SPEAKING MATHEMATICALLY: FRAMING PRACTICES, HEARING REGISTERS, LISTENING FOR SILENCE

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Over the years, I have learned about David Pimm's interests outside mathematics and this taught me to share my own "outside" interests. One of these, which I share with David, is poetry—in my case, primarily poetry in Spanish, my first language. I grew up memorizing poems by famous poets like Ruben Dario, Alfonsina Storni, and Garcia Lorca. As a young adult, I started reading contemporary poetry in Spanish; one of my favorites is Mario Benedetti. As an example of speaking my mind through poetry that counts, I share excerpts from two of his poems, both included in Benedetti (1978). This is from *Hagamos un trato* (Let us make a deal):

Compañera
usted sabe
que puede contar
conmigo
no hasta dos
o hasta diez
sino contar
conmigo

Compañera,
you know
that you can count
on me
not up to two
or up to ten
but count
on me

(my translation)

This poem reflects who I am in multiple ways. I seem to be obsessed with mathematics, although what I find most interesting about counting is the number zero or the question "Where do we start counting?" Only in writing this article did I realize that the poem also reveals my obsession with language—with the meanings that language can both convey and obscure.

In this poem, "counting on" is used in a polysemous manner, to refer to counting with numbers and to "counting on" as depending on someone. One can count using numbers and one can "count on" one's fingers, something that David has referred to in his work. One can also "count on" or depend on someone else (and yes, "counting on" is also a phrase for an invented strategy that children use). But there is yet another kind of meaning that this poem exemplifies for me. Poetry is incredibly difficult to translate from one language to another. These particular lines are rare in that I was able to easily translate them and describe the multiple meanings, assuming that many readers will have counted with numbers, or counted on fingers, or counted on someone. This makes me especially happy, since I can then share this favorite poem's polysemic twist across my two language communities.

The excerpt from the second poem I have chosen, *Te quiero* (I love you), is also about love [1]. It is much harder to translate due to the social context and the practices it draws on.

Si te quiero es porque sos
mi amor mi cómplice y todo
y en la calle codo a codo
somos mucho más que dos

If I love you it is because you are
my love my accomplice and everything
and on the streets, elbow to elbow
we are much more than two

(my translation)

This poem has a broader social justice flavor. The point of the phrase "elbow to elbow" is that, for many of us from Argentina, it conjures the image of the social and cultural practice of marching in the streets together for political demonstrations (which people seem to do extremely well and often in Argentina). The double meaning here is that when we march, we are more than two, because we belong to a collective or a social movement. This, then, is one context (or figured world, Holland *et al.,* 2001) in which one plus one is (or can be) more than two. Here, the polysemy depends on shared socio-cultural practices.

This introduction raises several themes that I explore in this article: language, two languages, polysemy, and practices. I will use these themes to describe how David's work has influenced my own and the theoretical trajectory I see as I look back. The fact that I felt comfortable including this personal introduction reflects the kind of community that David has helped to create: welcoming and inclusive of so many of the multiple languages, meanings, practices, figured worlds (and why not registers), that constitute who I am. I do not exist in Spanish or in English, but in both, and I have found both celebrated in the community of scholars David brings together. I love and appreciate not only mathematics (and science), but also language and poetry, and David's work certainly celebrates all of them.

A more academic introduction

In David Pimm's 1987 book *Speaking Mathematically*, he described two issues that have continued to arise in discussions about language and mathematics in research settings. One is what exactly the mathematics register is; the other is whether or not mathematics is a language like other languages.

One question I hear often is "Isn't speaking math principally about vocabulary?" I went back to read the book (it was a bit dusty and moldy; I was not allergic in 1987 but I am now). And found this quote:

> Thus it is not just the use of technical terms, which can sound like jargon to the non-speaker, but also certain phrases and even characteristic modes of arguing that constitute a register. (Pimm, 1987, p. 76)

The example of the use of "if, then" in informal vernacular register and formal mathematics register is used to illustrated this idea.

The other repeated question I hear is whether and how one language is better or worse than another language for speaking mathematically. This comes up especially when people notice that in one language there may be one single word to express an idea and claim that word does not exist in another language. Once again, I found this relevant quote:

> In Russian there is no article marker for distinguishing *a* from *the*, the definite from the indefinite. Yet this language contains a sophisticated mathematics register fully capable of distinguishing the meaning "there exists" from "there exists unique". It is very suspect, I feel, to go from observations about the marked surface features of particular dialects or languages to conclusions about the conceptual thought that can be carried by them. (p. 81)

Over 30 years later, although mathematics education research now includes books, articles, conferences, and working groups focused on language, many issues and concepts related to speaking mathematically still need attention. In this article, I address two issues regarding the concept of mathematical language: how we imagine the mathematics register and how participation in mathematical practices can be silent.

