The Perry Development Scheme: a Metaphor for Learning and Teaching Mathematics

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"When I ask a question," the student said angrily, "I want an answer, not another question."

"I told my class that there were three equally good algorithms for solving such problems, and one student demanded to know which one was best. When I insisted that they were all equally valid, he went home and told his father, who happens to be on the Board of Trustees of this Bible college, that I was ungodly" (Adapted from Perry, 1981)

"Now I know why my teacher won't tell us the answers: she wants us to learn to find them out for ourselves."

The world of teaching and learning is full of events in which communication between the participants is less successful than we would like. Many reasons can be cited for these misunderstandings — for examples, a lack of motivation or patience or effort. One structure that has helped my communication with students is the Perry Development Scheme

The scheme

Unlike many other theories in developmental psychology, this one grew entirely out of patterns observed in long, open-ended interviews with students — college students, actually, although I think the scheme applies to interactions among other categories of persons as well. William G. Perry, Jr., while a counselor at Harvard University, began to see patterns in data from such interviews made through the 1950's and 1960's. After he described the patterns, they were confirmed by independent judges, and since then have been found in a variety of persons around the Western world.

What is the structure Perry has brought us? Perry's full theory contains nine "positions" from which persons view aspects of their worlds. Most persons interviewed over time seem to move through these positions in order, although some of them "backtrack" — a phenomenon not strictly "allowed" by other developmental theories.

The nine positions can be condensed into four categories, the first of which Perry calls "dualism". From this perspective, a person believes that every question has an answer, that there is a solution to every problem, and that the role of an authority is to know and deliver those answers and deliver those truths. Any authority who doesn't do so either is incompetent (which equals morally evil) or, at best (in the case of teachers), is trying to set the students to learn to find those answers from themselves. This view of the world, illustrated in the three quotations above, is quite common, though not necessarily predominant, among entering college students in the United States. Younger students are even more dualistic

The diversity of many twentieth-century environments is not conducive to absolutism, however, so students do not remain dualistic, especially after entering college. For social reasons, they cannot go around openly condemning all those who differ from them, so they must at least pretend that "different" is not the same as "wrong". But opening the door to diversity leads eventually to a sense of freedom. No longer does a person believe that there is a Right Answer to the question about interpreting this poem. And no longer is the authority expected to present the tablets in the sky. Where no one knows, anything goes! Everybody has a right to his own opinion, and all of them are equally good. What a relief!

In mathematics, this "multiplistic" view of the world might be exemplified by turn-of-the-century formalism. Everybody has a right to his own axiom system, and they're all equally good since mathematics is only, after all, a collection of meaningless strings of marks

Persons with a multiplistic view of the world are quite fond of the phrase "everything's relative", but it is the third category of positions to which Perry attaches the "relativism" label. In his terms, "contextual relativists" have come to see several reasons that not all opinions are equally good. First, there are standards such as validity, internal consistency, and consistency with observed data. Second, validity depends upon context. One proof of a theorem, for example, is superior to others because of its elegance. Another proof of the same theorem, however, is superior in the context of teaching, since it doesn't involve such high-powered concepts. A third proof may be superior from an historical perspective. And so on.

Recognizing set standards and contexts, however, does not return the student to the absolutes of dualism. For even after one discards those opinions to which "nobody has a right", there remain — and will remain in the forseeable future — several equally valid viewpoints. Now the student finally grasps the ways his teachers have been thinking all along, and the standards by which work was being evaluated.

Such equilibrium is not permanent, however, since a relativistic stance brings with it several demands. Now that alternative perspectives can be considered valid, one can look at one's self from a more external viewpoint, and can think about his or her own thinking. Also, a person

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must now examine consequences of assumptions. When these two demands are linked, life can become difficult. How can I choose which stand to take? More seriously, how can I decide on a career? relationships? religious viewpoints? One parallel in mathematics might be the implications of Gödel's theorems concerning consistency and completeness. Are our efforts grounded in sand?

