ON METACOGNITION

ALF COLES

This journal has published several articles about teaching strategies, heuristics and problem-solving (*e.g.,* Schoenfeld, 1983; Clement & Konold, 1989; Zack & Reid, 2003) but none explicitly on metacognition. Supporting learners' metacognitive capacities is an important issue in the learning of mathematics. A consistent research finding has been that levels of metacognitive knowledge and skill are good predictors of mathematical performance in primary and secondary school (Schneider & Artelt, 2010, pp.158-9). There was considerable work within this area in the 1980s and 1990s (notably, Garofalo & Lester, 1985; Schoenfeld, 1987, 1992) that led to many ways of viewing or categorising the concept of metacognition. Veenman, Van Hout-Wolters & Afflerbach (2006) list sixteen terms for overlapping aspects of metacognition developed by researchers, including: "[m]etacognitive beliefs, metacognitive awareness, metacognitive experiences, metacognitive knowledge, feeling of knowing, [and] judgment of learning" (p. 4). Such a proliferation of terms has led reviewers of the field to conclude there is marked inconsistency (Brown, 1987, p. 105) and even contradiction (Moseley *et al.,* 2005, p. 378) in the way metacognition has been conceived.

In this article, I offer a re-framing of the concept of metacognition, away from the cognitivist roots that appear to have led it into a proliferation of categories. I suggest an alternative is to see metacognition as a *way* of teaching and learning, at a *meta*-level to the type of category above. This re-framing arises from my analysis of data from one mathematics classroom in the UK.

Conceptualisations of metacognition

As a starting point, I take metacognition to refer to our knowing about our own cognitions. The most common distinction in metacognition research is the separation of metacognitive knowledge from metacognitive skills (Veenman *et al.,* 2006; Depaepe *et al.,* 2010; Schneider & Artlet, 2010). Metacognitive *skill* describes how we are able to influence and alter the way we approach tasks or problems. Metacognitive skill is "procedural" (Sneider & Artlet, 2010, p.150), it is our metacognitive "know-how" or the monitoring of the strategies we employ to solve problems, also described as "regulation of cognition" (Depaepe *et al.,* 2010, p. 206). Metacognitive *knowledge* describes the extent to which we are aware of, for example, how we are approaching a task or the feelings provoked. Metacognitive knowledge is "declarative" (Schneider & Artlet, 2010, p.149), it is our metacognitive "knowing-that", the awareness, for example, that we are in the process of exploring different options, or stuck, or not clear about the task. For the purposes of this article, I use the distinction between a procedural, know-how, regulatory, "skills" component of metacognition and a declarative, knowing-that, "knowledge" component. I note, however, that there are further complexities and disagreements. Mevarech *et al.* (2010), for example, use the words "procedural" and "declarative" (p.196) to describe aspects of metacognitive knowledge, which they separate from "regulation of cognition" (they see regulation of cognition as developing earlier than knowledge of cognition, in humans).

Viewing metacognition as comprising of two distinct categories leads to problems in planning for the teaching of metacognition and it is for this reason I am interested in offering an alternative perspective. Schoenfeld (1992) raised the following issue in relation to Polya's (1957) classic text about problem-solving in mathematics:

> the critique of the strategies listed in *How to Solve It* and its successors is that the characterizations of them were descriptive rather than prescriptive. That is, the characterizations allowed one to recognize the strategies when they were being used [...] [but] did not provide the amount of detail that would enable people who were not already familiar with the strategies to be able to implement them. (Schoenfeld, 1992, p. 353)

Schoenfeld seems to be suggesting that a prescriptive approach is preferable to a descriptive one; yet the difficulty then, as he goes on to point out, is that any single description of a strategy must be translated into a dozen or more prescriptive routines and before long, a list of, say, six heuristics turns into close to one hundred prescriptions. In other words, as a teacher, I can choose a small set of heuristics that seem important ("trying out simple cases" might be one) and I might emphasise their importance to students. This approach may allow my students to recognize such a strategy when they see it being used but, according to Schoenfeld, is unlikely to mean they are in a position to know *when* would be a good time to use it, nor *how* to use it. It is such monitoring that is the mark of metacognitive skill. So, taking a descriptive approach, students would gain metacognitive knowledge but not skill. If I split the heuristic into more prescriptive routines (for example, "if you have an algebraic function, look at when $x = 0$") then my students may develop metacognitive skills; the "if" clause implies some monitoring is taking place. As a teacher, however, I end up with an unmanageable list of scenarios to cover and for students to memorise. It appears that the splitting of metacognition into the dichotomy of skills and knowledge leads to a teaching dilemma. Do I focus on "knowledge" and risk students not developing skill, or do I focus on "skill" with the risk of overwhelming students with lists of specific strategies?

