Student-generated Problems: Easy and Difficult Problems on Percentage

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1 Introduction

In order to teach mathematics in a way that respects the ideas of students, it is essential to know what the perception of the students is, for instance, of problems about percentage. If they are encouraged to express their perceptions certain aspects of students' mathematical development can be brought to light. And, what is more, these could help the teacher acquire some degree of insight into how to proceed with the course at hand, and perhaps even to anticipate in the actual teaching the future understanding of the students. Problems generated by the students themselves may reflect in an informal manner, some sort of anticipation of their future learning so serving as a guiding principle for future teaching [cf. Streefland, 1985].

This article reports on a study in which the knowledge and ideas of fifth grade students about the contents of what they are learning was investigated within the domain of percentage. The research not only reveals the range of comfort the students had in the instructional sequence, but also gives indications for the development of assessment problems.

2 Some first examples

What is an easy, and what a difficult, problem about percentage for a fifth grader? The examples in Figure 1 offer a first look at the possible range of easy and difficult problems for students who have just been confronted with percentage in class for the first time. The problems in this sample are of a common kind. Both teachers and researchers who are involved in mathematics education at this grade level will recognize the problems and their classification. The easy problems show simple calculations of parts of a whole, or the percentage if a part and the whole are known. The particular percentages involved are also of a simple kind, like 50% and 25%. The difficult problems not only involve more difficult percentages, such as 20% and 75%, but also more complex situations. For instance, the making of comparisons. However, the sample also shows some overlap between the two categories. Depending on the individual's level of knowledge a problem can belong to either category.

For teaching purposes it is more interesting to know why students consider certain problems to be easy and others more difficult than just to know how problems are classified. Figure 2 shows various reasons why these problems could be classified as easy or difficult.

Easy	Difficult
What is 50% of 80?	What is 10% of 50?
George had a race. 100 people entered, 50 people dropped out. What was the percentage of drop-outs?	George had another race. 80 people entered, 75% of the people dropped out. How many is that?
10% of 7,500 is ... than 300 1% of 7,000 is ... than 300	A hat is usually sold for $300.00. The sale price is 10% off. Another hat is marked $200.00 and the sale price is 20% off. Which is the better buy?

Figure 1
Some examples of easy and difficult problems on percentage

Reason why Easy	Reason why Difficult
Because it is half	Because some 5th graders would think that is hard
Because you just divide the bar in half	Because you have to think harder to get the 75%
Because you just divide for your answer	Because you have to figure it all out before you have got an answer

Figure 2
Reason why the problems in Figure 1 are easy, respectively difficult

The relevance of Figure 2 for this article is that it reflects the opinions of the students. Not only have the students expressed their ideas about the nature of these problems, but since they were the designers of the problems, the classification provides us with an index of the range of comfort students feel about percentage problems. It also reveals their awareness of why problems are considered either easy or difficult.

3 A change of perspective

In *Realistic Mathematics Education* as it was developed in The Netherlands over the last decades, students are considered as important contributions to the teaching-learning process, through their constructions and free productions. [1]

This means a marked shift from what was, and still often is, the case, namely that students are considered as passive learners to whom the knowledge and insights of the teacher and the textbook are dished out This change of perspective is also one of the leading philosophies of the *Mathematics in Context* project [Romberg (Ed.), in preparation], a NSF-sponsored curriculum development project designed to create a comprehensive set of mathematics materials for grades 5 through 8 The project is a joint effort of the National Center for Research in Mathematical Sciences Education at the University of Wisconsin-Madison and the Freudenthal Institute of the University of Utrecht in The Netherlands.

