe or rationally explain teaching phenomena which more usually give rise to empiricism or opinion rather than reasoned discourse?

At a date not quite so far in the past, on the occasion of the first conference of our group in France (in 1981, in Grenoble), G. Vergnaud introduced the theoretical and methodological trends of French research by calling to mind not its revolutionary nature but how it differs fundamentally from foreign research. My paper will sketch the evolution of this research from this date, by presenting of course its original aspects, but also by trying to link it to the concerns which inspire foreign research, as it seems to me (thanks largely to the work carried out within the PME Group) that objective links have been created between our approaches, or at least our questions, and the approaches and questions developed outside France.

I cannot of course present all of French research, in which about a hundred researchers are involved. I have made deliberate choices in order to keep only certain themes. One of the concerns widely shared within the French community is that of setting up an original theoretical framework developing its own concepts and methods and satisfying three criteria: relevance in relation to observable phenomena, exhaustivity in relation to all relevant phenomena, consistency of the concepts developed within the theoretical framework [Brousseau, 1986] A wide consensus also exists on the methodological requirement of using experimentation in dialectic interaction with theory: the experimental paradigm is devised within a theoretical framework, the observations of the experimentation are then compared to this framework and may well modify it in the light of the three preceding criteria The ambition of such a programme is obvious and the development of such a framework cannot be the work of a single researcher or a single team, it can only be the fruit of interaction and co-operation on a national scale. Several national institutional sites, where such exchanges take place, have been set up in France: a CNRS (National Scientific Research Centre) research group, a seminar (three times a year) and a two-week summer school (every two years)

II—The relationships between teaching and learning

What we call *didactique des mathématiques* in France covers the study of the relationships between teaching and learning in those aspects which are specific to mathematics. One widespread ideology presupposes a connection of simple transfer from teaching towards learning: the pupil records what is communicated by teaching with perhaps some loss of information. Numerous studies conducted within PME have shown clearly how mistaken this point of view is by highlighting in particular the characteristics of the concepts constructed by pupils concerning arithmetic, algebraic or geometric notions which are not contained in the teaching discourse: these concepts are local, partial and even false. These observations forewarn us of the complexity of the links between teaching and learning *This complexity is the origin and at the core of our research.*

In its first phase, French research undertook the study of these links, concentrating on the learning of mathematics knowledge whose processes, linked to the disciplinary contents, were still little known. Even though a study of the levers controlling the evolution of conceptual acquisitions had been undertaken and a theory of didactic situations already developed [Brousseau, 1981], these questions were taken up by French research — which, above all, centred itself on the cognitive activity of the subject; in a word, restricting itself to the knowledge-pupil subsystem For eight years French research has continually accentuated a trend towards the study of the dependencies between teaching and learning, attempting to answer such questions as: how can we characterize the conditions that must be put to work in teaching to facilitate learning which has certain *a priori* characteristics? Which elements in the description of a teaching process ensure that it can be reproduced from the point of view of the learning it allows the pupils? Such questions presuppose that the criteria for characterizing learning have been chosen in order to judge the possibility that a teaching process can be reproduced. The fundamental criterion which conditions our approach to these questions is that of meaning: what is the meaning of the knowledge which we want the pupils to acquire? What is the meaning of the concepts constructed by the learner during a teaching process?

French research in didactics has shown a desire to apprehend teaching situations globally, to develop a modelization which encompasses their epistemological, social, and cognitive dimensions and which attempts to take into account the complexity of the interactions between knowledge, pupils and teachers within the context of a particular class, or more generally of an educational group. These aims can be approached in several ways. One could attempt to pick out "good" teaching situations and try to extract the expert knowledge of "experienced" teachers in order to characterize the values of the input variables (in teaching) which guarantee "good" learning by the pupil as the outcome. That is not the approach chosen. It consists rather in describing the functioning of a teaching system as that of a system dependent on *choices* and subject to *constraints (the didactic system)*; in extracting these constraints and choices; and in observing how different choices give rise to different ways of learning from the point of view of meaning — that is, the construction by the pupils of the different meanings of the notions taught.

III—Choices and constraints of the didactic system

One of the axes of research in didactics consists in extracting the constraints which influence the didactic system and analyzing how they function. The most important of these constraints are:

1 — the characteristics of the knowledge to be taught, in particular the dependence between the mathematical objects which must be taken into account in the creation of a coherence in the content to be taught;

2 — the social and cultural constraints which act within the educational project to determine the teaching content;

3 — the temporal characteristics of teaching fixed by the syllabi, in particular its linearity;

4 — the pupils' concepts, their modes of cognitive development which condition access to new knowledge;

5 — the teacher-learner asymmetry in relation to the knowledge embedded in teaching situations (*didactic contract*);

6 — the teachers' knowledge, their ideas and beliefs about mathematics, teaching, learning, and their own profession.