Schoenfeld (1992) reported on training his students to use metacognitive self-questioning, as a way through this dilemma. Mevarech and Kramarski (1997) drew on Schoenfeld's work to develop a model of metacognitive instruction called IMPROVE (the acronym represents the teaching steps that constitute the method: Introducing the new concepts, Metacognitive questioning, Practicing, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment). In this method, students are trained to ask four *kinds* of metacognitive, self-addressed questions: comprehension questions, connection questions, strategic questions and reflection questions. The asking of questions such as "what is this problem all about?" (a comprehension question) is an example of a procedure or metacognitive skill. The approach taken is, therefore, on the "prescriptive" side of Schoenfeld's dilemma and there are indeed several questions that students are trained to use within each of the four categories. Mevarech and Fridkin (2006) report on the use of IMPROVE leading to impressive gains in the development of metacognitive knowledge and skill, improved mathematical reasoning and improved attainment on standard tests. They suggest future research involving direct observation of metacognitive instruction is still needed, to throw light on its benefits. This article offers such direct observation and, in the process, I suggest an alternative view of metacognition, away from both the knowledge/skills dichotomy and the descriptive/prescriptive teaching dilemma.

Dealing with dichotomies

Reid and Brown (1999) discuss three mechanisms for escaping dichotomies in mathematics teaching and learning, moving to a position where dichotomies are not seen in conflict. The first mechanism is "both" (p. 17): if conflict arises when extremes are seen as either/or, they can instead be seen as both/and. In Reid and Brown's example, they avoid conflict between their roles as teacher and researcher in a classroom through seeing them as complementary. The second mechanism is "fork in the road" (p. 17): sometimes a choice does have to be made but there are always routes between the branches of the fork at a later time. The third mechanism is "star" (p. 18), which draws on Varela (1976), cited in Hampden-Turner (1981, p. 193) (see Figure 1).

If the left hand set of words in Figure 1 is considered to be "it", the right hand set can be seen as "the process of becoming it". Hampden-Turner states:

> Varela proposes that dualisms or dialectical "contradictions" such as mind/body, whole/part, context/text, territory/map, being/becoming, intuition (right brain)/ logic (left brain) or environment/system should be conceived of as "stars". All stars consist of "the it"/"'the process of becoming it" where the slash or oblique stroke means "consider both sides of". Hence, we must consider both it and the process leading to it […] By looking at the star from one side or the other, "it" or "the process of becoming" are seen as emerging from the context of its opposite. (1981, p. 192)

I propose viewing the "metacognitive knowledge"/ "metacognitive skill" distinction as a "star" in just this way. Metacognitive skills are written about (perhaps unsurprisingly) as actions, or processes, as in, for example:

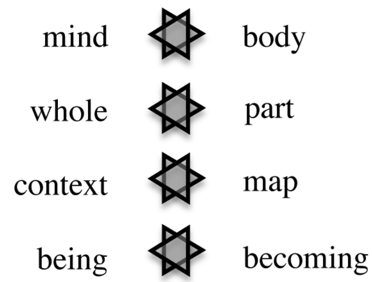

Figure 1. Varela's "stars".

"planning", "monitoring" (Van der Stel *et al.,* 2010, p. 220); "build a mental representation of the problem", "decide how to solve the problem" (Depaepe *et al.*, 2010, p. 206); "uses a model to support cognition" (Whitebread & Coltman, 2010, p.168). I see these skills on "the process of becoming it" side of Varela's star; the actual plan or representation or model used is likely to be different in every problem encountered; what is the same is something about the process of arriving at the plan. Metacognitive knowledge is generally seen as the more stable side of the distinction; the words used to describe it are relatively static, as in the following examples: "knowing about one's tendency to commit easy errors" (Sneider & Artlet, 2010, p.150); "I do not easily give up", "I get frustrated if I do not understand the problem" (Yimer & Ellerton, 2010, p. 255); "personal knowledge of strengths and weaknesses" (Whitebread & Coltman, 2010, p.167). These examples are states of being, "the it", on the left hand side of Varela's star; from solving one problem to the next there will not often be significant change in, say, one's knowledge of personal strengths and weaknesses.