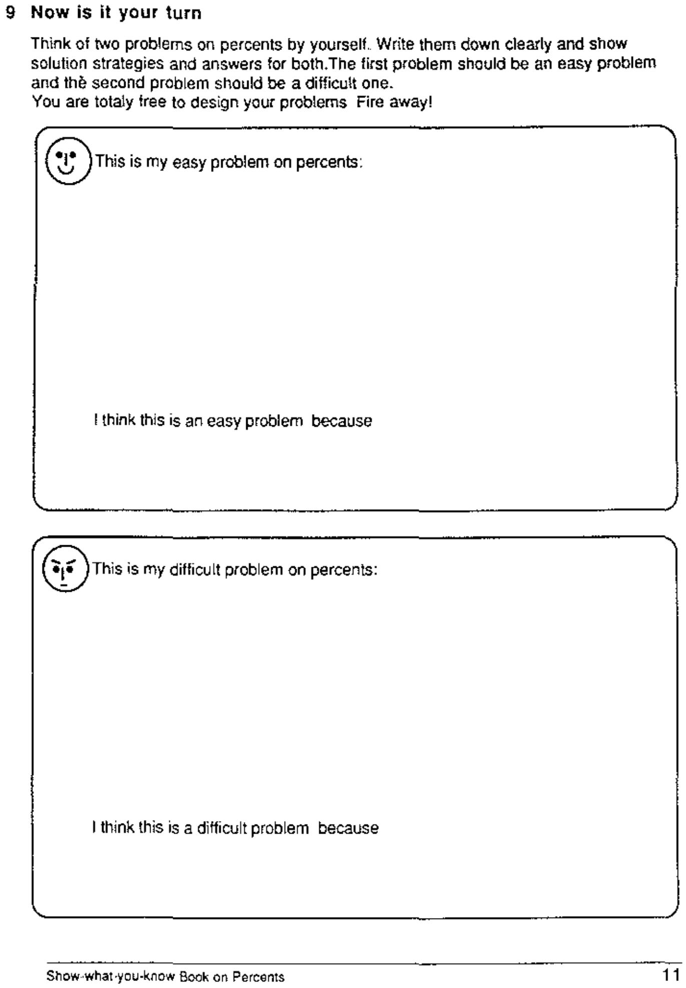
Since innovations in education cannot succeed without new forms of assessment [Romberg, Zarinnia & Williams, 1989] the *Mathematics in Context* project also contains a research component that focuses on assessment. Among other aspects, attention is paid to the creation and testing of new kinds of problems for assessment that are better suited to what we actually want children to know and be capable of [see Streefland & Van den Heuvel-Panhuizen, 1992; Van den Heuvel-Panhuizen, 1993a, 1994, in preparation] The current research reported in this article is situated in the development of a grade 5 teaching unit *Per Sense* [Van den Heuvel-Panhuizen, Streefland, Meyer, Middleton & Browne, in press]. This four-week unit is intended to be the students' first formal exposure to the concept of percentage

In the unit, students are immediately confronted with problems that require the use of their knowledge of percentage. Unlike the usual approach, it starts off by exploring the informal knowledge of the students. Algorithms are avoided. Instead of learning various calculation procedures, more attention is devoted to the so-called qualitative approach to percentage, in which estimations and the connection to simple fractions and ratios play an important part.

To support this line of thinking the percentage (or fraction) *bar*—that later on becomes a *double number line*—and the *ratio table* are used [for a discussion of the latter, see Middleton & Van den Heuvel-Panhuizen, 1995] Activities are developed that show students that they must learn to use a convenient kind of standardization for comparisons between different quantities (different parts of different wholes). The problem situations, or contexts, used in this unit were chosen in such a way that the students feel a need for standardization, while at the same time these situations become models that can support mathematics activities such as estimation or calculation

Development of the assessment problems intended for this teaching unit, which is also the core issue of this article, occurred in two cycles [Van den Heuvel-Panhuizen, in preparation]. A first set of problems was designed, tried out, and revised on the basis of students' responses. That resulted in the *Show-what-you-know Book on Percents* [Van den Heuvel-Panhuizen, 1993b].

It contains ten problems on percentage, ranging from finding a percentage or a part of a whole, to comparing different part/whole situations. One of these, Problem 9, is illustrated in Figure 3. It is aimed at enabling students to express the range of their knowledge. Students are asked to develop their own easy percentage problem, followed by a difficult one.


