"THE HAVING OF MULTIPLE METAPHORS": OPENING IMAGINATIVE SPACES FOR MATHEMATICAL NOTICING

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Most people first encounter the idea of metaphor as a figure of speech used in poetry and studied in language arts classes at school. Metaphors are introduced as an implicit or explicit comparison of two things that might be thought of in quotidian life as very different: "a likeness of unlike things" (Pimm, 2010, quoting Mulcaster). For example, Ezra Pound's (1913) short poem *In a Station of the Metro* draws attention of the "likeness" of two unlike things: the glimpsed faces of subway riders and blossoms after the rain:

> *The apparition of these faces in a crowd;*
> *Petals on a wet, black bough.*

The striking verbal imagery of a strong poetic metaphor has the power to open our minds to new connections, possibilities and imaginings. For example, Pound's metaphorical poem might open our imaginations to a consideration of others we encounter in the subway as beautiful, natural, tender, fragile, fleeting, or worthy of aesthetic appreciation. What might this poetic lyricism have to do with the learning of mathematics—a pursuit that is commonly, and prosaically, treated as literal rather than literary, methodical, logical, linear, factual, and accurate to a fault?

Pimm (1987) writes that "metaphor is as central to the expression of mathematical meaning, as it is to the expression of meaning in everyday language" (pp. 10–11). Along with many examples of the metaphorical use of language in mathematics classrooms, Pimm offers an example of a young Seymour Papert, who reported using a familiar image of physical gear systems as a metaphor to help build his understanding of linear equations. Pimm writes:

> My main reason for citing this example is not the particular image, but rather that his comprehension involved a metaphor which enabled Papert to develop a tool for thought. It is also important in that it provides a clear example of a child constructing personal knowledge. Would that more pupils could find functioning images of this sort which connect the ideas of mathematics with objects and processes that they feel they know and understand. (Pimm 1987, p. 97)

It is these sorts of learner-constructed metaphors and images, in this case of the graphs of mathematical functions, that I consider in this article. With Zwicky (2010), I am concerned with metaphors and their role in opening up imaginative spaces in mathematics learning. Sfard (1994) has made an explicit connection between the ways that mathematicians and mathematics learners come to understand mathematical ideas through metaphor; some of the research mathematicians she interviewed asserted that they could not understand or work with new mathematical objects without creating an appropriate metaphor (Sfard, 1994, p. 48). A contemporary example comes from 2014 Fields Medalist Manjul Bhargava, who tells these two stories about metaphor and his own mathematical understanding:

> My grandfather was a professor of Sanskrit and ancient Indian history. A lot of the mathematics I learned as a child was through what he taught me of the rhythms of Sanskrit poetry […]
>
> One key to solving hard mathematical problems that people have been thinking about for many years is to think about them in a totally different way. One of the most exciting moments of discovery for me occurred when I was a graduate student at Princeton. Those days I'd been thinking about Gauss composition […] At one moment, it just clicked—it was around midnight when I usually go to sleep—that, if I take a Rubik's cube and I cut off the top layer, and I put numbers on the remaining little cubes, that would lead to a cubic analog of Gauss composition. I went on to discover twelve more analogs of Gauss composition over the next couple of years, but that first one suddenly opened this door. And that door was opened in part because I was playing with toys. (Bhargava, 2014)

Bhargava is open to bringing metaphors and analogies from perceptual sources as diverse as the rhythms of classical poetry and games like the Rubik's cube to his analysis of mathematical ideas and patterns. He credits an element of his success and innovation to these unexpected metaphors and "totally different ways" of thinking about mathematics. I suggest that similar kinds of original thinking through surprising metaphors may contribute to learners' deeper understanding of mathematical ideas that are new to them. An inclination to generate such metaphors may be something to value and work with in mathematics teaching. At present, students are not often encouraged to generate metaphors in mathematics class, and may be actively discouraged from doing something so far from the classroom traditions of school mathematics.

