SONG ABOUT TWO GOOD PEOPLE: LETTERS TO DAVID

SEAN CHORNEY

Dear David,

When learning how to play an instrument, it is not that uncommon to begin by learning scales. Musical scales are easy to teach, easy to learn, and their progress can be easily measured. But when scales become the focus, thinking about which notes are in the scale and which are not, for example, the music can be overlooked. Learning and mastering scales can lead to a quagmire of notes without any guarantee of getting closer to music. This problem has an uncanny connection to mathematics, which I will come back to later.

I remember well discussing with you whether someone is actually *doing* mathematics when it looks like they are. You said that it depends on what the doer is thinking, which called into question my assumption that carpenters, computer programmers, or economists are doing mathematics whether they like it or not. The familiar refrain that mathematics is all around us is misleading. Mathematics is somewhere only when it is beckoned. It is only there if you want it to be there. That description was the crux of what mathematics meant to you, a relationship between a person and their way of seeing. Your ideas have always been challenging and thought-provoking.

I remember sitting in your class as a student. You were commenting on the relationship between content and form. You suggested that the meaning of a text was influenced in a significant way by *how* it was presented, such as in the usage of modality, hedging, or deixis. You suggested that form has at least as much weight as content. Form is a framing of content; if the frame is weak, the content is compromised. If we think of scales as the form, the music as content is lost. Back then, I disagreed with you. Today I agree with you. Our conversations have always been one-sided like that, haven't they?

I am writing this letter to thank you for your commitment, patience, and insight. You have supported many of us through your editing, feedback, and ideas. You have always been committed to good teaching and good thinking.

In the "Editorial" of *For the Learning of Mathematics* in November 1997, a section of the Bertolt Brecht poem "Song About the Good People" was used as a dedication to David Wheeler. One of the lines of that poem is "One knows the good people by the fact / that they get better/ when one knows them." I think this is true for many who know you; it is surely true for me.

Thank you, David!
Sincerely,
Sean

Reflections

During the spring term of 1996 at Simon Fraser University, David Wheeler, the founding editor of FLM, was my instructor for a master's course, Educ 846, Foundations of Mathematics Education, in the secondary mathematics education program. I had been teaching high school mathematics for six years and had started the master's program a year and a half earlier.

During the course, we submitted weekly reflections. The instructions on the syllabus were:

> One page of informal comment, written after the conclusion of each class, and handed in at the following meeting. These pages could become the basis of a "course diary," but you are not required to write them with that purpose in mind. Any aspect of the class may be commented upon (Length: "one page" is 300 words minimum).

What motivated me to write this article is David's feedback on my reflections. I still have all twelve of these reflections with David's handwritten responses, over half of which exceed 100 words. His responses were full of insight and their length revealed the commitment of a renowned educator to a young teacher. I believe my commitment to reassess my teaching practice throughout my 20-year high school teacher career was largely inspired by David Wheeler. I still use some of the problems he gave in that class.

Just a few months before I took Educ 846 with David Wheeler, he passed the editorship of FLM to David Pimm. Twenty-four years later, I am an assistant professor at SFU, home of the David Wheeler Institute, and David Pimm is my colleague. This paper draws together reflections on my journey as a teacher, my reactions to the feedback of David Wheeler and what I learned from it, and my interactions with David Pimm. I write this more from the perspective of a mathematics teacher than that of a researcher, partly because my reflections were written when I was a young teacher and David was responding to the teacher that I was. Over twenty years later, I feel I have the credibility to speak of a mathematics teacher's experience.

This article is an homage to David Wheeler, written in the form of letters in response to four of his feedback excerpts. [1] The feedback covers a range of topics that include calculator use (a more pressing issue in 1996), what it means to do mathematics, and what it means to teach mathematics, with all its challenges. The balance of pedagogy and philosophy in David's responses are epitomized by the statements, "Some students will need nudging some of the time";

"Maybe we could show [students] how to use test papers as important sources of feedback"; and "It would be fine if every student knew but…".