9 Now is it your turn

Think of two problems on percents by yourself. Write them down clearly and show solution strategies and answers for both.The first problem should be an easy problem and thè second problem should be a difficult one.
You are totaly free to design your problems Fire away!

This is my easy problem on percents:

I think this is an easy problem because

This is my difficult problem on percents:

I think this is a difficult problem because

Show-what-you-know Book on Percents 11

Figure 3
Problem 8 from the
Show-what-you-know Book on Percents

Asking students to describe what they consider easy and difficult problems sounds fairly straightforward. Yet this is rarely done, either in research or in practice. The underlying reason for the inclusion of such problems in assessment tasks is based on the idea of problem posing, which is receiving increasing interest and emphasis in mathematics education research [see, among others, Freudenthal, 1973; Treffers, 1987; Streefland, 1987, 1990; Van den Brink, 1987; Silver, 1994]. Problem-posing is becoming more and more of an established activity in the math classroom. It is seen, for instance, as a dynamic, student-centered approach to mathematics problem solving instruction [see Winograd, 1992; Silverman, Winograd & Strohauer, 1992]. Moreover, by including student's own problems, the actual curriculum, by which we mean the shared knowledge of the classroom community [Goodland, 1977; Romberg, 1987], becomes situated in the student's experiential basic and previous knowledge, providing ready cognitive anchors for instruction.

This change of perspective, from problems that are almost exclusively generated by adults to student-generated problems, has also had its particular influence on assessment. An illustration thereof is the increasing attention to self-assessment [see, among others, Stenmark, 1989] and student-constructed tests [see Clarke, Clarke, & Lovitte, 1990; De Lange & Van Reeuwijk, 1993] Although the above mentioned Problem 9 is a self-assessment task of sorts, it also has a different purpose.

4 Student-generated problems: a source for developmental research on assessment problems

Due to their openness of character, student-generated problems can be a rich source of information for both teachers and researchers Besides the information that teachers can obtain about the difficulties encountered by their students, they can also collect information about the budding knowledge of their students. This means that student-posed problems can serve as a springboard for future instruction Moreover, the extent to which student's problems reflect teaching strategies can tell the teacher something about her or his effectiveness in facilitating mathematical problem solving.

In our current research the focus is on the guidance that student-generated problems can give for the development of assessment problems. Student-generated problems are exploited for the improvement of assessment problems and for bringing to the fore new theoretical notions about assessment in mathematics education To achieve this, Problem 9 was used to (i) investigate what students think are easy problems and are difficult problems, (ii) investigate whether students are aware of why a problem is easy or difficult, and (iii) investigate student's ideas about percentage problems: for instance, what they think such problems should look like

The *Show-what-you-know Book on Percents,* which includes Problem 9, was administered in two self-contained fifth grade classrooms (44 students in total) during the *Mathematics in Context* project, following completion of the *Per Sense* unit.

4.1 Results of the analysis of student-generated problems on percentage

The range from easy to difficult

In the analysis of student-generated problems we found a great variety of differences between what students considered to be easy problems and what they considered as being difficult. It should, however, be noted that not all of the problems, and especially the difficult ones, were solved correctly by the students. Hence the categories "easy" and "difficult" refer only to the problems themselves and to what the students said about them. As was already illustrated in Figure 1, a problem that is easy for one student can be difficult for another On the whole, the collection of problems covered the range of ability levels from grade 5 onward fairly well Not only did the student productions offer a cross-section of the understanding of a class at a particular moment, they also gave an indication of the longitudinal trajectory students might follow generally: from operations with simple percentages and even numbers, [2] to operations with difficult percentages and odd numbers; from simple one-step problems to complicated multi-data-problems; from simple comparison situations to problems which demand reasoning backwards Figure 4 shows a selection of what was found in one class It almost covers the contents of a complete curriculum for grade 5.