The worldview in which a person realizes that such decisions can be made only on the basis of uncertainty, and takes the risks to do so, is called "Commitment" (The capital C is to distinguish this term from reference to the many commitments persons make at all times in their development.) From Commmitment positions, a person is aware that knowledge is not something "out there", to be "absorbed", "exposed to", "gotten", or even "acquired". A person's knowledge is something that he or she must build alone. It is a very personal structure for interpreting experience. (The philosopher Polanyi [1972] calls it a "personal knowledge". Henderson [1981] refers to "mathematical imagination".) My knowledge of mathematical functions is different from yours, and that's OK, although I happen to be enthusiastic about my viewpoint enough to share it with you. Likewise, I can find meaning from your metaphors and imagery (See Brown [1981]) But ultimately, you are not the agent of my learning; without my active participation, I cannot learn

Of necessity, this account of the Perry Scheme has been quite abbreviated. I hope it is sufficient, however, to convey a feeling for the logic of the movement and for the realism of the metaphor. For more details, see Perry [1970 or 1981]. Let us turn now to some fairly straightforward applications of the scheme to the teaching/learning process

First order implications

One point made especially clear by Perry's work is that persons at radically different positions have difficulty communicating effectively with each other. I think the hostility evident in the first two opening quotations illustrates the problems a teacher might have in trying to introduce to dualistic students a conception of mathematics other than one entirely absolute

A colleague of mine demonstrated this difficulty quite effectively when she asked a group of college mathematics students to read and respond to an essay I had written about relativism in mathematics. I had used numerous fairly simple examples to argue that "right answers" were no more legitimate in mathematics that in other disciplines The responses were fascinating, and are described (without editorial comment) in Copes and Buerk [1981]. Here are two of them that illustrate the complexities of the communication gap. Sara managed to interpret the essay comfortably within her worldview:

> I think the essay was great! Most points I can relate with and really do agree with. My only problem with some approaches like this is that I become impatient in not knowing whether I'm right or wrong. Even more so when I don't exactly know if I'm headed in the right direction or if I'm totally off the road.

Mike's response also shows a willingness to agree despite an underlying dualistic conception of mathematics:

> I felt that this paper was extremely interesting and quite logical. They make the point in the first paragraph about that there is no right answer to a question. This is quite true. The only exception to the rule is in math, because you have to be precise. The idea that $1 + 1 = 2$ is only an assumption. It depends on what variables you are using, which determines what you will end up with

The most obvious conclusion that somebody might want to draw from Perry's observations is that we should be careful to teach our classes as dualistically as possible We should "present the material clearly", and make assignments that do not give students much inkling that they have to be active in their own learning We should avoid discussions in class, for the idea that worthy ideas may come from someone other that the teacher can be quite disturbing to a dualist If there is to be any hope of students' mastering the skills we wish to transmit, we must communicate with them effectively

Moreover, we must do our best to keep our students at ease. We must make sure there are no conflicts between authorities, such as ourselves and the textbook. For if students are uncomfortable, can they really learn what we want them to? In fact, I think not. Dualistic students are quite uncomfortable with any approach other than the authoritarian. For many this discomfort will have negative effects on their "retention" of facts. If our goal is to bring students to the point of being able to regurgitate a "body of knowledge", then the case can be closed right here

Reaction

Since I did not stop there, you may conclude that I do not believe retention of information should be a major goal of mathematics teaching. A major reason for my stubbornness is that I think such a conception of mathematics is intellectually dishonest. I believe the history of mathematics (pieces of which were used as illustrations of the various points in the Perry scheme description above), as well as numerous analyses of what mathematicians do (see Poincaré, [1914], Hadamard [1945], Davis [1967] and Brown [1978], for example), vividly show that mathematics is a dynamic, creative process rather than merely a collection of facts. Teaching only a dualistic view of mathematics is dishonest. It is also rather boring.

What then do we do? We cannot communicate effectively with dualistic students about relativism, but to do otherwise is a sacrifice of our integrity. The only solution to this dilemma is to help students change their conception of the world to one that is sophisticated enough to deal with more complexity

Now Perry doesn't give us much help about how to do this. He proposed his scheme as a summary and metaphor for what he observed, not as a prescription for what should be. To move from the "is" to the "ought", as Hume put it, will require breaking new ground.

Or will it? Is it really "new ground" to take as a goal of education the preparation of students to handle a complex world? Can an argument for objectives that include self-awareness and Commitment within relativism be taken as previously unexamined ideas? Hardly! An entire line of philosophers, from Socrates through John Dewey, has defended similar aims for "liberal education". Of course, this philosophy is not the only valid way of seeing the world; but surely it is possible to Commit oneself to such a perspective within a relativistic context.

For arguments for such a world view, I refer you to Socrates, Locke, and Dewey, complemented by Thomas Kuhn's [1960] and Imre Lakatos's [1976] structures of history. For various contrasts, see Plato, Hobbes, Popper [1972], and Scheffler [1967]. Rather than dwell on this discussion, I'd like to accept Perry's notion of Commitment as "good", and examine some implications of that axiom

Second order implications

Although Perry's work did not directly address the question of environments conducive to the development he described, some observations by other developmental theorists can lend us some insight. For my own teaching, I find one principle particularly useful.

