Re-viewing Graphing: Traditional and Intuitive Approaches

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> As many experienced teachers are well aware, creating graphs and interpreting data from them are skills not easily acquired by most students [Padilla, McKenzie and Shaw, 1986]

One way of responding to a task you see as being difficult is to break it down into smaller bits that you feel more confident that you can tackle successfully. This technique is often used by mathematicians to good effect.

One way in which teachers respond to tasks which they perceive as being difficult for pupils is to break the task down into small steps, and have the pupils tackle these in order, so that their success will be reinforced at each stage. This step-by-step approach is deeply rooted in the pedagogic tradition of mathematics education, and can be clearly seen embodied in the pages of textbooks and the activities in classrooms. It is less clear that it is an effective technique for supporting children's mathematical learning.

The introduction of computers may challenge many aspects of both the content of the mathematics curriculum, and the ways in which mathematics is learnt and taught. For example, computers offer the possibility of performing in seconds, and by means of a few key presses, many tasks which have formerly required a range of valued mathematical skills: long division, solving quadratic equations, and drawing graphs are just three examples. For many teachers this is unsettling. This disquiet is expressed overtly in concerns about the loss or undervaluing of "basic" skills. It may also be more deeply rooted in the challenge which is presented to traditional pedagogies.

In this article, I describe work with primary school children (aged 8-10 years old) which follows what I have called an *intuitive* rather than a traditional approach to teaching a "difficult" topic; the use of line graphs

The perception of graphing as a difficult topic is not unique to teachers. There is considerable research evidence to support this view. The statement at the beginning of this article comes from a paper reporting research on graphing skills with North American pupils in grades 7-12. In this study, pencil and paper tests were used to investigate success rates in a number of different skills relating to line graphs. The results are somewhat depressing. The researchers found that, although 84% of pupils were successful in reading and plotting points, only 57% were able to interpolate and extrapolate, and only 32% were able to scale axes successfully. Swatton and Taylor [1994], reporting on tests of graphing skills within the Assessment of Performance Unit studies in the U.K., observed similar levels of competence with children at age eleven: 78% success at reading points, but only 35% success at interpolation. Both of these studies were undertaken from starting points in science education, but similar indications of children's difficulties are evident in studies within mathematics education: Foxman *et al.* [1980] reporting on the APU Primary Survey, Kerslake [1981] reporting on the Concepts in Secondary Mathematics and Science studies, and the more detailed study reported by Sharma [1993] are well known U.K. examples.

This article is based on one aspect of the work of the Primary Laptop Project, which has the long-term aim of studying the effects of continuous and immediate access to high levels of computer provision on young children's learning of mathematics. One focus of our work within the project is on children's use of spreadsheets, and the access that this gives to the instantaneous production of a variety of graphs. We are interested to explore how children respond to difficult kinds of graphs when they can by-pass the process of drawing by hand, and produce ready-made graphs from data collected on a spreadsheet.

Background to the study

The Primary Laptop Project is based in a combined school (for children aged 4 to 12 years), and involves a collaboration between researchers from the Mathematics Education Research Centre at Warwick University and teachers in the school. At the time of this study, one Y4 class (ages 8-9 years) and one Y5 class (ages 9-10) years) were equipped with one portable computer (an Apple *Powerbook*) for every two or three children. They had access to these machines throughout the school day, and took them home overnight and at weekends. The software available included *LogoWriter* and *ClarisWorks*, a commercial package containing word processor, database, spreadsheet and graphics facilities.

During the study, two researchers were working alongside class teachers, focusing on work in mathematics and science. This involved each of the researchers being responsible for teaching one class, and acting as a researcher in the corresponding lessons in the other class. Field notes taken in the classroom (directly onto a portable computer) were supplemented by examples of children's work, recorded interviews with children, and discussions between researchers and class teachers. While the researchers were teaching, the class teachers were encouraged to act as researchers, observing lessons and making notes on particular groups of children.

At this stage of the project, the research was in an exploratory phase. The activities used in the classrooms

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were planned jointly by the class teachers and researchers, following the thematic approach normally used in the school, but building into this opportunities to exploit the potential offered by the children's high levels of computer competence. Our agenda was to find out what was possible, and identify interesting areas for future research, rather than to address specific research questions. What follows is the story of our developing insights in one particular area.

Initial evidence of children's institutions

As part of a series of activities linked to the theme of *Growing and Shrinking,* the Y4 class had been researching their own body measurements, and how these had changed since they were babies. They used a spreadsheet, including its graphing facilities, to enter their own body measurements in a table, and draw bar charts to compare heights, foot sizes, etc. In an attempt to extend the children's experience of graphs, an activity using line graphs of children's growth was introduced.

I began by showing some data about the height of a baby between birth and age three. From this data I produced a line graph as shown in Figure 1, and invited the children to comment on what they saw.

	A	B	C
1	Name	Age	Height
2	Lilian	0	54
3		0.5	72
4		1	77
5		1.5	83
6		2	89
7		3	98

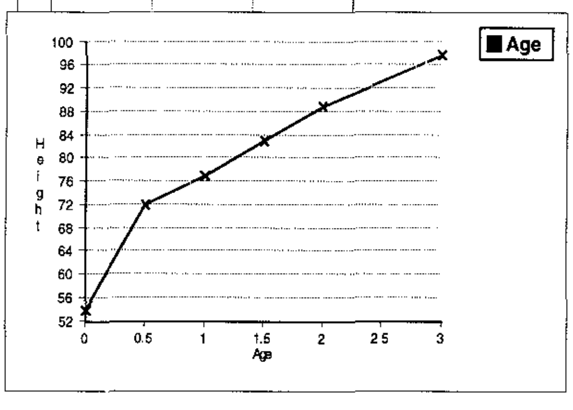


Figure 1
Graph drawn from spreadsheet data

Immediately Emily said: *It says two and a half there* (i.e. on the axis) *but that wasn't in the spreadsheet.* I asked if they could work out how tall Lilian was when she was two and a half. Several ideas were offered.