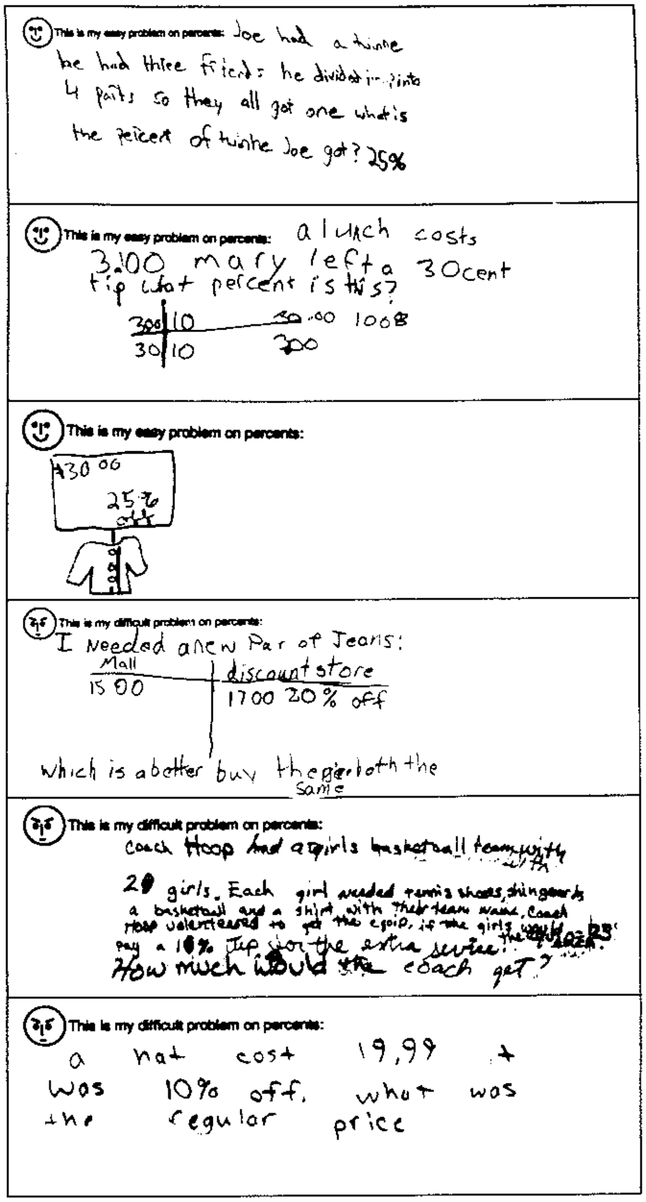


Figure 4
Examples of student generated problems
found in one class

In Tables 1 and 2 of Appendix A a more complete overview is given of what students considered to be easy percentage problems and which they found difficult. Besides their use for assessment, these problems can also provide the teacher with interesting teaching material which is embedded in the knowledge of the class itself In our view, assessment connects seamlessly with instruction.

Awareness of why a problem is either easy or difficult

In Tables 1 and 2 one can also observe why students found a particular problem easy or difficult. The ease or the problem generally had to do with the ease or simplicity of the percentage or the numbers (benchmark percentages, and easy-to-divide whole numbers). The difficulty of a problem was mostly due to the presence of large or awkward numbers, difficult percentages, and to the complexity of the structure of the problem (reasoning backwards, comparing situations, too much data, missing data). Another factor that can make a problem difficult is when it must be solved without a calculator. (Note: students mainly focused their attention on the computation of percentages within the scope of their definitions of easy and difficult.)

The examples given in Tables 1 and 2 make it clear that in general students were well aware of what makes a problem easy or difficult. For instance, there was the student who generated the following problem:

> If there was a sale, 50% off on $100.00 (sweaters), what would you pay?

The student explained that this was an easy problem because:

> It is easy to find any % of 100. The percent = that number

The descriptions show that sometimes students reached the limits of their insight. As did the student who generated the following difficult problem that requires reasoning backwards:

> Fred wrote a book. It costs $8.00 to buy. His royalties come to 35 cents. What percentage is that? Explain how you arrived at your answer

Asked why this problem is a difficult one the student answered:

> It is hard to explain what you did, and if you didn't get it the right way it is really hard.