These constraints act together and have only been separated in order to be exposed. They do not all occur at the same levels of the teaching process; constraints 1, 2, 3 and 4 particularly affect the determination of the knowledge to be taught (*didactic transposition*) in the upper part of the teaching process, whereas constrants 4, 5, and 6 operate more especially in the lower part, where the teaching is carried out. French research has contributed to the formulation of these constraints, the study of which is now giving rise to a wider interest in them in the international community. If the terms *didactic transposition* [Chevallard, 1985] and *didactic contract* [Brousseau, 1981] have left France to go beyond our frontiers, the necessity for an epistemological analysis has also been felt in the Federal Republic of Germany, and is currently established in research programmes such as the Research Agenda Project of the NCTM in the United States; the implicit rules which control the relationships between teachers and pupils in a class have been the object of ethnographic-type research (for example the notion of *Arbeitsinterim* [Krummheuer, 1983] presents definite links with that of didactic contract), the ideas of teachers make up a paradigm for research in Great Britain and in the United States (the report written by Cooney and Grouws [1987] can be consulted on this subject).

The possible choices in the setting up of a teaching process are made according to the constraints to which it is subject. These choices arise from generally implicit hypotheses on how to satisfy the constraints of the system, and the wish to highlight the existence of such choices contributes to a better knowledge of teaching phenomena. This has a strong influence in professional teacher training as it enables teachers to avail themselves of explicit tools for taking decisions in a class, as Romberg notes [quoted by Cooney and Grouws, 1987].

Three types of choice are fundamental, those relative to

- — the choice of content to be taught
- — the planning of interactions between learners and the knowledge to be learnt
- — the interventions and role of the teacher in a class situation.

The choice of teaching content and its organization is based on epistemological-type hypotheses and learning hypotheses [Arsac, 1989]. Let us quote as an example two choices which were advocated in France and carried out on a more or less large scale. The linear axiomatic presentations [such as one presented by Choquet, 1964], the result of organizational work carried out by mathematicians at the level of expert knowledge, which were proposed for the structuration of curriculum content and strongly influenced the so-called Modern Mathematics syllabi. They are based on the hypothesis that the meaning of mathematical knowledge arises from the logical hierarchical structure in which it can be placed; the learning hypothesis which is naturally attached to it is that of an accumulation of acquisitions, a new acquisition adding itself to the previous ones without challenging them or modifying them. These two hypotheses permit, in a very economic way, the satisfaction of the constraints of coherence of teaching content and linearity of teaching time.

As a reaction to this choice, the French Association of Mathematics Teachers recommended, from 1972, the organization of teaching content into *core-themes*, the core being made up of "fundamental notions which each pupil must have acquired by the end of the year," the themes being chosen by the teacher and the pupils, "either to motivate the introduction of fundamental themes, or to illustrate the uses of these notions, or to encourage supplementary research whose spontaneity would give the pupils a foretaste of the free studies they may perhaps undertake as adults" (*Bulletin de l'APMEP,* no. 300, Sept 1975). The potential themes are numerous — electricity bills, magic squares, tides, the daily cost of a dog in a city (*ibid.,* p. 457). In these propositions the meaning of some knowledge is given by the context of its use, and is learnt better as the same notion is encountered by the pupils in a great number of activities (*ibid.,* p. 419). An underlying hypothesis is that the pupil can by himself abstract mathematical knowledge from the contexts in which he has more or less implicitly used it. In *tool-object* terms [Douady, 1985] the tool aspect of mathematical notions is given greater importance; as a consequence the constraints of coherence, of knowledge dependence, and of linearity of time are difficult to satisfy (which probably explains why teachers abandon textbooks based on the principle of core themes so quickly). The constraint of adapting the teaching to the pupils is fulfilled by taking more into account the pupil's tastes, their familiarity with the chosen contexts than their knowledge and their cognitive capacities.

Different analyses of existing curricula and the underlying hypotheses relative to a given conceptual field have been carried out as a preliminary step to research on the teaching-learning of the notions concerned.

Another type of choice is made in the organization of the interactions between pupils and the knowledge to be learnt. An extreme choice (which is not generally made) consists in leaving the interaction completely to the initiative of the learner, presenting him with only the text of a decontextualized body of knowledge in a form which could be qualified as *concept-definition,* to use Tall and Vinner's terms [1981]. The underlying learning hypothesis postulates the appropriation of knowledge by the learner without any transformation of it by him, just as the knowledge was formulated by the teacher.