The principle has been stated in various forms by psychologists back to Piaget. Positions on a developmental scale can be described by the metaphor of "lenses" through which persons view their experience. A lens allows a great deal of data to be "fitted" into the person's structure, to be made "meaningful". For example, Sara had a lens allowing her to see only those parts of what I said with which agreement could be found.

This principle does allow for remembering data that appears only slightly out of focus through the person's lens. When enough of this kind of data has built up, the person is forced to change lenses for viewing the world. At first the new lens takes some getting used to, and experience that can't be seen especially clearly with the new lens is ignored completely. As the person becomes accustomed to the new way of viewing the world, however, some of that fuzzy data begins to pile up in memory, eventually leading to another change of structures

A major implication of the lens principle for teaching toward development is that, with any given student, there are times at which the student will be open to data that challenges his or her current viewpoint of the world. At other times, the student will need experiences that affirm and support a newly-found perspective. In terms of Perry's scheme, dualists are, in general, open to multiplistic challenges to their world view — especially if they receive enough dualistic support at the same time to keep them from rebelling. Some persons have just recently adopted this multiplistic way of structuring their experience, and need encouragement; others are comfortable enough with diversity that they need challenge to evaluate various viewpoints within contexts.

For dualistic students I find most effective a rather rigid course structure combined with challenges in other aspects of the course. For example, homework assigned with very clear deadlines can be quite useful in maintaining a feeling of security. Some of those assignments might ask for summaries of different approaches to the problem.

Also essential is being able to convince students who are very sensitive to deviations from their former mathematics courses that they really are learning something. Success stories are not easily accumulated, so I'll relate one here. Jim approached me after one week of classes last term and asked me to allow him to transfer to another section of the course where, he explained, the professor was known to "follow the book" more closely than I. Since Jim seemed to be saying that he was uncomfortable with the slight amount of multiplicity that had come out through student class contributions, I explained that my purpose was to help students learn to "find the answers for themselves", and that, in the long run, the diversity might be more valuable to him. Apparently this response fit into his dualistic conception of the mathematics learning world sufficiently well for him to become more comfortable with the course.

Other aspects of a course can be arranged to bring out a diversity of ideas — without requiring relativistic evaluation of these ideas. Generally I prefer to increase the degree of multiplicity gradually through the course. I'll mention here how I try to do so in four aspects of a mathematics course, leaving it to you to generate ideas that fit your own personality.

A major aspect of courses is the content to be discussed. I sometimes find that planning the order of things requires that I change the arrangement of topics quite a bit from what textbook authors think is ideal. Frequently definitions and axioms from early in the text lend themselves better to multiplistic treatment than do ideas appearing later in the book. I have argued elsewhere [Copes, 1979] that most presentations of mathematics are backward from their historical development; perhaps they are reversed from a developmental perspective as well.

The most noticed aspect of a course is the teaching method used in class sessions. It's noticed, that is, if it is at all different from the traditional inverse dentistry ("fill'em and drill'em") approach. I tend to start courses for students who are mostly dualistic with a combination of a lot of lecture and a little discussion, and move, through the term, toward a reversal of the balance. To initiate class discussion, I generally pose some sort of problem and then ask particular students how they would go about solving it. In such discussions I try to avoid (especially at the beginning of a course) much evaluation of student opinions, but stress rather the diversity of the approaches brought out. As you would guess, one of the trickiest aspects of this method is to steer students in productive directions. This often can be done by careful omissions when summarizing previously-mentioned ideas

Discussions can provide students with a great deal of quantity, which many multiplists appreciate. If all sorts of ideas are encouraged, even weaker students can acquire a degree of interest and momentum that might help them develop their thinking abilities. One student shared with me her perspective on the early part of her beginning calculus course, when I had been praising just about every student who dared open his or her mouth. Joan said that she had been nervous about saying anything until she heard how positively even "dumb ideas" were accepted Then she figured she had nothing to lose

A third area of a course in which I find it useful to introduce multiplicity is in assignments. Multiplists tend to equate quality with quantity, and I find that they equate learning with doing a large number of assignments To the extent that my time permits, I like to allow students to rework problems that I did not find satisfactory the first time they were turned in. They can redo problems as many times as they like up until the exam over the relevant material. This allows students who think that quantity is important to test that hypothesis. Unfortunately, of course, such an approach is quite demanding for the teacher, although it leaves little mystery about where students are having difficulties.