In this article, I offer examples of secondary school mathematics learners generating multiple metaphors for graphs of mathematical functions upon first encountering them. Some of these metaphors, like Papert's interlocking gear metaphor for linear equations, come out of a learner's physical, embodied experiences that are seen to be analogous in some way to the image of the graph. Others relate to images seen on movies and other media, or to other kinds of graphic images the learner has made or seen. Are these metaphors "appropriate" choices for creating imagery that will be helpful to these students in learning school mathematics around functions and their graphs? Undoubtably, some of these metaphors will prove helpful and useful in learning this mathematics, and many others will not. But my focus is on the initial process of *generating* and *holding in mind* multiple metaphors for a particular graph, even though these metaphors draw attention to different aspects of the graph and some of them will eventually be jettisoned as the learner becomes more familiar with graphs in association with topics in school mathematics.

My hypothesis is that the "having of multiple metaphors" (and images, and analogies) may be a prerequisite to finding apt metaphors for new mathematical ideas. I claim that the having of multiple metaphors also allows learners to *notice* and *recall* features of the graphs that might not otherwise be the subject of their attention, and to identify aspects of the graphs and functions that might prove to be mathematically salient (including symmetries, asymmetries, extrema, inflection points, slope, intersection points with axes and other lines, and piecewise aspects of functions).

My title is meant to recall Duckworth's (1972) influential education essay, "The having of wonderful ideas". Duckworth argues that vibrant and meaningful learning can only happen when there is space for learners to follow their own lines of inquiry, using all the intellectually creative means available to them; and where teachers provide occasions for explorations, and are willing to hear and accept learners' ideas. I suggest that eliciting, hearing and accepting mathematics learners' imagery—their "having of wonderful multiple metaphors"—and welcoming the introduction of these metaphors as part of the exploration of mathematical ideas may be an important element in welcoming students' creativity and joy as they develop mathematical understanding. Of course, there will be a further process of selection and choice of the metaphors most useful to a particular understanding, but without the openness to invention and perception at the initial stages, learners may feel there is no space for their creative involvement, no place where their ideas are welcomed.

The metaphors that occur to these learners spontaneously, upon first viewing and gesturing the shapes of these graphs, are often funny, witty, original, imaginative and unexpected. In the account that follows of their verbal narrative and gestural metaphors, I will connect the learner-generated imagery with ways of noticing features of the graphs that may have mathematical salience.

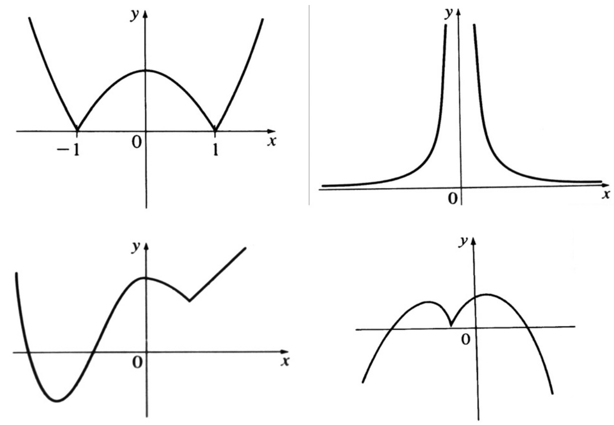

Figure 1. Graphs 1–4.

The context: Graphs & Gestures project

In 2007, I initiated a study that has formed the basis of an ongoing research program called Graphs & Gestures. In an early part of the project, secondary school students from three Vancouver public schools were asked to describe the shapes of the graphs of mathematical functions, shown in Figures 1 and 2, using metaphor and gesture. I have previously published findings from this research program relating to gesture and embodied ways of knowing (see, for example, Gerofsky 2010, 2011b, 2016), but there is another surprising finding related to multiple verbal/ gestural metaphors of the graphs. I will discuss this finding with evidence taken from videotaped data and interviews from the original study.

Specifically, of two dozen student participants in the study, I have selected eight to discuss in this article, in terms of the metaphors that they used to describe the graphs. Some were highly prolific producers of multiple metaphors; others brought forth just one or two, but these were noteworthy in their originality and in the ways that they highlighted particular features of the graph. These eight students were the same ones that produced the most bodily engagement in their gestures of the graphs: "being" the graph rather than only "seeing" the graph (Gerofsky, 2011b). They were also the students that were later identified by their mathematics teachers as the "top" students in their classes, in terms of both precision and creativity in their understanding of mathematical topics taught in their courses.

My hypothesis is that "having multiple metaphors" opens up imaginative spaces, allowing for flexible ways of attending to and noticing salient mathematical patterns. I suggest that the opening of imaginative space through the "having of many metaphors" may allow for creativity in approaching new mathematical ideas and patterns, and show flexibility of thinking that is a necessary prerequisite for in-depth mathematical learning.