Unlike a one-dimensional view of dichotomy as terms and their negation or complement, Varela's star presents distinct ideas as being at different logical levels: "it is a way to proceed from disjoint pairs to their unity at a metalevel" (Varela, 1976, p. 62). So, rather than construct a teaching dilemma out of the knowledge/skills distinction, I want to offer a view of metacognition that considers both sides as emerging from the context of the other. In order to ground this suggestion, in the next section I draw on a study that represents a "paradigmatic case" (Freudenthal, 1981, p. 135) of metacognitive instruction. I use Varela's star to analyse the metacognitive practices that the study exemplifies.

A study of one secondary mathematics classroom in the UK

For this article, I focus on data collected in the classroom of Teacher A, who was recognized within the school and by outside agencies as an expert practitioner [1]. There is evidently metacognitive instruction and learning taking place in her classroom, as I report below. After giving some details of the study from which I take this data and its methodology, I characterize what I see as taking place in Teacher A's classroom, focusing on teacher-student interactions during whole class discussions.

I took video recordings of year 7 (aged 11-12) mathematics lessons of Teacher A in one school where, at the time, I was head of the mathematics department. Four video recordings were taken in the first six weeks of the year (September to early October 2007), as I was interested in researching the role of the teacher in establishing patterns of communication during whole class talk. I believed, from my own experience of teaching, that these public conversations were significant in occasioning the emergence of particular ways of working mathematically within a classroom and I wanted to investigate. The other recordings (six in total, each one hour in length) were taken in January and June 2008, to give snapshots of changes over the year.

The methodology for the study was enactivist (Varela, Thompson & Rosch, 1991). In enactivist research, the instinct is always to look for relationships. For example, to view perception as saying something about my connection to the world, not about the world "itself" (nor indeed, about "me"), or to view an item of behaviour not as an expression of a personal trait or characteristic, but as one part of a sequence of on-going interactions. We are not pre-given minds passively receiving information from a pre-given world, rather, there is an "enactment of a world and a mind on the basis of a history of the variety of actions that a being in the world performs" (Varela, Thompson & Rosch, 1991, p. 9). Our history of interaction in the world determines what can count for us as a perception; faced with a long equation, a young student may see a mass of indistinct symbols, whereas I see a second order differential equation. To learn is to come to perceive and act differently, in a given context.

In enactivist research, practical knowledge linked to action is foregrounded, with propositional knowledge arising from awareness of action. As Maturana and Varela (1987) state, "cognition is effective action, an action that will enable a living being to continue its existence in a definite environment" (1987, p. 29). An action is effective if it is "good enough" to allow the maintenance of my relations in that context. Knowledge, which requires cognition, is also equated with effective action. Knowing cannot be separated from the knower nor the context in which the knower acts so that "[a]ll doing is knowing and all knowing is doing" (Maturana & Varela, 1987, p. 27). The aim of enactivist research is to arrive at theories that are "good enough for" (Reid, 1996, p. 208) the purposes of their users, thus supporting the development of new awarenesses or distinctions and effective actions.

A key methodological principle is to search for pattern, or "the pattern which connects" (Bateson, 2002, p. 10). I make an assumption that, within a classroom, patterns of talk become more established over time. This assumption leads to an enactivist method of analysis, which is to look at the last piece of classroom data collected, identify patterns and then trace the emergence of these patterns through the earlier data (Brown & Coles, 2011, p. 866). The aim here is to respect the complexity of the situation by avoiding giving any linearly causal account of what has happened through the data, but yet to be able to account for patterns observable at the end. I therefore began with the final recorded lessons (in June 2008) and segmented the recording into activity types (broadly distinguishing between periods of whole class discussion and individual, group or paired work) and selected only times of "whole class discussion". I then looked for patterns and transition points within whole class discussions to identify "episodes" (Levinson, 1979, p. 369). Different episodes were classified according to a predominant pattern of interaction. For example some episodes closely resembled the Initiation-Response-Feedback (IRF) pattern identified by several authors (Sinclair & Coulthard, 1975; Mercer, 1995). A student initiating a sequence of interaction by voicing a new idea could mark another episode. Once I had identified episodes of interaction, I looked at patterns, both within and across episodes, and patterns in the transition points. The patterns identified from the final lesson were then used as "lightings" (Gattegno, 1987, p. 44) on the rest of the data (which includes video recordings of lessons, interviews with three pairs of students and interviews with Teacher A).