David's responses to me reflect an educator who cared. His comments were not only insightful and challenging, but respectful and humble. His pragmatic and progressive ideas still resonate with me through my practice and in my continued contemplation. I have been lucky in my mathematics education. I have had some great mentors and educators, two of whom are named David. One was my teacher 24 years ago, while the other is my colleague, teacher, and friend. In what follows, I will start with a brief comment on what I wrote in each assignment, then present a copy of David's response to what I wrote. My letters to David then follow.

Mathematizing (Letter 1)

In my weekly write up of 7 February 1996, I was contemplating whether professional athletes use mathematics when they engage in their sport. I suggested that an outfielder in the game of baseball calculates the parabolic trajectory of a ball they are about to catch. David responded:

> The athlete "doing math without knowing it"—if you say this then I think you have to say the hawk swooping on its prey and the horse jumping the hedge are also "doing math". For me, this analogy shows the idea isn't very useful.

Dear David,

There is a story of Agnes Martin, the twentieth century abstract painter who, in conversation with a young girl who had just picked a rose, asked her if the rose was beautiful. The girl said "Yes". Agnes took the rose from the girl's hand and held it behind her back and asked the girl if the rose was still beautiful and the girl said "Yes". "You see", Agnes reportedly said, "Beauty is in your mind, not in the rose". Similarly, mathematics emanates from the mind and moves towards its subject (or object). I believe you call this directional enactment, "mathematizing". David Pimm (1995) refers to this directionality when he suggests it is important to teach students to "read a sum *into* the text, rather than extract one *from* it" (p. 158).

This directional aspect to mathematics has a humanizing effect on mathematical practice. Mathematics emerges from us, as people, trying to understand the world. In my reflections submitted to you, I took the position that an athlete was doing mathematics when they engaged in their sport. The basketball player, for example, calculates the trajectory of the ball as they throw the ball toward the hoop. In your response you indicated this was a questionable view by saying that if the athlete is calculating, then the "hawk swooping" or the "horse jumping" must also be doing mathematics. You could have gone farther and said that a piece of wood falling to the ground would be engaging in mathematics too. The carpenter laying diagonal boards across the base of a shed to make sure the base is square could simply be using a technique that was passed down from their mentor and have no direct connection to the Pythagorean relationship that states two diagonals of a square will be equal in length. I've had conversations with David Pimm, who argues that maybe the craft being practiced is simply a skill passed on from one carpenter to another.

However, in FLM **21**(2), you also point out that maybe the carpenter *is* doing mathematics if they are wondering why their technique works, or if they are contemplating the Pythagorean relationship when they lay down the boards (Wheeler, 2001). In fact, you suggest that mathematization could be "an adult noticing a building under construction and asking himself questions about the design" (p. 51). Is the act of noticing doing mathematics?

One of the examples you draw on in your writing is the story of Thales comparing his shadow with the shadow of a pyramid in order to calculate the height of the pyramid. Thales was not studying mathematics based on theorems; he was contemplating, questioning, and reasoning with the notion of scale. You write that mathematics is the name for the paths "transported from one realm to another" (Wheeler, 1993, p. 53); and in Thales' case, there is a shift from the realm of shadow to the realm of measurement. Freudenthal (1973) encouraged people to mathematize and thought it important to re-establish the starting point from which mathematical activity begins. Instead of students working from what others have discovered, mathematizing asks the student to organize information, create a particular pattern, and reason with a reduced set of results. So, one of the questions I posed to my students in a mathematics history course last term was "What was the first mathematical thought?" It is a nice question because it strips away all the discoveries, tools, and conventions we rely on and simply asks the question, "Given nothing, what is mathematical thinking?" This question aligns with the idea of starting not with theorems or discoveries but with an opportunity for the "transportation". Mathematics is a "ruse" created to see a situation in a specific way. You write that mathematization is the "the act of putting a structure onto a structure" (Wheeler, 2001, p. 51). Each student can according their ability work toward a way of understanding the situation at hand. Burkhardt (1981) describes the need for a student to have an active attitude, one that includes asking questions, choosing between approaches, and trying to answer the questions. This view draws attention to mathematics as being as much about the active-person process as about the actual mathematics. The mathematics of students may not be the same as the textbook, but it is a power they possess. You write that "everyone is entitled to be shown that he has the ability to mathematise" (Wheeler, 2001, p. 53). The shift for the teacher then and now is to provide opportunities for processing situations as well as for mastering already established content.