Occasionally the explanations clearly indicated a lack of understanding. For instance, one student wrote 50% on the worksheet and explained that it was a difficult problem because:

> I don't understand how to do 50% that well.

Besides clarifying the awareness of students about the degree of difficulty and ease of problems, student-generated problems can also be used for class discussion. There students can explain their ideas about easy and difficult and compare them to the various opinions held by others in class (e.g., Who agrees that this is an easy problem? and Why do you think this problem is easy?). Trying to determine for oneself what makes a problem easy or difficult, together with learning the various strategies others use, is a powerful instrument by which to achieve a higher level of understanding.

Diversity of types of problems

The variety of types of problems students considered about percentages provided us with information about their conceptions of how a problem looks. Several categories can be defined: word problems story problems, picture problems, bar problems, ratio table problems, and formal problems. Examples of each category are presented in Figure 5.

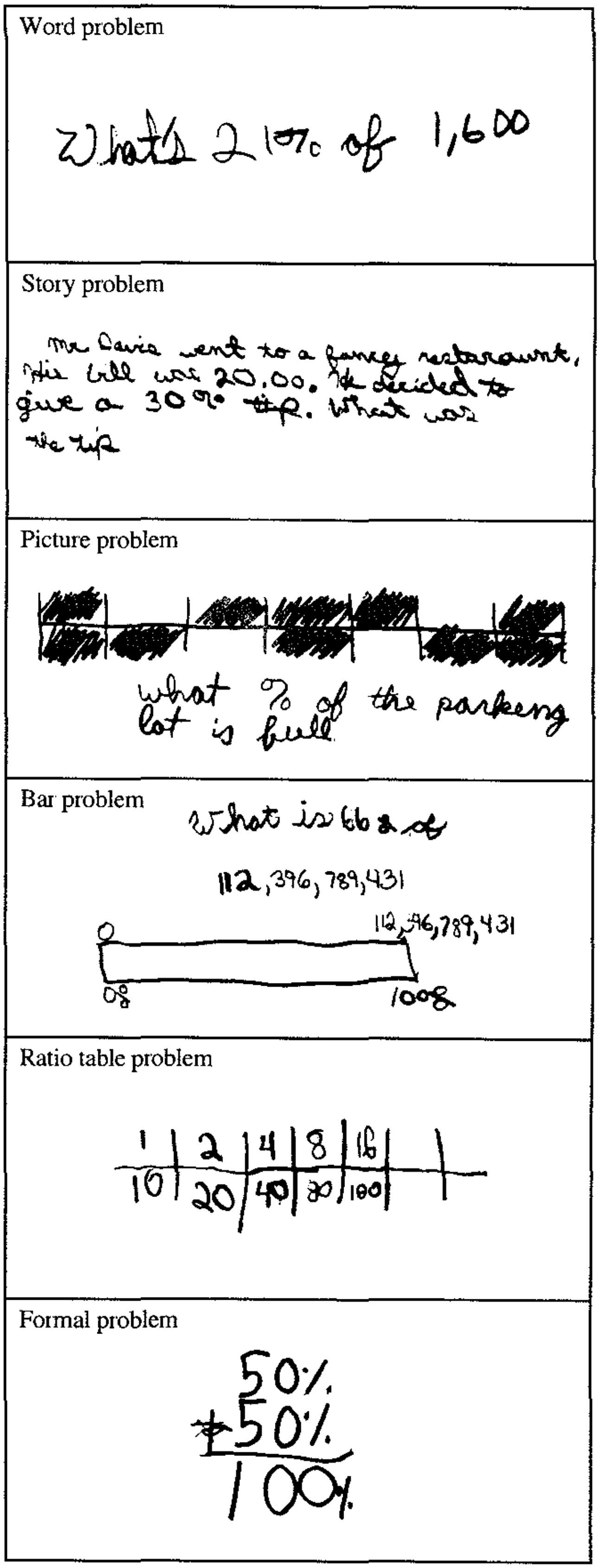


Figure 5
Diversity of types of problems

Interestingly enough, the story format was chosen, in the majority of cases, for both easy and difficult problems. (See Table 3.) Furthermore, the difficult problems tended to include more purely word problems, while the easy problems tended to contain more picture problems.