A pattern I observed in the final video recording of Teacher A is the use of the word "conjecture". For example (for transcription conventions, see [2]):

> *Teacher A:* I want you to try what has been suggested (.) so can you find a counter-example to the conjecture that all the differences will be eight
>
> *Teacher A:* okay (.) could you try some different starting numbers for your conjecture then

Having identified a pattern, I then looked through the rest of the data to trace the emergence of this word. Starting with "conjecture", my analysis of other lessons threw up three more words that were used frequently in the same episode, namely "counter-example", "proof" and "theorem".

At this stage in my analysis, I was not explicitly focusing on metacognition; looking at "conjectures" had arisen as an issue in the manner described. It has only been later that I have recognised the language of conjecture as being metacognitive. The way the use of these words emerged seemed to be different from anything I have found in the research literature, which provoked me to look for an alternative conceptualisation of metacognition. I focus, in the next section, on the emergence of the language of conjecture, in Teacher A's classroom.

The classroom of Teacher A

The first video recording I have of Teacher A with her year 7 class is their second mathematics lesson of the year, and these students' second mathematics lesson at secondary school (ages 11-18). In this lesson, the word "conjecture" is used forty-two times in whole class discussion, six times by students and thirty six times by Teacher A. There is one extended whole class discussion that lasts twenty minutes of the one hour lesson, when most of these occurrences take place. I present below three of Teacher A's contributions to the discussion (numbered for later reference), that represent the range of ways in which she uses the word "conjecture".

This first contribution is early in the lesson, as Teacher A sets up an initial task for students:

> what I want you to try and do now is *while* you are doing these sums be *thinking* about the conjectures we talked about last lesson (.) be *thinking* about whether you think they are true (.) be thinking about whether you can explain why they are true and why you don't

think they are true and also writing down all of these things (TA, Lesson 14-9-07, #1)

It is clear that the word "conjecture" had already been introduced in the first lesson of the year and that some conjectures have already been found by the class. The transcript also suggests a connection between conjectures and writing. In interviews, Teacher A described the importance she placed on students writing, linked to developing a process of thinking mathematically. Teacher A described writing as a mechanism by which students are forced to consider what they are doing and that hence supports them in making the connections that form conjectures.

During the one extended whole class discussion in this lesson, there is a disagreement amongst students about whether a zero should be retained at the start of a number, in the middle of the process of a number trick (in which all three digit numbers can end up at 1089). The issue is in fact one where both options could be mathematically correct. Rather than close off this ambiguity and force one interpretation on the students, Teacher A leaves the decision to the students and uses the opportunity to make a point about working mathematically:

> so (.) I think what you need to be clear about as somebody working on mathematics as a mathematician (.) you need to decide what you're going to do and be consistent about that [...] J's conjecture might not apply to your rules (.) so when we're making conjectures we need to be clear about the rules we're using (TA, Lesson 14-9-07, #2)

Teacher A makes a further comment later in the lesson, relating conjectures and counter-examples to how mathematicians work, in response to two students who changed their conjectures:

> this is what mathematicians do (.) they develop their conjectures so they begin with something they believe to be true and then they might change their minds having got some results (.) so it's interesting that I've got the same two people in the group (.) whose conjectures have actually changed (.) but what you need to be doing as mathematicians is thinking about starting testing these conjectures (.) and it might be that leads you to develop conjectures of your own (.) it might be that it leads you to disprove one of these (.) and what T has given us an example of (.) and what J has given us an example of (.) is where this doesn't hold to be true (.) called a counter-example (.) an example that doesn't fit the conjecture (TA, Lesson 14-9-07, #3)

In this instance and the one before, it can be seen that Teacher A uses and expands on the word "conjecture" at the moment when a student exhibits a behaviour that fits the notion of what it means (in this classroom) to be an effective mathematician. In interviews, Teacher A reported having told students in their first lesson (the one before this recording) that the purpose for the year for all of them was "becoming a mathematician". It is also evident in transcript #3 that the word conjecture is linked to such a purpose: "this is what mathematicians do".