In the practice of teaching there is generally a process of contextualization of the knowledge taught — that is to say, the organization of a context which *situates* this knowledge, in which the activity of the pupils can operate. The interactions between knowledge and pupils operate through the context, the *milieu* [Brousseau, 1986]. The anticipated interactions can arise from different choices.

Let us then compare three situations which introduce the notion of symmetry: in the first, frequently used in textbooks, a figure is folded along its line of symmetry and the pupil is asked to make as many comments as possible; in the second, he is asked to draw a line so that when the figure is folded along this line the two parts of the figure are superimposed; the third situation differs from the second only in that the rigid material on which the figure is drawn cannot be folded. The cognitive processes implied by the pupil's responses to these three situations are different. In the first an inductive approach is sought from the pupil [Joshua, 1987], hoping that by simple observation he will naturally extract the properties which are relevant from a mathematical point of view; in fact the criteria of relevance are not provided by the situation, only the teacher knows them. The pupil is partially freed from the responsibility of his answers and part of his work will consist more in seeking signs of relevance outside the situation than in analyzing the figure: he is in maths class and therefore should pay more attention to equalities, parallel or perpendicular relations than to the colour of the figure, for example. This situation must necessarily be summed up by the teacher, who will give a list of comments. A theoretical analysis of this type of situation has been developed by Voigt [1985] who points out the frequency of such episodes in ordinary teaching.

In the second situation the pupil finds in the situation itself (by folding) a way of establishing whether his answer is correct or not, but if folding is an instrument of validation, it is also an instrument of solution: the pupil can have direct access to the solution by successive attempts at folding and adjustment; the question has been transformed, now all that remains is a perceptual activity of matching two figures.

In the third situation, the perceptual verification is blocked as the pupil can no longer fold the material; the question now is to put into real effect the properties of symmetry in order to draw the line of symmetry. These are the properties which form the instrument for the solution. However, the material situation no longer provides the instrument of validation of the pupil's answer and because of this it does not allow him to become aware of an incorrect reply (for example, if in the case of a rectangle he drew a diagonal) and to try and modify it. The situation in itself does not really allow the evolution of the pupil's answers. In a way, the situation can only be concluded by the teacher, as in the first situation.

These variations on one example were chosen to point out that in this organized interaction between knowledge and pupils, two criteria play a decisive role as to the target learning:

i) the question to which the pupils actually reply;
ii) the feedback which the situation offers to the pupils answers.

The interactions between the situation and the pupil can arise directly from the material environment, as in the case of folding to find symmetry; they can also spring from the pupil's own acquisitions — for example, as the situation contradicts his previous acquisitions. The pupil's acquisitions act as *validity criteria* [Margolinas, 1989] for his answer. As a mathematical notion progresses along its learning path, the situation can call more on validity criteria and no longer needs the use of a material environment. The same evolution takes place in progress in schooling. The knowledge constructed by the pupils acts as a lever for the evolution of these same acquisitions.

A lot of French research has studied the conception of teaching processes in the determination of such situations, the chosen epistemological and learning hypotheses consisting in giving *the problem* a central place both in the significance of mathematical knowledge and in the pupil's learning [Vergnaud, 1981]. Mathematical knowledge takes its meaning from all the problems to which it provides instruments of solution; when confronted with problematic situations the pupil will put into action his previous acquisitions and if they do not allow him to provide an effective solution, he will construct a new answer adapted to the problem through an interactive process of equilibration and desequilibration. In the background here a constructivist learning hypothesis can be recognized. In the study of the relationships between teaching and learning concerning various mathematical notions, one of the didactician's tasks, therefore, has been to design problem-situations which conserve the meaning of the knowledge to be learnt, in which the concept to be used and constructed by the pupil is entirely justified by the situation (criterion i) and which allow a fruitful interaction with the pupil (criterion ii). The pupil still has to grasp the problem in order to take charge of its solution "alone," and has to understand it and seek a solution in the manner expected of him This is the level at which the teacher can intervene (see below)

The theoretical analyses at the basis of the construction of situations are validated by the confrontation between the observed phenomena and the expected phenomena. This means that for each situation-experiment, a theoretical analysis of every possible reaction by the pupils who are confronted with the situation is indispensable. Moreover, it is this analysis which allows a meaning to be given to the reactions observed in the pupils, which take on significance in relation to other possible reactions.