Show-what-you-know book Problem 9	Total grade 5 N = 44	Class A grade 5 N = 27	Class B grade 5 N = 17
Easy problem			
Type of problem			
word problem	8 (18%)	1 (4%)	7 (41%)
story problem	19 (43%)	15 (56%)	4 (23%)
picture problem	10 (23%)	9 (33%)	1 (6%)
bar problem	4 (9%)	0 (0%) (+1)*	4 (24%) (+7)
others**	3 (7%)	2 (7%)	1 (6%)
Difficult problem			
Type of problem			
word problem	12 (27%)	5 (18%)	7 (41%)
story problem	21 (48%)	17 (63%)	4 (23%)
picture problem	3 (7%)	3 (11%)	0 (0%)
bar problem	3 (7%)	0 (0%) (+3)	3 (17%) (+7)
others	5 (11%)	2 (7%)	3 (17%)

*The number of students that used a bar for solving another type of problem
** 1 formal problem included

Table 3
Types of problems of that were made by the students

Because the problems revealed what the students picked up from the instruction, they provide feedback for teaching. Analysis of the student-generated problems reveals that they illustrate the influence of instruction on two levels. First, the contents and intentions of the *Per Sense* unit were clearly reflected in the work of the students. Examples of these were: the use of parking lots, auditorium seating arrangements, tipping in restaurants, fraction/percentage bars, and especially the demand for an explanation of answers.

Secondly, a closer look at the two classes revealed that, despite that fact that both teachers taught the same unit, the students produced different types of problems. In class A more students came up with story problems. In class B word problems were more common. Also, students from class B referred to the fraction/percentage bar more frequently, both in their problems as well as their solutions.

Students as professionals?

The student-generated problems discussed here suggest that it is not such a bad idea for teachers and/or researchers to call upon the professional help of students in the development of assessment problems. [3] Students have immediate access to their own knowledge and strategies, and by creating assessment problems they can assist the teacher in understanding the range of variability in knowledge and the stages in the learning process the class has achieved. Discussions of how problems were developed and solved can provide multiple ways of coming up with math topics from which the entire class can learn. Wiggins [1992] has even suggested that students be asked for help in devising a scoring system. By working on a scoring system the students make evaluation a thing of their own. It makes clear to them that judgments need not be arbitrary. It also becomes possible for students to achieve higher standards as a result of clearer and more reasonable criteria. Even if teachers feel reticent about allowing students to create assessment problems for evaluation (i.e., grading) purposes, allowing them to create a pre-test for study, prior to the regular assessment, could be an acceptable alternative. This still provides a good opportunity to involve the students in the instruction and to enable them to develop more ownership of the mathematics. The student-generated problems collected in this study indicate that students are actually quite good at posing problems. Figure 6 provides three examples that reflect a certain level of professionalism.