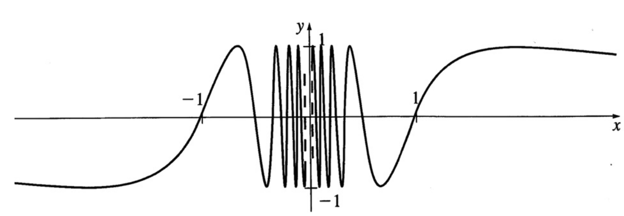

Figure 2. Graph 5.

Examples of students' "having of wonderful metaphors" for the graphs of mathematical functions

In this section, I quote the learners' words and brief descriptions of the gestures that accompanied them. Of all the student participants, Annie, a Grade 8 student, was by far the most generative and inventive producer of metaphors. Without additional prompting, she came forth with a seemingly endless flow of multiple metaphors and analogies for the graphs. Each metaphor emphasized and perhaps enhanced her noticing of particular features of the graph. Annie's teacher noted in a later conversation that although she excelled in mathematics and science, Annie had a preference for the arts and language arts as school subjects. Her tendency to generate images and analogies as part of the learning process might serve her well in both arts and sciences.

Three of Annie's metaphors for Graph 5 ($y = \sin(\pi/x)$) make analogies with physical objects that have folds or bends that are tightly packed at the centre and looser at the edges:

> It sorts of looks like [*pause*] crazy thing. Like maybe string, long string crammed together, so that the middle's more stuck together, and then stretched out a little bit.
>
> It looks now like a springy, a springy like this [*gestures vertical spring*], but then slanted [tilts to right], one part stretched close together, and one's being pulled apart kind of, so it's straight.
>
> That looks like those accordions, but started [*pause*] the accordion flaps started being straightened out. Maybe someone started stretching it out or playing it wrongly.

Annie's string, spring and accordion metaphors show a holistic perception of the shape of the graph and a comparison with "crazy things", including a broken spring or accordion bellows. She notices that on both the left and right sides of the graph, the folds or coils are straightened out or pulled apart. Where the image of the piece of string involves a softer material in a static image, the accordion and especially the spring metaphors indicate something with some tension in the materials that are tightly packed at the centre, and something wrong or broken with the straighter folds or coils at the left and right edges. Interestingly, her prototypical image of the spring is vertical, perhaps from science class experiments with vertical springs illustrating Hooke's Law. She indicates that the "springy" is slanted so that it will conform with the horizontal springlike shape of the graph.

Annie's metaphors do not capture the fact that this graph is low on the left (near –1) and high on the right side of the graph (near +1). Her gestures of the graph that accompany these particular verbal metaphors also ignore this mathematically salient feature of the graph. However, in her third take, using gesture without words, she shows her attention to this distinction, as she gestures with two palms together from right to left, starting from a higher point on her right and ending on a lower point on her left.

Eva, a Grade 11 student, has a similar image of tightly-packed folds at the centre of Graph 5, with a connection to her own physical experiences in sewing:

Eva [*in a quiet aside while gesturing the graph with her index finger*] It's gathered up in the middle, so,

Interviewer You said it's gathered up the middle. Can you tell me more about that?

Eva It's just that when it's starting to ascend up to the negative one, it's not as, like, it's more spaced out. But once it hits into, it heads toward the zero, zero points, it just becomes more compressed and thinner. It's still in the same. It seems like it's in the same, slope, but smaller. Smaller slope, and then as it goes back out, it just changes, gathering. It's like. I do sewing, and sewing has gathers.

Interviewer That's very interesting. That's very cool because it looks like you're doing pleats or gathers like a shirt or a smock.

Eva Yeah.

Interviewer When you see graphs, do you sometimes see an association with something else? Like on this one you did.

Eva Not necessarily. Usually when I describe it I will compare it to something else, but when I look at it, like Graph number 4, it does look like a McDonald's sign. But the other ones, like Graph 2 or Graph 5, if I speak it I'll associate it with something, but when I look at it, like I'll just look at it as a graph itself.