Why should mathematics education researchers take time to carefully consider the concept of mathematical language? Although some research since 1987 has used the phrase mathematical language, there are many different definitions and uses of the concept. We need clear definitions, examples, and more research on this topic that avoid two pitfalls: separating mathematical practices from language and dichotomizing everyday and school ways of talking. Separating language (or thinking) from the practices in which language is embedded treats language as a static cognitive object. Connecting practices to language is essential for analyzing how meanings arise and change, because meanings are embedded in practices. For example, the meaning of being more than two in the second poem excerpt is embedded in the practice of marching in the streets (elbow to elbow). I have previously described how the borders between the everyday and school mathematics registers are not rigid but overlap (Moschkovich, 2010). Here I reiterate that we have evidence that both serve as resources for communicating mathematically, not only for learners but also for professional users of mathematics [2].

A careful consideration of the concept of mathematical language also has implications for practice. Developing theoretical clarity and examples grounded in classroom data will help to develop clarity in conversations with practitioners. The two pitfalls described above have serious implications for teaching. Separating language from practices, for example by pre-teaching vocabulary divorced from participation in mathematical practices, has implications for students learning the language of instruction. Separating the two registers has social justice implications, since this hierarchy creates a new deficit that some learners need to overcome—their everyday ways of talking.

A Vygotskian perspective on mathematical language and practices

My own exploration of mathematical discussions in learning mathematics started from a cognitive perspective that, early on in my career, did not fully address issues of language (Moschkovich, 1996). David's work, by documenting and analyzing how learners and teachers speak mathematically in one language in classrooms provided a foundation for my own research. As I developed my own research agenda and theoretical framing (Moschkovich, 2002), I used David's work to more deeply consider language, particularly classroom discourse, using the concept of register to analyze mathematical discussions among bilingual mathematics learners.

When I first started working as a doctoral student in 1986, I spent time trying to connect new research on mathematics cognition (Schoenfeld, 1992) to Vygotskian and neo-Vygotskian theories of learning (*i.e.,* Forman, 1996; Vygotsky, 1987). My goal was to reconcile my theoretical commitments to Vygotskian perspectives with the study of mathematical thinking. I initially struggled to answer several (fundamentally Vygotskian) questions, especially where one finds mediation by social-cultural artifacts. Today I see that, for me, the answer to this question lay in the concept of practices: clarifying practices, connecting practices to mathematical discourse, and using appropriation to describe how practices and language are acquired. A focus on practices differs fundamentally from a cognitive perspective, which treats cognition (thinking, mathematical reasoning, conceptual understanding, *etc.*) as individual and mental, and embedding thinking in social and cultural practices, which are collective, not individual (for a more detailed discussion of the concept of practices, see Moschkovich, 2013). In plainer words, we learn to speak mathematically by hanging out with others who not only speak mathematically but also invite us to participate in different mathematical activities (those valued by that particular mathematical community). Using Vygotskian theories and work found in sociolinguistics, I have spent the last 30 years analyzing discussions of mathematical problems among students or between a learner and an adult. My analyses have focused on identifying and describing central aspects of mathematical discourse practices.

Vygotskian perspectives, and in particular the concept of practices, can frame the study of mathematical language. What assumptions do we make about the nature, origin, development, and acquisition of mathematical practices? Are mathematical practices individual, collective, cognitive, discursive, social, and cultural phenomena? From a Vygotskian perspective, mathematical practices are socio-cultural phenomena in the sense that they are higher order intellectual activities and originate in social interaction. They are first constructed interpersonally and then appropriated to become part of the repertoire of practices that an individual will use. A Vygotskian theoretical framing can contribute to clarifying and articulating the concept of mathematical practices. First, it provides a practice perspective on cultural activity. Second, it provides a connection to discourse as a central aspect of practices. And lastly, it describes how practices are acquired through appropriation.