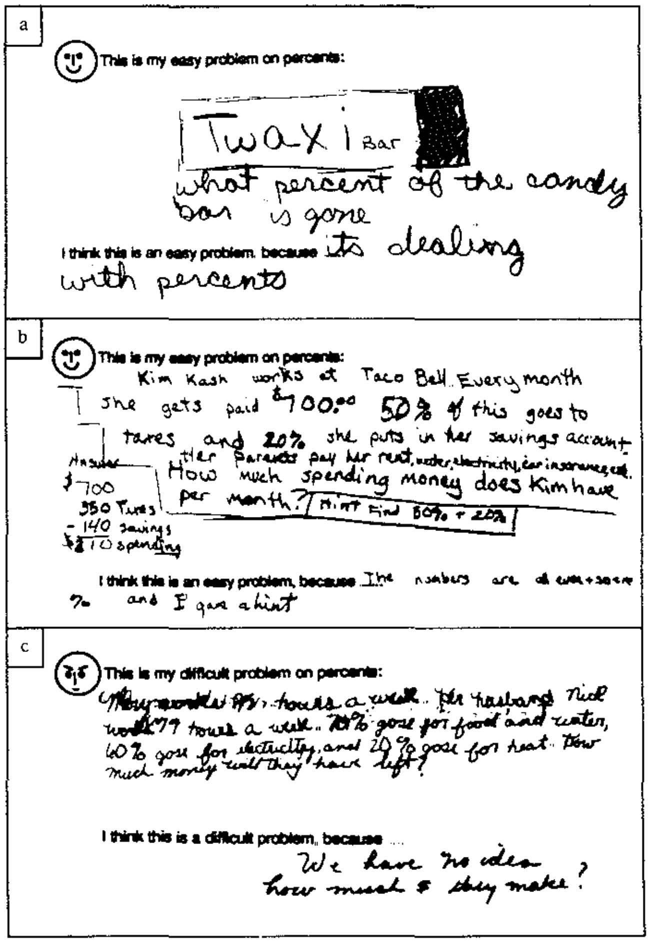


Figure 6
Examples of "professional" problems of students

The first problem (a) indicates that one must not forget that a student's understanding of percentages has its non-numerical, contextual roots. The second problem (b) illustrates that in assessment problems, giving help should not necessarily be ruled out. This is clearly illustrated by the student who explained that the problem was easy because:

The numbers are all even + so are %, and I gave a hint

And the third problem (c) about spending monty (70% for food and water, 10% for electricity and 20% for heating) illustrates that students can even come up with problems that are problems for teachers The first sentence of this problem, "Mary works 95 hours per week" turned out to be a perfect distractor for the student's teacher, who did not catch on to the problem immediately Her literal reaction to the question about the amount of money left, was:

> We have no idea how many dollars they make?

Obviously it slipped her mind that in this case the question could be answered without knowing this In reaction we can only repeat the words of Silverman *et al* [1992, 6]:

"It is no fairy tale—student-generated problems are novel, interesting, and challenging."

5 In conclusion

Summarized, student-generated problems are a rich source for developing assessment problems Firstly, analyzing the perceived range from easy to difficult provides a framework of sorts which can be helpful in developing problems with a degree of difficulty that suits the students being tested Moreover, the information about what students find easy or difficult can contribute to a higher face validity of the problems In addition, students can recognize their own ideas in the problems, which can provide them with a sense of ownership over the mathematics class Their contribution can expand the kinds of problems that are included in assessment tasks

The students' problems in this study show that assessment problems need not be restricted to word problems and formal problems, or exclude other representations of mathematics and reality And, finally, students are capable of creating some really excellent problems Teachers and/or researchers can enlist students as professionals in the effort to improve assessment

Notes

1 The ideographic approach to teaching and learning is actually not new Dewey [1902], for example, clearly worked within a student-centered educational model a century ago

2 By even numbers the students meant numbers that are divisible by several numbers By odd numbers they meant numbers that are only divisible by a few numbers