Eva's metaphor of gathered cloth connects with her embodied experiences of making pleats, gathers or smocking in sewing. It is interesting that she draws on multisensory cognitive resources from her lived experiences outside of her mathematics class to communicate the shape of the graph metaphorically (whether with herself, *sotto voce,* or with others). The image of gathers may be a helpful one in visualizing folds that are increasingly tighter together on either side of the y-axis. Even though physical gathers in textiles will have a limit based on the thickness of the fabric, as do the spring coils or the accordion bellows pleats in Annie's metaphors, the image of gathers in fabric is more flexible in terms of pointing towards the possibility of the number of folds tending to infinity. It is not quite clear what Eva means by "slope" here (certainly not the standard meaning of slope), but her gestured versions of the graph, like Annie's, show attention to the fact that the graph is low/ below the x-axis on the left and high on the right.

Annie and others also interpreted Graph 5 as a function over time, and compared it with images of a heartrate monitor they had seen in hospital stories on TV shows and movies:

Annie Looks like a heartbeat and it's like dying, and then more heartbeat, and then starts fading again.

Frank (Grade 11 student)	So with the last one. It's like one of those monitors you see at the hospital kind of thing. It goes, it starts at the left, comes down low, up high, and there's a bunch of little squiggles, they go up, and back down, and back up, and it goes, flat again.

The heartrate metaphor brings with it an emotional charge, as the image of the graph corresponds with a dying heartbeat, followed by a flurry of heart activity (a heart attack?), and then fading—perhaps the last moments of a person's life as shown in many films via the heartrate monitor flatlining. Does the emotion that might be associated with this image make it more memorable? Could the association of this metaphoric narrative with this graph make the graph (and function) more interesting—or something to be avoided?

Jack, a Grade 11 student, also perceived Graph 5 metaphorically as a function over time, but associated it with two different kinds of monitors: a polygraph test and a seismic monitor:

> It kind reminds me of a lie detector test. It's all calm and then jagged and then relaxed. It starts out calm, and then goes crazy, then calms down again. It's kind of like when something went wrong on the test, kind of like, even like the Richter scale. [The other part] is more calm and peaceful, that's the best way I can describe it.

Jack's emotive associations with this graph identify the tight vertical loops in the centre with jaggedness, craziness, or something going wrong (for example, someone caught lying on a polygraph test, or the occurrence of a seismic tremor). The near-horizontal portions of the graph are identified as calm, relaxed and peaceful.

It is interesting to note the identification of vertical lines with an unsettling, exciting or possibly threatening motion, while horizontal lines are identified with calm. Similar emotive associations emerged in interviews with students around their symmetrical, two-handed gesturing of Graph 2. Students who gestured this graph from the outside edges to the middle, finishing the gesture with arms upraised, said they chose that because it felt like saying "hurray" or having arms upraised in victory. Those who gestured it from the centre outward, finishing with hands moving horizontally, said they preferred that movement because it was like saying "ahhhh…": a "Zen" moment, calm and balanced [1].

Taking this graph metaphorically as a representation of movement over time (whether the movement is heartbeats, or changes in skin conductivity, or seismic tremors) afforded these students a way to read the graph as temporal and an example of "fictive movement" from the left to the right side of the graph. This kind of graphic interpretation is one of the conventions of many of the graphs students see in the intersection of physics and mathematics. Annie's metaphors for the same graph as both static, holistic/ gestalt images and an image of movement over time may offer her more flexibility in thinking about functions and their representations.

Frank offers a further series of metaphors for Graph 5, all of which involve tilting the graph (or your head):

Frank	Another thing is, it's like, it's almost like a comb if you look from the side. So it comes upwards, a lot of squiggles that go up and down.
Interviewer	Can you just explain, you said it's like a comb when you look at it from the side?
Frank	Yeah, if you tilt it [*turns paper with image of graph*] from this way and move it this way [*gestures turning the paper 90 degrees to the left*], it's kind of like a comb, the way the squiggles go up and down like this [*gestures*]. Or they look like, thorns! Or, or, like on a snowy day, the rooftops, they have [*gestures*] the icicles that come down? I think it's kind of like that too, if you tilt your head.

These metaphors all emphasize the pointy, sharp, jagged nature of the centre part of Graph 5—the teeth of a comb, thorns, or icicles. They complement Frank's earlier, more holistic image of a heart monitor by offering a focus on just the middle "spiky" part of the graph.