I began with a Vygotskian definition for a *practice* perspective. I use the terms *practice* and *practices* in the sense used by Scribner (1984) for a practice account of literacy to:

> highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems. (p. 13)

In using the terms *practices* in the sense used by Scribner, I make a distinction between the concept of *practices* and other common uses; for example, *practice* as repetition or rehearsal, or practice as in "my teaching practice". This definition requires that practices are culturally organized and involve technologies or symbols systems. From this perspective, mathematical practices are social and cultural, because they arise from communities and mark membership in communities. They are also cognitive, because they involve thinking and they are also semiotic, because they involve semiotic systems (signs, tools, and their meanings). Mathematical practices involve values, points of view, and implicit knowledge.

Many researchers who have used the concept of mathematical practices define them as the "taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas" (Cobb *et al.,* 2001, p. 126). In contrast to social norms and socio-mathematical norms, mathematical practices are specific to particular mathematical ideas. Cobb *et al.* make a distinction between two types of mathematical practices: those that are "normative ways of acting that have emerged during extended periods of history" and mathematical practices that are emergent in classroom activity. In my own work, I have framed mathematical practices as simultaneously emergent in current classroom activity and the result of socio-cultural historical activity [3].

Beyond the assumption that practices are social and cultural in origin, a Vygotskian perspective has several implications for the concept of mathematical practices:

- Social interaction that leads to learning principally involves joint activity (not just any type of interaction),
- Goals are an implicit yet fundamental aspect of practices,
- Discourse is central to participation in practices,
- The meanings for words are situated and constructed while participating in practices,
- Appropriation is a central metaphor for describing learning (but learners do not merely imitate practices, they sometimes actively transform them).

The central features of appropriation, as described by Rogoff (1990), are that appropriation involves achieving a shared focus of attention, developing shared meanings, and transforming what is appropriated. Rogoff suggests that intersubjectivity may be especially important for learning to participate in practices that are implicit.

I have used this Vygotskian perspective to frame the study of mathematical practices. We should think carefully about how we see the origin of the mathematical practices that mathematics educators value and, in particular, the role of social interaction in learning to participate in these practices. Interpretations and applications in teaching will depend on how we frame the concept of mathematical practices. Without a Vygotskian framing, we might assume that some mathematical practices are individual in the sense that we typically accomplish the goals for these activities alone (for example when we persevere in solving problems, reason abstractly and quantitatively, model with mathematics, or look for and make use of structure). In the same vein, we might think that critiquing others' reasoning is *really* social because critiquing the reasoning of others requires other people. However, from a Vygotskian perspective, all mathematical practices are socio-cultural phenomena in the sense that they are higher order intellectual activities that *originate* through social interaction. Children and adolescents learn to participate in mathematical practices first interpersonally and then come to appropriate the practices as these become part of the repertoire of practices that an individual will later use (either alone or in the company of others).

How does this theoretical framing matter for instruction? If we leave behind the assumption that mathematical practices are socio-cultural in origin and see these practices as purely individual and mental, we will continue to see some learners as deficient because they have not yet developed proficiency in these practices and others as talented because they have (supposedly) developed these practices all on their own. The question to ask is not whether a mathematical practice is always or primarily carried out through social interaction, but whether social interaction was involved in the origin of the practice, even if the practice is now accomplished alone. Another question that is relevant to instruction is what kind of social interaction is conducive to learning (appropriating) mathematical practices. Without a Vygotskian framing one might think that the social interaction necessary for student learning is telling students that these practices are important, or reminding students that they should engage in these practices often, or modelling these practices at the board. However, from a Vygotskian perspective, these examples of social interaction do not include an active learner or involve joint activity and, therefore, do not support learner appropriation of goals, focus of attention, or shared meanings for language.

How does this theoretical framing matter for language? In summary, mathematical talk is not disembodied, it is embedded in practices. Language, utterances, or meanings are not mathematical in themselves, they are embedded in mathematical practices. Although practices are a fundamental framing concept, I cannot write about language practices without invoking the concept of register.

Hearing registers

The concept of register has been with me ever since I first read *Speaking Mathematically.* While the mathematics register adds complexity to how language is conceptualized, this notion also presents challenges. The notion of register as proposed by Halliday (1978) is much more than a set of lexical items and includes also phonology, morphology, syntax, and semantics as well as non-linguistic behavior:

> A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. (p. 195)

Some examples of registers are legal talk and baby talk. In mathematics, since there are multiple meanings for the same term, students who are learning mathematics have been described as learning to use these multiple meanings appropriately. My favorite example of such multiple meanings is the phrase "any number", which means "all numbers" in a mathematics context (Pimm, 1987).