In assignments through the course I like to include questions that ask students to write about what we discussed in class. At first these questions merely require description: what ideas were raised? Later, I begin to ask for more of the thread of the discussion: How did the falling body problem lead to the concept of antiderivative? A typical reaction is that of a very capable but rather oppositional student this past semester. "How come we have to do all this writing?" Jill asked repeatedly. "I can do math, but you're making this into an English class." My usual reaction is that students should understand what they're doing, not just be able to do it. This response was apparently enough of an "explanation" to her that she stuck with the course, albeit noisily, until she began to catch on "what I wanted" in the way of explanations.

If this method of giving assignments is used, exams serve no real evaluative purpose. Students do need to have opportunities to "prove themselves" more formally, however, so I give them several tests Multiplicity can be introduced into these by giving different questions to different students, depending on how well they already have demonstrated their understanding of the topics discussed. Some students need a "minimal" exam to show that they can indeed solve fairly basic problems with straightforward algorithms. A few students have shown through homework assignments and office discussions that they could almost teach the course, and can be challenged on exams to think about ideas that have not yet been encountered in the course. Most students are in the middle somewhere, and would be given a third set of problems, overlapping with both extremes.

Such testing, as you can imagine, is frequently challenged by students, especially those not receiving the "simplest" exam. Most dualistic students can be convinced of the validity of this approach Some are convinced, or at least appeased, by my saying (quietly) that they can be proud of having received a less trivial exam, and that the difficulty of the test will be taken into account in considering their final grade. (Usually this is unnecessary; raw scores correlate quite positively with the complexity of the exam.) Those more in tune with multiplism can usually buy the "different strokes for different folks" argument, and the notion that they are not competing with other classmates but with themselves.

Teaching multiplists

I have illustrated several ways in which dualistic notions of mathematics can be challenged even within an environment that is structured enough to give some certainty to fall back on. But what happens to students who are already multiplistic? They can receive a great deal of support in such a course — from the same elements that prove challenging to the dualists. But how can they be challenged?

Here the lens principle becomes most evident: dualistic and newly-multiplistic students will often be able to filter out the relativistic ideas that challenge students more comfortable with diversity Once in a skill-oriented computer programming course we were discussing file-manipulation techniques. I was describing how programmers must check by hand their programs' results on a small subset of the files to be used, and should try to include in that subset examples of extremes and special cases. But, nevertheless, I had rambled on (more or less unaware of what I was saying), in order for programmers to be positive the program would work for all records in the file, they would all have to be checked manually, defeating the work-avoiding purposes of programming work. That statement apparently made an impact on some students, for the room became unusually quiet. Then one student offered more of a statement than a question: "You mean you'll never know for sure if the program is OK?" When a horse is led to water unintentionally, it's especially nice to watch him drink

The ultimate uncertainty of knowledge does come up as class conversations become more spontaneous and comfortable. There are even times when the instructor can intentionally lead the class to moments like the above without completely losing those who cannot perceive much of what is happening Assignments can be individualized, too, as students comfortable with multiplicity are offered problems asking them to compare and contrast different approaches rather than just to describe them.

Our oppositional friend Jill got a fascinating glimpse of relativism on an exam when she was assigned a series of questions that were to lead her to seeing a parallel between hyperbolic functions (which we had encountered in her course) and the trigonometric functions of complex variables, which were almost entirely outside her realm of experience. She griped off and on through the entire exam about how awful the problem was, approaching me several times with her frustration. Fortunately for our relationship, she overcame the obstacles (and her own mind set) and saw that, by golly, these things do relate! One summer vacation later, she still lets down her opposition when thinking back to that experience

Similar techniques apply to the few relativists we encounter among our students. They receive support from the relativism we offer late multiplists, and can be challenged through assignments, exam questions, and office discussions to go beyond the passivity of the other students. We can share our personal images with them, we can encourage them to pose new problems after solving old ones, and we can ask them to take stands within their uncertainty. They often are impatient with the ways we approach the course as a whole, but can be offered structures, such as Perry's, for understanding why we're behaving this way and even can be asked to help. They are willing to treat us as peers, and we can strive to deal with them in that way without being threatened.

Reservations

Skeptical readers probably have several reservations about what I've said here Let me begin to address some of the more likely of these.