3 One of the teachers involved in the study has already done this

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Appendix

Show-what-you-know book Problem 9, Easy problem grade 5, n = 44		
What makes the problem easy?	**N**	**examples***
a Easy numbers/percentages	30	
- 50%	11	- 'What is 50% of 80?'; it is easy because it is half - Made a drawing of a concert hall which is colored half; wrote below it: 'How full is the concert?'; 'I think it is easy because it is obvious that is 50%'
- % of 100	6	- 'If there was a sale, 50% off of $100.00 [sweaters] what would you pay?'; is easy because 'it is easy to find out any % of 100. It is that percent = number' 'Try to find half of 100. What is it?'; drew a bar and filled it in; it is easy because 'all you have to do is to divide it in half'
- others (10%, 25% 100%, 'even number' $200)	12	- 'Mary went out for lunch at Bob's restaurant. It costs $3 50. She gave him a 10% tip. How much did she gave him?'; answer '10% of 350c is 35c All you have to do is to divide' - 'Kim Kash works at Taco Bell. Every month she gets paid $700 00. 50% of this goes to taxes and 20% she puts in her savings account Her parents pay rent water, electricity car insurance etc How much spending money does Kim have per month? Hint: find 50% + 20%'; it is easy because 'the numbers are all even + so are % and I give a hint'
b Easy tool (bar)	2	- Drew bar; split it in four parts; wrote below it 0 25 50, 75 100; and above it 0, 25 50 75; colored the whole bar; it is easy because 'all you do is fill the whole thing in'
c Analogy with other problems	1	- '50% + 50% = 100%'; it is easy because 'it's the same as normal +'
d Done before	1	- 'The Tigers had a game. There was 20 Tiger fans and [for every] 5 Lions fans. If there were 200 Tiger fans, how many Lions fans would there be'; made a ratio table: T/L, 20/5, x10/x10, 200/50, 'or to find out percent'; 1% of 200 = 2, 10% of 200 = 20, 1% of 50 = .5 10% of 50 = 5'; it is easy because 'We did it in the book. You just take % (1) and what its of (50) So you would x 50 by 01 and get 5'
e No % problem	6	- Made a drawing of two parking lots; table has to be filled in; it is easy because 'I know that everybody in my class knows how to count squares and spaces'
f Unclear	5	- 'I bought a toaster for 17 99 including the discount of 30% off from the original price. What is the original price'; answer: '19 99 original price'; it is easy because 'It's not very long or difficult to figure out' - Drew a bar; called it Twaxi bar; shaded a part of it; wrote below it: 'What percent of the candy bar is gone'; it is easy because 'it's dealing with percents

Table 1: What are easy problems and why?

Show-what-you-know book Problem 9, Easy problem grade 5, n = 44		
What makes the problem difficult?	**N**	**examples**
a Difficult number/ percentages (10%, 25%, 50%, 75% $12\frac{1}{2}$ % 15%, 60%, 21% 18% 66%)	20	- 'What is 66% of 112,396,789,431?'; it is difficult because '67% is an odd percent, plus it takes a little bit longer because it's a bigger number' - 'What percent is 8692 to 3255'; it is difficult because there is an uneven number that don't come even at all [xxx]' - 'Jason went to East Town to buy a pair of boots They cost 35.00. A sign said it was 25% off How much were they?'; it is difficult because 'it took more than one step to do it on a bar graph' - '50%'; it is difficult because 'I don't know how to do 50% that good'
b Complex problem structure	10	
- Backwards reasoning	2	- 'Fred wrote a story It costs $8.00 to buy and his royalty is 35c. What percent is this? Explain how you get your answer'; it is difficult because 'it is hard to explain what you did and if you don t have the right way it's really hard'
- Comparing situation	3	- 'I need a new pair of jeans [in schematic notation:] Mall 15.00 Discount store 17.00 20% off. Which is a better buy'; it is difficult because 'It took more figuring'
- Much data	4	- 'There's a VCR [xxx] for $389 99. How much would it cost if it was 39% off? Tax is $5 to the hundred. $245 50 with tax'; it is difficult becasue 'It's not dealing with even numbers like $400, or 50% You have to think a little harder'
- Lacking data	1	- Schematic drawing of two concert halls; above it is written: 'Which concert is more full'; it is difficult because 'I did not tell you anything about the concerts'
c Difficult tool (ratio table)	3	- Mrs. Johnson went to a store She bought a pan [xxx] for $5.00. Tax in her town is 5¢ / a dollar. What was her total bill + tax'; it is difficult because 'I used a ratio table to figure it out'
d Without tools (calculator paper and pencil)	4	- Drew a parking lot with 14 spaces; a part of them are colored; wrote below it: 'What % of the parking lot is full'; it is difficult because 'you can t use a calculator for it'
e Not done before	2	- 'Chris has bought some land. If he buys it, he realizes that he should have spent 1,128,000 instead of 960,000. What percent off is that'; it is difficult because 'it is harder than in the book'
f No % problem	4	- 'There are 14 bins on a ship with 26 crates of books. Each crate has 80 books in them. How many books are on the ship?'; it is difficult because 'more figuring out
g No problem	1	- Don't know

Table 2: What are difficult problems and why?