Most importantly the notion of register includes the situational context of utterances. Although words and phrases do have multiple meanings, these words and phrases appear in talk as utterances that occur within social contexts, and speakers use situational resources to derive the meaning of an utterance. For example, the phrase "give me a quarter", uttered at a vending machine clearly has a different meaning than saying "give me a quarter" while looking at a pizza. When imagining that students face difficulties with multiple meanings in mathematical conversations, it is important to consider how resources from the situation, such as objects and gestures, point to one or another sense, whether "quarter" means "a coin" or "a fourth".

A second challenge in using the notion of register is that, although it is easy to set up a dichotomy between the everyday and the mathematics registers, these two registers should not be treated as mutually exclusive. During mathematical discussions, students use multiple resources from their experiences across multiple settings, both in and out of school. Forman (1996) offers evidence of this in her description of how students interweave the everyday and academic registers in classroom discussions. Thus, everyday meanings should not be seen only as obstacles to participation in academic mathematical discussions. The origin of some mathematical meanings may be everyday experiences and some aspects of everyday experiences may actually provide resources in the mathematics classroom. For example, climbing hills is an experience that can be a resource for describing the steepness of lines (Moschkovich, 1996). Other everyday experiences with natural phenomena also may provide resources for communicating mathematically.

While differences between the everyday and mathematics registers may sometimes be obstacles for communicating in mathematically precise ways and everyday meanings can sometimes be ambiguous, everyday meanings and metaphors can also be resources for understanding mathematical concepts. Rather than emphasizing the limitations of the everyday register in comparison to the mathematics register, it is important to understand how the two registers serve different purposes and how everyday experiences, meanings, and registers can provide resources for mathematical communication and conceptual change (Moschkovich, 2010).

Researchers in mathematics education have examined many topics related to the mathematics register. One contribution that is especially relevant to word problems is a shift from seeing the mathematics register as merely technical mathematical language, as the following word problem illustrates:

> A boat in a river with a current of 3 mph can travel 16 miles downstream in the same amount of time it can go 10 miles upstream. Find the speed of the boat in still water.

The complexity involved in making sense of this word problem does not lie in the technical mathematical vocabulary, but in the background knowledge (Martiniello & Wolf, 2012) for understanding and imagining the context or situation. In this case, the reader needs to imagine and understand that there is a boat traveling up and down a river, that the speed was measured in still water (presumably a lake), and that the speed of the boat increases (by the speed of the current) when going downstream, and decreases (by the speed of the current) when going upstream.

A glossary for non-mathematical words such as upstream, downstream, and the phrase "in still water" would certainly help. However, much of the language complexity is not at the word level, but at the sentence and paragraph level, in the use of the passive voice without an agent and in the multiple subordinate clauses and nested constructions (Cook & MacDonald, 2013). In contrast, we can imagine a conversational way of describing the same word problem using the everyday register:

> This boat goes x miles/hour. That's the speed if the water isn't moving. Next, imagine that boat going up the river. The river has a current of 3 miles/hour. Going up the river, the boat goes against the current, so the boat's speed is $x - 3$ miles/hour. Now imagine the boat going down the river. When it goes down the river, it goes with the current. The current helps move it. That makes its speed going down the river $x + 3$. Imagine that the boat goes 16 miles downstream. Then, in the same amount of time, it travels 10 miles upstream. How can we find the speed in still water? How can we write the equation?

In this version, there are many short sentences, and each one contains just one idea. The sentences are connected together by the repetition of words from the end of one sentence at the beginning of the next sentence. To construct the meaning of this paragraph, a student could stop to understand each single idea, one at a time, and then later link all the ideas together.

For students learning mathematics, informal language and everyday registers are important, especially when students are exploring a mathematical concept or first learning a new concept, or discussing a mathematics problem in small groups. Informal language can be used by students (and teachers) during exploratory talk (Barnes, 1992, 2008) or when working in a small group communication context (Herbel-Eisenmann *et al.,* 2013). Such informal language can reflect important student mathematical thinking (for examples, see Moschkovich, 2008). In other situations, for example, when making a presentation, developing a written account of a solution, using more formal academic mathematical language becomes more important. Lastly, we cannot assume that informal language is a deficit or something to overcome as one becomes more proficient in mathematics, since informal language has been documented to be used by scientists in professional settings (Ochs, Gonzales & Jacoby, 1996).