A third category of choice is situated in the management of the teaching process in the class by the teacher He is the one who, by the information he gives and withholds, allows a problem situation to be passed [*dévolue*, Brousseau, 1986] to the pupils without changing the meaning for them Numerous examples of change of meaning brought about by the teacher's intervention have been reported in various studies Steinbring [1988] presents a case of reduction of meaning brought about by a teacher who, having noticed that his pupils could not manage to solve the problem of calculating the perimeter of a rectangle of variable dimensions x and y, segmented the problem by asking the question in the case where the dimensions were concrete numbers, then asked for a generalization of the result obtained using x and y. The solving process was modified by this intervention: the teacher resorted to an algorithmization of a process which functioned on numbers instead of working on variables. The meaning of the concept of variable is therefore modified: it is only given the value of a substitution instead of the value of an object This example illustrates well the paradoxical nature of the didactic contract: everything the teacher attempts in order to make the pupil produce the reaction he expects, tends to deprive the pupil of the conditions necessary for the understanding and the learning of the notion aimed at [Brousseau, 1984].

Grenier [1988], in a study of the learning of orthogonal symmetry, analyses the case of a situation in which the pupils have to draw the line of symmetry of an isosceles trapezium using only a set square and an ungraduated ruler. The constraints of the situation were chosen to encourage the use by the pupils of certain properties of the line of symmetry (incidence and orthogonality) and to block the use of the property according to which the line of symmetry passes through the middle points of the parallel sides of the trapezium. But the pupils diverted the ruler from its usual use and either graduated it or used its section as a measuring instrument to enable them to construct the middle points, a procedure they knew well. The teacher, noticing the change in the problem made by the pupils, intervened to pose the initial problem again and insist on the precision required for the construction But, whereas for the teacher a construction using intersections and orthogonalities is more precise than a construction based on approximate measurements, for the pupils the simple fact of using a measure is itself enough to guarantee the precise nature of the construction; their procedure of resorting to the middle points was in fact reinforced by the intervention of the teacher.

Douady [1985] has pointed out the rôle of the teacher in the institutionalization of knowledge, that is to say the transformation of a concept into an object of knowledge, this concept having been used by the pupils as a tool for the solving of a problem. A concept used implicitly to solve a problem is not recognized by the pupils, it has to be extracted by the teacher who points out the aspects which should be part of the knowledge to be learnt so that it will be reusable later In a teaching process, the teacher finds himself therefore at both ends of the problem-situations given to the pupils: he must ensure the devolution of the problem and institutionalize the knowledge at work He therefore plays a primordial rôle in influencing the meaning of the knowledge constructed by the pupils. Although theory has highlighted this rôle, the conditions and variables on which its exercise depends are less well known. For this reason they are beginning to become objects of research.

Indeed, research has tended to neglect the rôle of the teacher. However the gaps between what theoretical analyses foresaw should be the result of teaching sequences and the effects actually observed have led researchers to choose to study the rôle of the teacher in the class. Robert and Tenaud [1988] have studied the effect of interventions by the teacher during group work by the pupils. Robert and Robinet [1989] have sought to analyse teachers' ideas about mathematics and their teaching which have a definite influence on the spontaneous decisions they make in class, and which for the moment theory hardly takes into account. Grenier [1988] has shown how much the teacher's room to manoeuvre is linked to the activity of the pupils and to his interpretation of the pupils' activities. The assessment phases when the teacher sums up the pupils' activities are particularly sensitive from this point of view.

IV—The computer as an environmental constituent

In the organization of interactions with the knowledge to be learnt, the rôle of the computer has been the object of much research in the last few years. Indeed, a didactic analysis has been made necessary by the existence of new teaching situations created by the change in the environment brought about by the use of the computer: research on the new significance of mathematical concepts mediated by the computer, research on the new interactions made possible between knowledge and pupils, research on the change brought about in the didactic contract by the use of the computer as the respective rôles of the pupils and teachers are modified [Gras, 1987]

Let us point out however that the simple fact of including the computer in a class does not imply a unique idea of teaching and that different uses inferring different epistemological and learning hypotheses are possible. One of the original possibilities of a software such as Cabri-Géomètre, that of the production of dynamic representations of mathematical objects, and therefore the visualization of mathematical properties as invariants in the transformations carried out on the screen, can be used to induce different approaches by the pupil: induction of properties from observation, or validation of conjectures by simulation of the screen. Thus in Cabri, the dragging of figures constructed by the pupils disqualifies constructions built on guesswork as the software only takes into account the geometric properties that are used explicitly