Gen, a Grade 11 student, offers a further, seemingly embodied metaphor for Graph 5. She begins by comparing it to the images of sound waves she has seen in science classes, and then introduces the idea of "a freezy thing":

Gen	Well when you see, like the sound, sound device, there's a freezy thingy on here. So it starts from, slowly, like the line over here, and it goes up, and there's a sound thingy on here, and it goes down, and slightly up, and straight.
Interviewer	And you said something about, I didn't hear what the word was, the freezy thingy, or frequency?
Gen	Yeah, freezy like [*gestures the graph*]
Interviewer	Freezy like, freezing cold, like that kind of freezy, or?
Gen	Yeah, like, when you feel it and you go like you know [*gestures huddling for warmth*], you're kind of freezing, like [*gestures huddling in toward the centre of the body*]
Interviewer	Like, shivering, that kind of thing?
Gen	Yeah, so like it can feel like [*gestures the graph*]

If my interpretation of Gen's words together with her gestures is correct, her metaphor for the graph has something to do with the movement of squeezing arms and shoulders toward the core of the body, as people do when huddling for warmth in cold weather. Although both verbal and gestural cues were somewhat confusing here, I believe Gen's description focused on a "squeezing" together of lines from the outside edges to the centre of the graph—an embodied metaphor accompanied by verbal and gestural images that showed her squeezing her arms into the centre of her chest as an analogy for the loops of the graphs squeezing in towards the y-axis. The combined metaphors of a sound wave and a "freezy thing", together with a literal description of the up, down and straight lines of the graph seemed to jointly capture a description that was memorable and telling for Gen.

Annie generated a great number and variety of metaphors for Graph 3 ($y = |x^2-1|$) and Graph 4:

> [Graph 3] It looks like a W, or, vampire teeth coming out [*gestures in front of face*] and kind of like, a crazy cat smile [*gestures in front of face*] and then, a frown, or a backwards M. Looks like a round sunset, and then things shooting up. Kind of like fireworks? But straight line.
>
> [Graph 4] Looks like a kindergarten M, or, a birdie you draw, like, crows in the sky, round, two hills again, kind of, and then two jello blobs, one big, one small, beside each other. Yeah. Looks like a 3 but then turned, turned somehow. And then looks kind of like, there's this little kind of bomb thing, and then it shoots out, it shoots outward [*gestures symmetrically from middle outwards*].

There is much that can be unpacked here. Annie describes both graphs in terms of the letter M, and at first glance, one might think that she saw the two graphs as the same, without noticing the features that differentiate them. But with multiple metaphors to describe each graph, it is clear she noticed more than the M or "backwards" (or upside down) M analogy. In Graph 3, she gives attention to the sharp points where the graph changes direction (vampire teeth, crazy cat smile), and in gesturing the graph in front of her own face, makes a personal, embodied connection with both teeth and smiles. Her image of a round sunset with straight-line fireworks shooting up in both directions highlights her noticing of the smooth, rounded curve of the centre portion of the graph and the portions of the graph on either side that appear to be straight.

Annie's metaphors for Graph 4 focus on images she remembers drawing (a kindergarten M, a child's drawing of a crow, a rotated numeral 3), recalling the gestures of drawing two smooth curves with a sharp change of pencil (and gestural) direction at their connection point. The metaphoric images of two hills or two jello blobs (one large, one small) give a sense of two objects stuck or squashed together, a relevant image if this is a piecewise graph of two partial parabolas meeting at a shared root. Her final image for this graph, of a bomb exploding outwards from that central root point, emphasizes the near-symmetry of the graph and a fountain-like fictive movement outwards to both right and left. Again, her multiple metaphors offer alternative ways of noticing and remembering features of the graph that may have mathematical salience as she learns about the functions.

Several of the students used "hopping" or "bouncing" as a metaphor for graphs that made a sharp turn upward, especially when this turn happened at the x-axis, as with any graph of the absolute value of a function. One student said that the graph of an absolute value reminded her of a bunny in a cartoon, going "boing, boing, boing", with the speed lines drawn in behind (looking like the graph). Jack, a Grade 11 student, mentioned something similar with Graph 1:

Jack	The graph bounces.
Interviewer	Can you tell me about "bounces"?
Jack	To me, a bounce is when something hits a solid object and then comes right back up again [gestures last section of Graph 1].