Research and practice need to avoid dichotomies such as everyday/academic or formal/informal. Classroom discussions draw on hybrid resources from both academic and everyday registers. Most importantly for supporting the success of all students, mathematical discussions need to build on and link with the language practices students bring from their home communities. Therefore, everyday ways of talking (and gesturing) should not be seen as obstacles to participation in academic mathematical discussions, but as resources teachers can build on to support students in developing the more formal mathematical ways of talking as they learn mathematics.

Listening to silence(s)

Does school mathematics privilege speaking over observing? Participation in mathematical discussions has been framed primarily in terms of student talk. What about students who are silent during a mathematical discussion? It is especially important to consider how silence, and not only talk, can be an important form of participation for marginalized students. In some communities, language practices for children may be different than the norms of a classroom and may focus less on talk and more on silent, yet intent (in the sense of observation), participation. There are multiple reasons why a particular student may be silent during a mathematical discussion. Some of these may be more about what are typically labeled “individual” differences and can range from a quiet personality, to a bad day, to a personal predilection for thinking without talking. More complex reasons include avoiding speaking in groups or embarrassing situations. Marginalized students who are learning the language of instruction may be afraid to speak due to their different accent or to hesitancy to make mistakes in the language of instruction. It is not so important to know the reasons for a particular student’s silence. What is important for the purposes of research on supporting marginalized learners during mathematical discussions is to consider how silence might impact how we perceive a student’s participation in such discussions.

To illustrate how to consider silence during mathematical discussions, I revisited a short episode from a classroom discussion where one student was silent much of the time (see Moschkovich, 2008, 2015). The episode is from a classroom discussion between two students, Carlos and David, and a teacher, as they compared two graphs. In Moschkovich (2017), I used suggestions made by McDermott, Gospodinoff and Aron (1978) to consider not only talk but also other evidence of participation such as posture and gaze. I used the following questions: How were students active even when silent? Was there evidence of intent participation (as described by Rogoff *et al.,* 2003)? When I counted the total number of turns in the discussion (a total of 44, not counting gestures as separate turns), it was clear that David was quieter than Carlos since he uttered 16% of the turns, while Carlos uttered approximately 46% of the turns, and the teacher 37%. David actually stopped talking after line 61 (the whole transcript is about 90 lines). If we counted words, David contributed approximately 34 words by line 61 (depending on how words are counted), while Carlos uttered approximately 96 words by line 61. If we considered only talk, it would seem that David was not participating in this discussion after line 61. However, when I looked at posture and gaze, the story of David’s participation changed.

Although David spoke fewer words overall and took fewer turns than Carlos (both before and after line 61), he was still engaged and participating in the discussion but in silent observation. His participation can be seen, for example, when he stood up to look at the two graphs that the teacher had turned towards herself (line 74). After line 74, David continued to orient his body towards the graphs and while looking intently, although silently, at the two graphs after he stood up as the discussion proceeded between the teacher and Carlos (lines 74 to 87).

If we include posture and gaze as part of participation, then David can be described as still participating in the discussion even after he stopped contributing talk. The discussion between Carlos and the teacher (in lines 74 to 87) involved pointing to the graphs through multiple gestures (at least 5 gestures by the teacher), so there was certainly ongoing meaningful mathematical activity for David to observe by focusing on the graphs and the gestures.

It is crucial to include the possibility that some students may choose to speak less, for whatever reasons, and to consider how students participate besides contributing talk. Intent observation seems important for making sense of mathematical discussions (especially those that involve inscriptions), and may turn out to be the other (positive) side of some silences. If that is the case, those silences should not be interpreted as the absence of participation. The example illustrates how silence may be overlooked or undervalued as a way of participating in mathematical discussions.

Thus, my focus on practices and communication has expanded to include “para-language”, such as gestures, posture, gaze, and material tools, such as the graph. This expansion fits well with David's own recent interest in extending the mathematical register to include gestures (Pimm, 2020).

An academic closing: back to mathematical practices

So, how does the concept of register connect to mathematical practices? From a Vygostkian perspective, discourse is central to joint activity and the multiple meanings for utterances are an important component of any mathematical practice. In general, mathematical talk is not disembodied talk; it is embedded in practices. Most importantly for mathematical practices, language, utterances, or meanings are not mathematical in themselves but are embedded in mathematical practices. This is one connection between mathematical registers and practices. Moreover, mathematical practices involve not only meanings for utterances but also focus of attention. The notion of focus of attention also comes from a sociocultural framework that uses appropriation to describe learning. Mathematical practices are not simply about using a particular meaning for an utterance, but rather using language in the service of particular goals while coordinating the meaning of an utterance with a focus of attention. Thus, mathematical practices involve not only language but also perspectives and conceptual knowledge. Words, utterances, or texts have different meanings, functions, and goals depending on the practices in which they are embedded. Language occurs in the context of practices and practices are tied to communities.