In subsequent video recordings of Teacher A, the word "conjecture" is used less frequently. It seems as though the process of looking for connections rapidly becomes part of what students expect to do in lessons with Teacher A and, as such, the word "conjecture" perhaps only needs to be mentioned infrequently. In the fifth recording (in January 2008) there are four uses of the word, twice by Teacher A and twice by students; in the sixth recording (June, 2008) there is one use of the word, by Teacher A. However, in both these lessons (and in fact across all six recordings) there is a common pattern of students making statements of generalities, for example:

> *Student:* with the rhombus or kite (.) whatever one we were doing (.) you could um cut it in half and then on the sides you get a little box and it's like half (Lesson, 5-1-08)
>
> *Student:* the two answers in the little circles no matter what you start with will always be the same (.) because if you times five times two it's like times ten (Lesson, 6-6-08)

In both of these examples, students make general statements, in the latter case, with a reason to justify the thinking. Both statements could be called conjectures in Teacher A's classroom, but by January, and then June, there is little explicit need to mention the word; making statements such as these and discussing them have become part of what students do, unprompted, in whole class discussion.

The decrease in frequency of the use of the word "conjecture", with very high occurrences of teacher use in early lessons, mirrors a result reported by Schoenfeld (1992) that he needed to pose metacognitive questions less and less as his courses on problem-solving progressed. It also seems to be significant that the word "conjecture" is barely defined by Teacher A. It is used in *response* to student behaviours. The teacher notices and names the conjecturing of the students and supports these effective actions in the classroom. As the year progresses, different aspects of what it means to conjecture are commented on by Teacher A (*e.g.,* considering the "rules" behind a conjecture, looking for counter-examples). The word therefore acts as a placeholder for an expanding variety of actions that students can usefully perform in mathematics lessons.

The description by Teacher A at the start of the year of how mathematicians work, by developing conjectures, testing them, and finding counter-examples, fits almost exactly with the students' descriptions of what it meant to do mathematics, when I interviewed them in the summer of 2008. I asked them what they had learnt about becoming a mathematician. In all three pairs there was a response typified by the interaction below:

> *Alf:* what have you learnt about thinking mathematically or how mathematicians work
>
> *Student 1:* they always like come up with conjectures
>
> *Alf:* what's a conjecture

Student 1: like a theory (.) and then they see if they can prove themselves wrong

Student 2: or try and prove it right

Student 1: and then they find out other people's conjectures and like test them

Student 2: test 'em yeah they test 'em and from other people's conjectures they also make new ones

The word "conjecture" has come to symbolize a complex pattern of ways of working in mathematics; looking for connections or patterns, then trying to prove yourself wrong, trying to adapt other people's ideas to make your own connections and trying also to prove yourself right. The student responses above do not translate the word "conjecture" neatly into a set of propositions, nor link it directly to specific skills learnt. In other words, these responses link "conjecture" to a complex mix of metacognitive knowledge and skills. The meta-language of conjecture seems to "hold" the process of doing mathematics with Teacher A, linked to the notion of "becoming a mathematician".

From my interviews with students, it is clear that they were introduced in the early lessons to a new way of working on mathematics compared to their primary schools, where the focus (according to them) was far more on working through textbooks. In Teacher A's classroom, students acted in ways that were unfamiliar to them. The language of conjecture and counter-example had been introduced in response to student actions. The words became short-hand for these new ways of acting. All six students spoke in interviews of enjoying mathematics. The language of conjecture, counter-example and proof is subsumed underneath the purpose of "becoming a mathematician", a purpose that is embodied and enacted in the person of Teacher A; the word gets linked to vivid and common experiences as it is initially introduced and used in response to what students do and say.

Re-framing the dichotomy

The process by which the students of Teacher A take on metacognitive practices is highly efficient. By the second lesson of the year, some students were using the new (to them) word "conjecture" and engaging in metacognitive activity (looking for patterns in results, making and testing conjectures, looking for counter-examples). Teacher A used this meta-language linked to an explicit process of "becoming a mathematician", which was offered to students as their purpose for the year. The activity of writing was emphasized to encourage students to think about and become aware of what they noticed. The words of the meta-language (conjecture, *etc.*) were introduced and used at the moments when students themselves exhibited behaviours that Teacher A judged as fitting with what it means, in her classroom, to be an effective mathematician. It is apparent in Teacher A's interactions in the second lesson of the year (#2 and #3, above) that she does not respond to the *content* of student responses. This is a pattern observable in all her responses to students in the early lessons. For example (in #2), rather than responding to a student dilemma in terms of its mathematical source, she responds from the perspective of how mathematicians resolve such difficulties. From interviews and the video recordings of lessons it is clear that Teacher A has a well established (though not fixed) framework of what it means, to her, for students to work mathematically, which comprises the types of activity I reported that students can be seen doing in lessons (asking questions, noticing patterns). Also, from interviews and video recordings at the end of the year, it seems that students had entered into a different relationship to mathematics since starting secondary school and that the language of conjecture, though not often needed in class by the end of the year, "held" the process of working mathematically with Teacher A.