Reservation 1: "But all those different kinds of students are in the same class. How can we deal with that?" Beyond what I've offered above, I have only two things to say. First, if you are teaching, you already do deal with a large variety of students They may even be closer together in Perry's terms than they are in other ways. Second, you also are probably aware that they all see you and the course through different lenses We can depend on these lenses to shelter students from our most devastating challenges But we should try to be careful not to force the issue. I thought student Leslie was performing uncommonly well on several exam questions that started "Explain in your own words ..." Then I realized that she had memorized long paragraphs from our textbook to use as "her own words". She had no sense of herself as an active learner; would I have helped matters to have accused her of plagiarism?

Reservation 2: "But won't teaching non-dualistically affect my course evaluations?" This is quite possible. College administrators are trying to dismiss two of my favorite friends from the faculty of a school that is so dedicated to the student satisfaction that it uses student course evaluation as the sole criterion for retaining non-tenured faculty members. In these two cases, the teachers, acknowledged by their dean to be among those most concerned about teaching, have been teaching toward development, and too many students have been made uncomfortable with newly-found challenges and world views.

Can such a tragedy be avoided? If I didn't think so, I probably would have left teaching long ago. But the success of one's efforts depends greatly on how much one's own colleagues and administrators are threatened by non-traditional goals and teaching styles.

Better methods for assessing course effectiveness also would help. Standard questionnaires may go a long way toward soliciting student opinions about the course and teacher without asking any questions that give the student a chance to define his or her terms. What does it matter if the student "strongly agrees" that the instructor "made the objectives of the course clear" at the beginning? Does the student mean that a day-by-day syllabus was handed out ("good" to a dualist, "not good because not flexible" to a relativist) or that the teacher and class together set the goals as "learning to pose significant mathematical problems" ("bad" to a dualist, "fascinating" to a relativist.) It is not difficult to go through many such questionnaires and give, from a dualistic perspective, terrible ratings to relativistic teachers. An alternative might be a pair of essay questions: 1)What do you think an ideal course would be like? 2)How close did this course come? When specifics were elicited, such essays provide a large amount of information about the student's structure and ideas — but only if they are read by one sensitive to student development. Even then, they are hopelessly useless for summarizing a teacher's competence by a number.

Reservation 3: "But who has time for a two-hour interview with each student to find out what his or her Perry position is?" Fortunately, I can reassure you here, for there are both shorter formal, and less formal, ways of determining position.

A major written instrument for glimpsing the position of students not yet into relativism has been developed by Knefelkamp [1974] and Widick [1975]. It consists of several essay questions, which can be scored by trained raters. Another instrument, which can be integrated with other information — gathering techniques (such as a beginning-of-course questionnaire) is being developed now. In fact, just about any essay you can get students to write can provide some insight into that student's perspective.

Less formal but perhaps just as effective "rating" can come from careful listening to students. After all, they may be in different positions with respect to various areas of their lives; what is probably most important to you is their position with respect to the current topic of conversation. Research has shown [Goldberger *et al*, 1978] that teachers who know the scheme and their students well can give ratings that match pretty well those acquired by more formal means.

Such familiarity with our students can lead to very moving experiences in the classroom, too. Perry [1978] describes an interview in which a Harvard student is talking about what "real learning" means to her:

> One informant expressed regret that no such experience had occurred for her in this her first year at Harvard. She had learned some things, surely, but these had been "just additions", expansions of expertise and valuable as such, but leaving her with a sense of flatness. What stood out for her as an example of really learning was her experience of the "Ames window illusion" in high school. She recalled that her teacher in Social Studies had shown the class the experiment in which, she explained, the window revolves on an axis "I mean it revolves and you see it revolve and you know it revolves. Then the lighting changes and the window doesn't revolve, it oscillates, and you know it oscillates. And then the light goes back to normal, and there it is, revolving."
>
> She then reported how the teacher, unlike those who present such items as one more quirk of perception, stopped and asked as if of the air: "what do you make of that?" All of a sudden, she reported, it came to her how much we bring with us to our perception of reality, how much we create of all we "know" It all "opened out" to her how much we build our worlds. If this could be true of windows, how about people? how about oneself? how about ...? "I mean, I really learned

how different things can look in a different light, so to speak. It was terrific."

Our informant then returned to her dissatisfaction with this present year, but our interviewer grew restless and soon invited her to expand further on her feelings in that experience in high school

"Oh well, of course, it was awful I mean my world was shattered! I guess it's sort of naive to talk like this here, but it was like I lost my innocence I mean, nothing could ever be for sure — like it seems, I mean — again."