The challenge for a teacher is to determine whether someone is doing mathematics. Pimm (1995) writes, "There are serious difficulties in ascertaining when someone, who appears to be engaged in other activities, is "actually" doing mathematics" (p. 157). Teachers desire to know how to evaluate students' mathematical activity. If mathematics depends on what the mind is doing during an activity, it becomes a challenge to assess. Pais (2013) asks "How we can say what certain people are doing is mathematics, if they do not recognize it as mathematics" (p. 3). While teachers want to see mathematical practice, it is hard to see it without having access to the student's mind.

Kind regards,
Sean

Application of mathematics (Letter 2)

The new curriculum introduced in British Columbia in 1995 included a shift towards applications of mathematics. Advocates argued that it would prepare students for the workforce. Here are parts of David's responses to my notes of 27 March 1996 and 24 January 1996:

> This argument goes down well with me, as you would guess, but it's a difficult matter to see how one can successfully challenge the current trend. The "train for the workforce" goal seems admirably hardheaded and realistic—and of course it has always been well-represented in the past as the major purpose of mass education.

> Isn't there a good deal of anxiety concerning the jobs that will be available for graduating students? When jobs are plentiful (*e.g.,* in wartime, ironically enough) it is easier to get a hearing for the less immediate purposes of education. When jobs are scarce, who can wonder that there is a greater emphasis in schooling as job-preparation? The tactical problem is how to accept the emphasis while still offering the broader view.

Dear David,

One of the rationales for giving mathematical questions based on real life is that students will be motivated by the thought that what they are doing is potentially useful. However, as Confrey *et al.* (2010) note, students may not always get motivated: "Students are typically told that they must study mathematics in order to keep their options open to pursue quantitatively-oriented careers in mathematics, science, technology, or engineering. For most of them, this is a very distant and abstract motivation" (p. 19). But how do we teach skills for jobs that do not exist yet? I don't think either of us would have guessed what kind of jobs there are in 2020. With the new curriculum, there was a sense of relief from many that mathematics would finally get real. The debate between applied and pure has been around for a long time. Xenophan believed that mathematics need not be studied for reasons beyond what is useful for civic life: "enough geometry to measure the land, enough astronomy to choose the right season for a journey. Anything more complicated, he says, is a waste of time" (in Burnyeat, 2000, p. 4).

A problem with bringing "real" applications to students is that they often end up being presented with overly simplified problems. Applied problems in mathematics textbooks are overly contrived and typically have less to do with an application than with eliciting a procedure (Chorney, 1997). Teaching mathematics through applications also often perpetuates questionable norms. In our class of 27 March 1996, one of the students suggested that applied mathematics trains students to retain the economic status quo. The curriculum implicitly perpetuates attitudes and societal values through the kinds of contexts and questions asked. What are students being prepared for?

For "critical mathematics" scholars such as Skovsmose (2000), mathematics applications should always be scrutinized. Although mathematics might seem apolitical, questions in mathematics textbooks often focus on making a profit. In my third year of teaching high school, I was teaching a quadratic optimization problem in a grade 11 (age 16–17) mathematics class which asked a question about maximizing revenue in a theatre. After solving the problem, a student in the back asked, "What if our goal was to fill the theatre?" At the time the question surprised me. This was a valid question, but my goal was to present a situation where we needed to "complete the square" to solve a problem—societal values were extraneous. The critical mathematics educator would argue that there is no empty context. A good way to address this is to simply follow more of David Pimm's (1995) advice, "Whenever that term [applied mathematics] is used, we should ask ourselves not only: what is being applied to what?, but also both how is it to be applied? And what is the means of application?" (p. 148).