While it may be possible to analyse developments in student metacognition in terms of skills and knowledge, the data above highlights the connection between terms on either side of this dichotomy. For example, in comment #3, Teacher A introduces the label "counter-example". It could be argued that she is pointing to the development of metacognitive knowledge (by offering students a vocabulary which could support awareness of their own or others' work in mathematics). Equally, however, the comment could be seen to be about metacognitive skill, in encouraging students to look for counter-examples (which they talk about in interview: "[mathematicians] see if they can prove themselves wrong"). To take another example, in the interview above, Students 1 and 2 linked conjectures to proof (proving a conjecture is true and trying to prove it wrong), testing, and making new conjectures. These statements display metacognitive knowledge (they are descriptions of what goes on in lessons), but equally it is clear from the lesson recordings that these are general statements of processes students engage in; these processes involve monitoring and regulation and hence are evidence of metacognitive skill.

The mechanism of Varela's "star" (Figure 1) entails viewing each side of a distinction as arising from the context of the other. Teacher A consistently links specific metacognitive skills (*e.g.,* about conjecturing and proving) to what mathematicians do. These skills ("the process of becoming it") arise from the classroom context of students being placed as mathematicians ("the it"). Equally, what it means to be a mathematician in this classroom ("the it") is constituted by the exercise of metacognitive skill ("the process of becoming it"). As students become more skilled, "the it" changes. Each side of the distinction arises from the other, each side is dynamic; in Teacher A's classroom, the meta-level, which bridges metacognitive knowledge and skills, is in the concept of "mathematicians". Students are, at the same time, placed as already "being mathematicians" (in the way Teacher A draws out aspects of mathematical practice from actions she observes in the classroom) and as still being in a process of "becoming mathematicians" (by developing the kinds of skills and awarenesses she highlights). This analysis points to an alternative way of viewing metacognition that does not emphasise the knowledge/skill dichotomy.

Final remarks

To summarise the perspective offered in this article, I suggest that metacognition can be viewed as a (meta)category that encompasses both knowledge and skills and need not be reduced to those components, or their combination. From the perspective of planning to teach metacognition, it would be an error of logic to see metacognition as defined by any list of knowledge and skills. Metacognition is a meta-category, or *a category of categories of behaviour* and, as such, dilemmas arise if it is *reduced* to categories of behaviours. This view, which arose from the analysis of one mathematics classroom, suggests a resolution to the teaching dilemma that follows from seeing metacognition as split between knowledge and skills. Metacognitive learning need not be seen as an inner event that is either an item of knowledge or skill, declarative or procedural, knowing-that or know-how. Rather, metacognition can be seen as a *way* of teaching and learning in the classroom, which encompasses *both* knowledge and skill through a focus on a meta-level that transcends the distinction (in this instance, what "mathematicians" do). There need be no dilemma, as a teacher, about choosing to develop either metacognitive skill or knowledge if attention is placed, instead, on offering students an explicit meta-level language to describe and support their learning. In Teacher A's practice, there is evidence of how a manageably small list of words can, over the course of a year, accrue a wide complexity of associated actions (skills) and awarenesses (knowledge), within a context of each student "becoming a mathematician". In the classroom data I have presented, the teacher focus on metacognitive aspects of working mathematically occasioned a transformation in classroom discourse and practice.

Notes

[1] The data was collected in 2007-8 as part of a Studentship funded by the UK's Economic and Social Research Council.

[2] Transcription notation: *italics* indicates emphasis; (.) indicates a pause of less than 1 second; (2) indicates a pause of 2 seconds; [...] indicates some words missed out to ease reading.

References

Bateson, G. (2002) *Mind and Nature: A Necessary Unity.* Cresskill, NJ: Hampton Press Inc. (Original work published in 1979).