Our interviewer waited and then asked, "How come you stayed with it instead of just laughing it off and forgetting it?"

"Oh, that was because of the teacher! You see, I trusted him, and I knew he knew. I mean we didn't talk about it really, but he just looked at me and I knew he knew — what I'd learned — and what I'd lost! Mostly it mattered he knew what I'd lost — and so I could stay with what I'd seen"

Third order implications

Here I'd like to mention three more issues that might have been phrased as objections in the above section. I shall intentionally leave them open, partly because I am less comfortable with any resolution of them for my own teaching, and partly because they lend themselves to a great many insights that ultimately will be personal.

A major strength and fascination about Perry's model is that it reveals complex interdependence among intellectual, ethical, and identity developments. The dualist sees "good" as equivalent to "correct", and "self" as "good". The relativist thinks of "good" and "correct" as relative to context, and a person in Commitment views discovery of "self" as the goal of Commitment (intellectual or otherwise) or, later, as found in the style of the Commitment process

Of course we're all aware of ways in which a student's family or romance can interfere with academic achievement. But our Ames window friend shows us that interactions among the intellectual, ethical, and identity can go far beyond such difficulties. Indeed, to the extent that we are successful in inducing intellectual development, we shall precipitate other kinds of development and associated crises. My student Sandra is a good example. Having come to view mathematics as consistent with her overall multiplistic worldview, she decided she liked it Since she was quite successful with calculus, I innocently encouraged her to consider continuing with mathematics. This came into direct conflict with the plans she had already made for her life, and has resulted in a good many emotional conversations "How can I ever know", she's asking, "what the right career for me is?"

A second idea comes from our tendency to think of good things as pleasant Since developmental growth is good (according to our axiom), partaking in it should be a happy experience. But persons encountering new ways of looking at things are not always — perhaps not often — joyful. They do not thank us for giving them this marvelous new lens. For embedded in their former way of viewing the world were all their hopes and dreams: their very identity. To take on this new worldview, must they abandon the old identity? Indeed, in some sense they must, and the associated grieving for the former self can be poignant

Like Sandra, my friend Cliff was brought to such a crisis by some mathematical experiences, but in a different way. His identity was embedded in his teen-aged dream to become a mathematician, believing that mathematics was the one island of certainty in a sea of relativism. (How many of us were attracted initially to mathematics for similar reasons?) His first shock came when someone told him the hangman paradox. Cliff decided to "solve" the problem, but within a few months had the misfortune of encountering Gödel's theorem As he recalls, his world, too, was shattered. He had to restructure his identity because he discovered that the foundations even of mathematics are built on sand.

Thus mathematical experiences can serve as powerful forces for development, perhaps in part because they commonly are not expected to do so. Through this observation, we come to our third idea: that we may see our own pedagogical roles through a lens somewhat different from that to which we're accustomed. We are used to evaluating our own success in terms of the mathematical information and skills our students have "learned". Perhaps an equally valid criterion for success would lie in how much these mathematical ideas have induced development along Perry's scheme, toward what dualists call the impractical wishy-washiness of relativism, and what we describe as liberal education. The figure becomes ground; "mathematics education" becomes educational mathematics.

Before we get carried away, let us consider how conflicting Commitments might shatter our own worlds From one perspective, it certainly can be argued that we have an obligation to change our focus in this way, if indeed we take Perry's description as our goal On the other hand do we have a right to bring about these crises, to force our students to do the kinds of thinking that will lead them to question their own value systems? As a parent of one college freshman once asked me in anguish, "What right do teachers have to destroy in a few months the values it took us all these years to inculcate?"

As I recall, in that incident I bowed to the pain of grief, and mumbled some multiplistic platitudes Perhaps she would have understood an answer that dealt with independent thinking, student's conceptions of authority, and Commitment. Maybe she would have recognized Leslie, Mike, Jill, or Cliff. Or perhaps she wouldn't have. Surely we can remain in awe of the dynamics of growth in parents and teachers as well as students.

Perry's scheme does not necessarily increase our "understanding" of those dynamics, any more than we "understand" falling objects by naming the phenomenon "gravity". But the scheme can provide us with an additional tool for interacting with others in a way that enhances all our lives What more could we ask?

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BETTER LATE THAN NEVER

We apologise to readers for the very late arrival of this issue. The delay has been due to a change in the typesetting procedure and to a search for additional revenue to ensure the completion of Volume 3. We believe readers will find that the general style and accuracy of the journal has been maintained in spite of some cost-cutting.

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