In your remaining feedback from 27 March 1996, you addressed the question of value:

> One of the few opportunities to shift the ground of this position, I think, is to try and tackle it in terms it appreciates, that of how we as a society can create more "wealth" in the future. We could—and should— argue that wealth is not only measured in money, a high standard of living, and so on, but also in "civilization", the values and ideals of the society. Wealth isn't just created by turning raw materials into manufactured goods (though that's important), but it's also created when thoughts are turned into ideas and into theories, and when sensations are turned into works of art, and so on.

It may seem that mathematics aligns better with some things than others. This might be why finances often appear in mathematics examples. In fact, in British Columbia, there is a focus on financial literacy in the mathematics curriculum from grade one to grade twelve. Financial mathematics seems like a natural fit because of topics such as counting and monetary value. But this fit has ethical implications, epitomized by the question asked by my student. What kinds of value should we appreciate? You challenge the common themes used to describe wealth by drawing on non-conventional values, such as those of works of art. This shift in perspective again highlights your non-standard thinking. It also sets in motion thinking about what perspectives we have in mathematics and what others we should consider. Content is important, but so is what mathematics is used for, how it is practiced, and when it is called upon.

Traditions are hard to change. Mathematics is still seen by some as calculation, efficiency, proof, and brevity. Applications in mathematics can muddy the waters; they can complexify mathematics. I adopt David Pimm's (1995) view that draws us back to thinking about mathematics purely rather than implementing it in the world. He writes: "But for some of us at least who are involved in mathematics education, our greater concern is with students encountering and engaging with mathematics than solely with their successful functioning in the outside world" (p. 134). I will apply this to my practice.

Best,
Sean

Digital technology (Letter 3)

From 28 February 1996:

> Although it's a strong point that the calculator replaces several earlier "mechanical aids", it has a quality the other aides don't have—it seems to respond. You do something, the calculator does something "in reply". You do something else, it responds again. It doesn't behave like a passive dictionary, say, in which you "look something up": it's more dynamic than that. You get *almost* a sense of interaction with a calculator—a sense that becomes much stronger still with a computer. There are pedagogical ("psychological") dimensions to this that aren't yet explored.

Dear David,

One of the many challenges teachers deal with is that their students are often uninterested in the mathematics they are studying. This is related to the point in my earlier letter when I referred to Freudenthal (1973), who believes students should not start with what has already been discovered. When mathematics is presented as a finished product, there are few opportunities for experimenting or developing ideas. Students in this situation have no voice, which contributes to apathy in the classroom. Mathematical objects are presented *to* them while the objects themselves are inaccessible to scrutiny.

As you indicate in your feedback, however, digital technology introduces new dimensions of interactivity. Students can interact with mathematical objects directly. For example, in the past a teacher would draw a square on the board and that square would represent all squares. But in a dynamic geometric environment, a properly constructed square can be dragged by its vertex such that the outcome is a visual display of unlimited examples. The squares vary in size and orientation, but some features of the square remain invariant. It is the invariant features that make a square a square. What was before presented as the object's properties are now invariances that come into existence through movement. Pimm (1995) writes: "If you cannot vary something, it is hard to get a sense of the possibilities" (p. 46). Digital technology provides the possibility of scrutinizing mathematical objects through interaction, allowing students to get a sense of their generality.

In this way, digital technology reconfigures mathematics as a quasi-empirical science, where experiments are conducted. This shifts the knowledge from one previously enacted in authority to one accessible through testing conjectures:

> The result is the opening up of mathematics by a powerful new Agent in the direction of an empirical science; a development that not only goes against its time-honored status as a purely theoretical pursuit, but is a dramatic and radical reversal of the last two centuries' effort to eliminate all traces of the physical world from mathematics' definitions and methods in the name of an abstract program of mathematical "rigor". (Rotman, 2008, pp. 76–77)

Students can test, make conjectures, and explore the generality of their conjectures by playing with a particular.