Brown, A. (1987) Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In Weinert, F. & Kluwe, R. (Eds.) *Metacognition, Motivation, and Understanding,* pp. 65-116. Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Brown, L. & Coles, A. (2011) Developing expertise: how enactivism reframes mathematics teacher development. *ZDM* **43**(6-7), 861-873.

Clement, J. & Konold, C. (1989) Fostering basic problem-solving skills in mathematics. *For the Learning of Mathematics* **9**(3), 26-30.

Depaepe, F., De Corte, E. & Verschaffel, L. (2010) Teachers' metacognitive and heuristic approaches to word problem solving: analysis and impact on students' beliefs and performance. *ZDM* **42**(2), 205-218.

Freudenthal, H. (1981) Major problems of mathematics education. *Educational Studies in Mathematics* **12**(2), 133-150.

Garofalo, J. & Lester, F. (1985) Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education* **16**(3), 163-176.

Hampden-Turner, C. (1981) *Maps of the Mind.* London, UK: Mitchell Beazley Publishers.

Levinson, S. (1979) Activity types and language. *Linguistics* **17**(5-6), 365-399.

Maturana, H. & Varela, F. (1987) *The Tree of Knowledge: The Biological Roots of Human Understanding.* Boston, MA: Shambala.

Mercer, N. (1995) *The Guided Construction of Knowledge: Talk Amongst Teachers and Learners.* Clevedon, UK: Multilingual Matters.

Mevarech, Z. & Fridkin, S. (2006) The effects of IMPROVE on mathematical knowledge, mathematical reasoning and meta-cognition. *Metacognition and Learning* **1**(1), 85-97.

Mevarech, Z. & Kramarski, B. (1997) Improve: a multidimensional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal* **34**(2), 365-394.

Moseley, D., Elliott, J., Gregson, M. & Higgins, S. (2005) Thinking skills frameworks for use in education and training. *British Educational Research Journal* **31**(3), 367-390.

Polya, G. (1957) *How to Solve It: A New Aspect of Mathematical Method* (2nd edition). Princeton, NJ: Princeton University Press.

Reid, D. (1996) Enactivism as a methodology. In Puig, L. & Gutierrez, A. (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education,* vol. 4, pp. 203-209. Valencia, Spain: PME.

Reid, D. & Brown, L. (1999) Fork in the Road . *For the Learning of Mathematics* **19**(3), 15-22.

Schoenfeld, A. (1983) The wild, wild, wild, wild, wild world of problem solving: a review of sorts. *For the Learning of Mathematics* **3**(3), 40-47.

Schoenfeld, A. (1987) What's all the fuss about metacognition. In Schoenfeld, A. (Ed.) *Cognitive Science and Mathematics Education,* pp. 189–215. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. (1992) Learning to think mathematically: problem solving, metacognition and sense making in mathematics. In Grouws, D. (Ed.) *Handbook of Research on Mathematics Teaching and Learning,* pp. 334-370. New York, NY: MacMillan.

Sinclair, J. & Coulthard, M. (1975) *Towards an Analysis of Discourse: The English Used by Teachers and Pupils.* Oxford, UK: Oxford University Press.

Schneider, W. & Artlet, C. (2010) Metacognition and mathematics education. *ZDM* **42**(2), 149-161.

Van der Stel, M., Veenman, M., Deelen, K. & Haenen, J. (2010) The increasing role of metacognitive skills in math: a cross-sectional study from a developmental perspective. *ZDM* **42**(2), 219-229.

Varela, F. (1976) Not one, not two. *CoEvolution Quarterly* **11**, 62-67.

Varela, F., Thompson, E. & Rosch, E. (1991) *The Embodied Mind: Cognitive Science and Human Experience.* Cambridge, MA: The MIT Press.

Veenman, M., Hout-Wolters, B. & Afflerbach, P. (2006) Metacognition and learning: conceptual and methodological considerations. *Metacognition and Learning* **1**(1), 3-14.

Whitebread, D. & Coltman, P. (2010) Aspects of pedagogy supporting metacognition and self-regulation in mathematical learning of young children: evidence from an observational study. *ZDM* **42**(2), 163-178.

Yimer, A. & Ellerton, N. (2010) A five-phase model for mathematical problem solving: identifying synergies in pre-service-teachers' metacognitive and cognitive actions. *ZDM* **42**(2), 245-261.

Zack, V. & Reid, D. (2003) Good-enough understanding: theorizing about the learning of complex ideas (part 1). *For the Learning of Mathematics* **23**(3), 43-50.