Lewis, a Grade 8 student, saw "bounces" as he was re-watching the video of his own gestures for Graph 4:

Lewis	It made me feel like a frog or something, a rabbit [*gestures this graph*], cause it looks like mini-jumps in between.

Lewis's gestures for all the graphs were by far the most kinetic of all the participants in the study, engaging his spine and head and simultaneously locomoting vigorously across the floor, where others tended to use primarily hands and arms. Like many of the students who produced multiple verbal and gestural metaphors for the graphs, Lewis connected his embodied and imaginative responses to the graphics with out-of-school life experiences; in his case, he compared his imaginative experiences of the graphs with swimming, diving, and riding a wave as well as hopping like a frog or rabbit. The ability and willingness to make connections between mathematical experiences and other memorable physical experiences (anything from sewing or diving to experimenting with springs, drawing, or shivering from the cold) seemed a hallmark of these students' ability to generate and consider multiple, varied metaphors that might help them make sense of potentially salient mathematical features of each graph. It also recalls Bhargava's connections with sources of metaphors from outside mathematics.

Finally, here are two original metaphors, from Bella, a Grade 8 student, and Dora, a Grade 11 student, that may have helped them remember features of particular graphs, while at the same time offering images that might possibly distract them from the mathematical intentions of the graphs—another conceivable side-effect of the ability to generate multiple metaphors:

Bella	[*Describing Graph 1*] it sort of looks like, well for me, it sort of looks like a kangaroo with an arched back. Like the kangaroo's knee, it has a back [*gestures*] sort of like arched [*gestures with two hands together*]. That's what it looks like to me.
Dora	[*Looks at Graph 3, giggles and sputters*]
Interviewer	Tell me why you're laughing.

Dora [*Gestures shape of graph with index finger*] Because it looks like a, bum! [*Giggles, then regains her serious demeanour*] With, open downward, and it looks like a W that's open down, like [*gestures graph with index finger*] and it looks like, a bum! [*Giggles*] Yeah.

Conclusions

It seems that whether or not they are prompted to do so, many students will produce and use metaphors in making sense of mathematical ideas. There is a natural human propensity to "see" new things in terms of other, known and familiar things, and to compare perceptions and experiences, and some learners seem especially tuned to this kind of perception. These metaphors can be generative for mathematics learners as ways of giving attention to mathematically salient features of new patterns and objects. They can also be ways of opening minds to new and innovative conceptions of mathematical ideas.

Mathematics teachers do not often encourage these ways of perceiving objects like the graphs of functions; in fact, students may be actively discouraged from expressing their metaphorical ways of noticing in mathematics classes. Such discouragement may be from teachers' fear of introducing extraneous, unproductive or distracting images in teaching and learning. Certainly, not all metaphors prove to be helpful in developing a deeper mathematical understanding, and some are eventually jettisoned or at least deemphasized as part of a process of familiarization with mathematical objects. But I contend that, without the encouragement for students to have, hold, and communicate multiple metaphors in mathematics class, they are deprived of potentially innovative ways of noticing and understanding mathematical ideas. The "banning" of rich and surprising metaphors from mathematics classes also reduces the possibilities for poetic, joyful and humourous approaches to the learning of mathematics. I call for educators to notice and value the wide range of metaphors that students bring to their mathematics learning. By hearing these metaphors, teachers can value learners' perceptions and generate openings to mathematical noticing and understanding.

Coda

David Pimm works with metaphor in his own poetry as well as in his mathematics education writing. I want to close with this playful and highly metaphorical poem, where a cat named Jazz plays (with) standards—recalling the NCTM standards as well as musical ones?—in riffing on the cat(ching) of a bird, "Trills/of yellow feathers/dizzy the porch" (Dizzy Gillespie, perhaps?)

The Call of Standards

I play standards so I know who I am.

—Wynton Marsalis

Jazz, that calico cat
you insist is mine,
appears at the patio door,

feather against fur,
goldfinch taut in her teeth.
Minutes before,

I'd watched her
stalk through grass,
each limb

slow as evolution,
supple as a wave,
ears quivered
as if far off
she'd heard the backbeat
of the Serengeti.

Trills
of yellow feathers
dizzy the porch.

(Pimm, 2013)

Note

[1] See Gerofsky (2011a) for further discussion of cultural and emotional associations with vertical and horizontal lines and movement.

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