This interplay between a particular and the general is exemplified in Euclid's proof of the infinitude of primes in Book 9, proposition 20. Euclid's proof is exceptional because it demands from the reader that they see the general through the particular; Euclid's proof begins by asking the reader to consider three primes, A, B and C. In Euclid's *Elements*, a prime number was a measure of a magnitude and would not be thought of as a variable since algebra had not been developed at that point in time in history. So the three primes asked for from the reader would have been most likely thought of as materially instantiated particulars. In addition, the request to consider a finite number, in this case, three, also moves away from the infinite. The proof goes on to show that there is another prime, G, that can be found. That's the end of Euclid's proof. There is no final statement of how to generalize with this newfound prime. One could generalize by noting that the number of primes is arbitrary and no matter what finite set of primes were listed one could always find another. Or, that if one were to repeat the proof with a different set of three primes, a new prime could always be found.

Euclid's proof is unique in that it not only conveys a result, but also provides space for a reader to think through the relationship between the particular and the general. Contrast this with more contemporary proofs which in my experience begin with a general statement and continue with the general through the complete proof. For example, in the text, *The heart of mathematics: An invitation to effective thinking* (Burger *et al.,* 2013), the first line of the infinitude of primes proof states: "Let M represent an arbitrary natural number". Both in the reference to M as a "representation" and the use of the word "arbitrary", there is a two-fold appeal to the general.

Logo, a computer code you were quite aware of, uses "variable", as both a number and a variable, a particular and a generality (Rotman, 1987; Pimm, 1995). The letters A, B, and C in Euclid's proof and ":variable" in Logo play a dual role. They play the part of both the general and the particular, giving students or computer scientists the freedom to move back and forth between them according to their needs at the moment. There is something profound in mathematics that provides the option of thinking generally and particularly. By providing this freedom, there is a sense that the reader will be able to establish their own pathway through the proof.

I wonder whether the unexplored psychological dimension you mention in your feedback has been partially addressed in the interplay of student voice and the scrutinizing of a mathematical particular that leads to an understanding of a general case. It is a powerful connection between voice and mathematical structure.

Virtually,

Sean

Practice (Letter 4)

In my weekly response of 20 March 1996, I posed the question, "Are mathematical skills learned by practising them a lot?" I suggested that practice was of limited help, because repetition is so unappealing to students.

I doubt if we will ever solve the problem of the "over-stuffed" curriculum. Especially when there is an element of competition which often rewards students who "know more".

I'd like to accept your argument about practice, but I'm not sure I can. It would be fine if every student knew what it was in their best interest to master at each point. Now, while I think that's a better assumption than to assume they don't know their best interest at all, I find it a bit too optimistic. Some students will need nudging some of the time.

Dear David,

I am intrigued by your comments about practice. I come from a tradition of practice. In more than 20 years of teaching, I must have assigned over 2000 problems from the textbook. This is the tradition in mathematics classrooms, particularly at the secondary level. It is an odd tradition, but teachers often do it to keep the students occupied during the later stages of an 80-minute class. In looking back, I am not sure if what I did was all that effective. While practice may be worthwhile, did I really do enough to make sure that the practice students were doing was helpful? Regarding practice, David Pimm (1995) asks, "what it is that is being done again?" (p. 177). A lot of high school mathematics seems to involve manipulating symbols. Once practice becomes manipulating symbols, practice becomes repetition. Once practice becomes repetition, mathematics becomes algorithmic. That is, the mechanics of the practice take over and the focus is on fluency with symbols rather than thinking about the objects behind the symbols. Pimm (1987), however, points out "the danger of the symbol being interpreted as the object itself" (p. 132). In the case of manipulating symbols, we might ask "Where is the math?" Is it in the symbol manipulation or in the thinking about the objects those symbols represent? Hewitt (in Pimm, 1987) states: "Algebra is not what we write on paper, but is something that goes on inside us. So, as a teacher, I must realize that notation is only a way of representing algebra, not algebra itself" (p. 136). If we take Hewitt's point, spending too much time on symbolic manipulation does not seem to hold much value.

Sfard (2017) holds a middle ground, proposing that mimicking and practicing repetition without thinking is an important first step, and that this "ritualized learning", as she calls it, provides a foundation for "explorative learning", where the true thinking about mathematics occurs. She writes:

> Rather than presenting mathematics as a bunch of rituals to be practised with the sole aim of adhering to some inexplicable social norms, I would like my students to be masters of their mathematical activity, that is, to be able to decide when and how to use mathematics for their own needs. (p. 125)

Practice can often be about manipulating symbols, expanding a product of two binomials using the first, outside, inside, last algorithm or using SOH CAH TOA for trigonometry questions; but at some point the objects for which these symbols stand need to be brought forth. But of course, we are back to the same problem. How much ritualized practice is too little, or too much?