My recommendations for research and practice are grounded not only in theoretical rigor and the deep analysis of data, but also from considering social justice issues in mathematics education. The ways that I suggest complicating mathematical language have important implications for equity in mathematics classrooms.

Research and teaching need to shift away from dichotomizing everyday and academic registers. Because classroom discourse is a hybrid of academic and everyday discourses, multiple registers co-exist in mathematics classrooms. Most importantly for supporting the success of all students in classrooms (not only the ones who are already perceived, heard, or framed as successful in mathematics), instruction needs to build on the everyday language and practices that students bring from their communities. Therefore, everyday practices should not be seen as obstacles to participation in academic mathematical practices, but as resources for engaging students in more formal mathematical practices. For example, the ambiguity and multiplicity of meanings in everyday language should be recognized and treated not as a failure to be mathematically precise but as fundamental to making sense of mathematical meanings and to learning mathematics with understanding.

Research and teaching also need to address student silences in multiple ways that are productive for as many learners as possible. Although speaking mathematically is important, engaged silence should be included in how we define and analyze participation in mathematical practices. More research is needed that explores whether and how silences can support mathematics learning. Concerns with a few talkative students monopolizing discussions can be informed by studies showing that engaged silence *is* participation that results in student learning (for example, see O'Connor *et al.,* 2016). If we use an expansive notion of register which includes not only talk but also posture, gaze, and focus of attention, then we will also expand our notions of what constitutes proficient or successful participation in mathematical practices.

A personal closing: back to poetry

In the introduction to this article, I raised several themes: language, two languages, polysemy, and practices. In the introduction, I used poetry to write about counting, translating, and to illustrate how polysemy is connected to social and cultural practices. Multiple meanings do not exist in a vacuum or create themselves in an individual mind and neither does mathematical language. Multiple meanings provide us with jokes and puns, require us to switch from foreground to background, and encourage us to see, hear, and experience multiple figured worlds (Holland *et al.,* 2001). Meanings are connected to the socio-cultural practices in which we use those meanings, not to the static definitions in a dictionary. By including excerpts from some of my favorite poems, I provided the reader with a window into some of the figured worlds that I inhabit, along with some (not all) of the multiple identities I enact in those worlds, and perhaps even revealed some of the values that underlie my theorizing, researching, writing (as well as my choices in poetry to share). Just and equitable mathematics education for me is concerned with much more than learning or teaching mathematics; it can offer learners doorways to multiple figured worlds that might otherwise be closed to them.

Notes

[1] Other themes in Latin American poetry are death or exile, and I thought love might be a better choice for this piece.

[2] It is unfortunate (and somewhat frustrating) that, in order to write about the need to not dichotomize, I first had to separate the two, but I do this in response to previous ways of defining the registers learners use in mathematics classrooms, not because I believe they are distinct.

[3] However, it is not my aim here to review the use of the concept or compare and contrast how different researchers have used the construct of mathematical practices. Instead, I only wish to briefly summarize what a Vygotskian perspective of mathematical practices entails.

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Meaning is about reference on occasion, but is also much more. Meaning is also about associations of all sorts (including verbal similarities). In particular, meaning is about unaware associations, about subterranean roots that are no longer visible to me, but are nonetheless active and functioning.

— David Pimm, from p. 35 of 'The silence of the body' in *FLM* **13**(1).

Metaphor is not solely, or even primarily, a decorative device of poetry. Rather it is one of the central linguistic strategies at our disposal with which to create sense of the world. Seeing through metaphors is essential in both senses; we cannot do without this process, yet it is important to be aware when we are using them.

(I have been accused of seeing metaphors everywhere. There is a delightful cartoon [...] comprising a bearded, bespectacled male in a country scene where all the objects carry labels. The tree wears a sign "metaphor for growth and change", the fence by the side of the road "metaphor for limits", while the two adjacent flowers are to be seen as a "metaphor for love". It provides a visual instance of seeing both the general and the abstract in the particular.)

— David Pimm, from p. 199 of *Speaking Mathematically: Communication in Mathematics Classrooms* (1987) Routledge & Kegan Paul.