A lot of research has been carried out on the interactions between knowledge and pupils, in particular on the design of situations which play on the computer's characteristics to allow the pupil to develop new strategies when faced with a problem and therefore with new concepts. As an example I will quote that of Osta [1988] who thought up a process for learning the notion of reference system in three dimensional space in which the evolution of the pupil's strategies is made possible by the constraints of the software used, in this case Mac Space. Mac Space is a conversational graphic editor which constructs graphic representations in perspective of 3D objects from three views on which the user can operate. Because the requirement of explicit communication to the machine of the coordinates of the solid objects to be represented requires the software's particular structuration of space to be taken into account, the problem to be solved by the pupils is precisely that of the construction of a reference system. The possibility of seeing the result obtained on the screen is a factor in the evolution of the pupils' strategies. A result in perspective which is not perceptually satisfactory encourages them to look for the reason for the mistake, this reason then bringing another representation of the solution . . Perception is used here as an instrument of validation and not as an instrument of solution, as the complexity of the Mac Space reference system and the objects to be constructed prevent adjustments by trial and error from giving a perceptually satisfying solution

As can be seen from this example the organization of situations including a computer is based on the same type of analysis as in a classic environment. The analysis is perhaps made more complex by the large number of strategies allowed by the same software, their conceptual complexity, and above all by the mental representation the pupil develops of the computer's functioning which changes during the task in interaction with the pupil's representation of the problem and his approach to the solution. This is an extra element of complexity which must be taken into account if significance is to be attributed to the pupils' behaviour and therefore to the nature of the learning

V—Forms of the functioning of knowledge

Brousseau [1981] has shown that different forms of the functioning of knowledge correspond to different organizations of the *milieu*: certain situations call for an implicit use of knowledge (*action situations*), others require the pupil to make concepts explicit (*formulation situations*), and finally a third category of situations requires the justification by the pupils of what they have made explicit (*validation situations*). Action situations have been the object of particular attention for a long time and have been mastered well, particularly from the point of view of the retroactive effect of the environment. The influence of the Piaget school is clear in the extent of researchers' preoccupations with the question of the relationship between conceptualization and action.

More recently, the links between conceptualization and formulation, and between conceptualization and validation, have given rise to a wider interest which goes hand in hand with the appearance of research concerning the higher levels of schooling (beyond compulsory schooling) which demands levels of conceptualization involving formulation and validation. Research has shown the dialectic links between the cognitive status of mathematical objects for the pupils and the formulations or validations used by pupils in relation to them [Balacheff, 1988]. In particular, the usual formulations of mathematical discourse require a certain level of acquisition of mathematical objects and relations: they must be sufficiently decontextualized and detached from the pupils' actions. Geometric objects such as points and segments must be sufficiently separated from their context and perceived as independent in order to receive a coding from the pupils through the use of letters: a segment seen only as the border of a region, that is to say linked to a region, is not coded by the pupils [Guillerault, Laborde, 1986]. Conversely the requirements of formulation lead the individual to envisage mathematical objects in another way: indeed the specific problems posed by the activity of formulation force new conceptual analyses to be made. For example, a lack of adequate vocabulary; the complexity of the discourse to be produced is too great because the relationships envisaged between objects are too complex. Awareness of the person for whom the discourse is intended leads to the explanation of data which was up till then obvious to the speaker, obliging him to become aware of implicit elements in his approach, to operate a distancing and decentration of the objects so that they can be understood by the other through verbal data alone

On the other hand, the question arises of the construction of teaching situations which allow the pupils to learn how to formulate in mathematics. Indeed, the spontaneous formulations of pupils in mathematics often contain implicit information and ambiguities. Ordinarily in teaching the pupil is asked to imitate the discourse of the teacher or the textbook. Situations allowing for the construction of precise and unambiguous formulations in response to a problem have been designed. Two characteristics are used to generate such a construction: the social dimension of the language activity and its finality. The pupil's formulation is aimed at a peer who needs it to carry out a subsequent activity which cannot be carried out without this formulation The fact that a peer is addressed encourages the pupils to be careful about the quality of their formulation so that their classmate can manage the activity which depends on it. One of the still unresolved problems of this type of situation is that the next step in the situation is carried out by the classmate who in some cases interprets false formulations in a way that the pupil who produced the formulation wanted

VI—Conclusion

French research is diverse but driven by the common attitude which I have attempted to communicate in this paper. It can be summed up in two words: audacity and reason, the audacity of wanting to construct a rationality of phenomena as complex as those that cover the relationships between teaching and learning. The results produced over the last few years lead one to believe that this audacity is not rash. The scientific debate, sometimes lively, within our community contributes without doubt to this advance. I hope that it will continue to open itself up to the international community to receive validation.

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