You suggest in your feedback that if students do not practice, they may not get a sense of the possibilities. "Some students will need nudging some of the time" suggests that practice is also a way of guiding the student to home in on something of value. It may be necessary to make students aware of what they practice so they do not resort to mechanical behaviour. Your approach makes sense: if they don't have a sense of what they are doing, how can they be inspired to move forward? But how can the shift to a more thoughtful intentional practice be made? How can we get students to practice with awareness?

This challenge brings us back to learning scales as a way to understand music. When practicing becomes about mastering scales, the scale starts to determine the way to play. Because there is a right way to play a scale, playing that scale correctly becomes the focus. Non-cliché music, however, does not follow rules. The musical expression of an artist breaks rules all the time; arguably, that's what makes it music. In mathematics, the linear progression of the rules is too often what we think of as mathematics. This is like when the musical scale is telling you what to play. Are you playing the scale or is the scale playing you?

The mathematizing that I have addressed throughout these letters in many ways points to a deeper sense of what it means to do mathematics. There seems to be an essence of mathematics. The essence is not knowledge but a relationship we have with the world. The notion of mathematizing helps me differentiate mathematical scales from mathematics.

Still practicing,
Sean

Final words

I still feel like I am David's student. As I read his feedback, I think back to when he was my instructor and I am left with the feeling that in 1996, I was not fully aware of the significant figure before me, one who loved mathematics and believed "everyone is entitled to be shown that he has the ability to mathematise" (Wheeler, 2001, p. 53). I called him on the phone once outside of class hours, to ask him a question about a particular problem he posed to us in class. There is nothing profound to report other than he answered my query. As Brecht (1976) writes in his poem "if one comes to them, they are there".

I am drawn back to the lines of Brecht's poem that I cited at the beginning of this article and that were dedicated to David Wheeler in David Pimm's editorial from 1997:

> One knows the good people by the fact
> That they get better
> When one knows them. (p. 337)

I appreciate the playfulness of Brecht in his use of "they" and "them". It is unclear who the good person is. In my experience with David Wheeler and in fact, David Pimm, I wonder who could be afforded the qualifier "good". My first letter of this article is addressed to David but I meant it for David Pimm. They have both inspired me to rethink what it means to do and teach mathematics. The title of this article widens the scope of David Pimm's first reference of the

This is fine.

You're right, of course, that we all benefit from the security of knowing that no society questions the importance of math for its technology and its culture; but it may be an instructive "thought experiment" to imagine how we would defend our position if we didn't have that security.

Figure 1.   One of David's first comments to me dated 17 January 1996

poem as a dedication to David Wheeler. I expand the dedication, 24 years later, to thank both Davids for their goodness.

The architect Juhani Pallasmaa (2009) writes, "Beauty is not a detached aesthetic quality; the experience of beauty arises from grasping *the un*questionable casualties and interdependencies of life" (p. 12). This quote epitomizes what I believe a mathematics education should be about: a human practice that arises from the interdependencies of life. I finish this piece with David Wheeler getting the last word. Mathematics is held in high regard in our culture. In my first submission on 17 January 1996 to David, I highlighted that mathematics is a mainstay in the curriculum. In his response, David posed a challenge to me, which I here pass on to the reader. How would you define teaching "for the learning of mathematics"? See Figure 1.

Note

[1] To simplify the layout of this article, David's handwritten responses have been typed verbatim; the final image, in Figure 1, shows one of David's responses in its original handwriting and form.

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How are we implicated in making ourselves (and others) unaware of certain connections?
— David Pimm, from p. 122 of 'Another psychology of mathematics education', in P. Ernest (Ed.), (1994) *Constructing Mathematical Knowledge: Epistemology and Mathematics Education.* The Falmer Press.