> Joanne: *Would it be half way along that part of the line?*
> Emily: *You could look at the numbers and work out half way between them*

A number of other children were moving their fingers in the air, apparently trying to read off the height on the vertical axis, although they found it difficult to articulate what is was they wanted to do. Discussion with the whole class gave an overall sense that the graph was meaningful to the children, and that they were able to make sense of the information contained in it, at least at an intuitive level.

In the next stage of the activity, the children were presented with tables showing the heights of four imaginary children at various ages between 2 and 16 years old. The task was to produce a line graph from one set of data, and to find out from it how tall that child was at age 3 and at age 13.

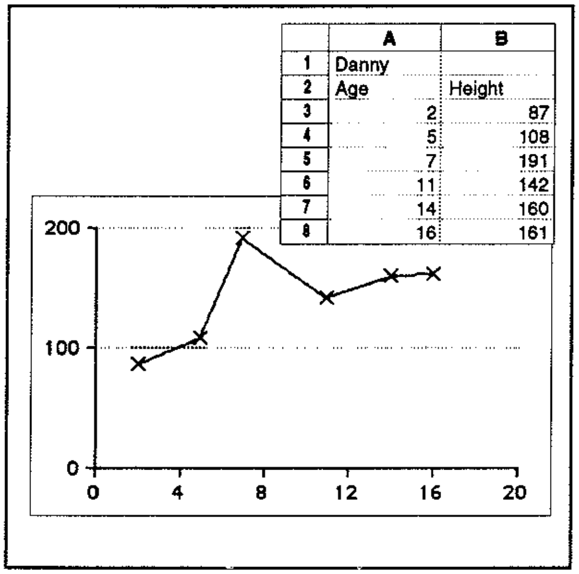
The task of entering data into the spreadsheet, and producing a line graph from it seemed to present few problems for the children. They were used to being shown new computer techniques, and helping each other to learn how to use them confidently. Most groups were also able to make a good attempt at interpolating the values for the heights at age 3 and age 13. They used a variety of methods for doing this: pointing with fingers, using the mouse pointer, using lines from the graphics tools. Observations of the children while they were working revealed their understanding of the meanings of the graphs they produced.

> Holly and Verity quickly produced a line graph
> Verity: (Pointing with the mouse to 11) *That's about 11 there and* (pointing again) *about 3 there.*
> Holly: (reading across the graph) *So it's about 80 centimetres at age three. Is that right?*
> They then went over to get a ruler to see what 80 cm looked like. They had a quick discussion about whether this could be the height of a three-year-old. In the end they decided their answer was reasonable.

> Emma and Phillipa showed their graph (see Figure 2) to the class teacher because it didn't look as they had expected. She asked them what the chart told them; they said that Danny had shrunk as he got older. The teacher asked: *Is that likely?* Phillipa thought that they had probably put in the wrong data. They went to check, and found that Phillipa had typed in 191 instead of 119 for the height at age 7. They quickly changed this entry, and were satisfied with their new graph.

Both teacher and researchers were excited about the events of this lesson because of the intuitive ease with which the children seemed to handle line graphs. The children's facility with the graphs was surprising for a number of reasons.

- This was the first time the children had met line graphs in school (although they would almost certainly have seen them used on television and other advertising media). The class teacher would not normally have introduced work of this kind to children at this age.
- By default, the software draws graphs showing only the main *horizontal* grid lines (as can be seen from the figures above), so very little support is available for interpolation.



	A	B
1	Danny	
2	Age	Height
3	2	87
4	5	108
5	7	191
6	11	142
7	14	160
8	16	161

Figure 2
Emma and Phillipa's surprising graph

- The software scales the graph so that it fits the frame available, and the scale then alters as the size of the frame is changed. This means that the children had to deal with a variety of scales, some of which are not ones which we would have chosen as being easy to read
- The children had had no explicit instruction in how to interpolate points on the graph, or how to read scales on the axes.

Exploring intuitive responses

In order to explore further the strength of the children's intuitive understanding of the work they had done with line graphs, individual structured interviews were conducted with about a third of the class (10 children), taking a cross-section of abilities, based on the teacher's assessments. In the interviews, each child was given new sets of data for the growth of two more imaginary children They were asked to produce a line graph on the computer from one set of data, and to read the child's height at 3 and 13 from the graph, as in the previous lesson. All of the children interviewed were able to repeat the original task of entering data into the spreadsheet and use the software to produce a line graph, and to make a reasonable attempt at reading off heights for the given ages

In order to explore the children's understanding of the structure of the graphs they had been using, we then put the computer away, and asked if they thought they could draw a similar graph from data about another imaginary child on a sheet of squared paper. Although we did not feel that the skills of drawing graphs were particularly important, an earlier experience had suggested to us that this task would offer opportunities to assess children's understanding of the conventions of graphing. In the original lesson two girls had chosen to produce graphs by hand rather than on the computer. Although they seemed to have a good understanding of what they were trying to achieve, they both got completely stuck because they did not know how to scale the axes or plot points.

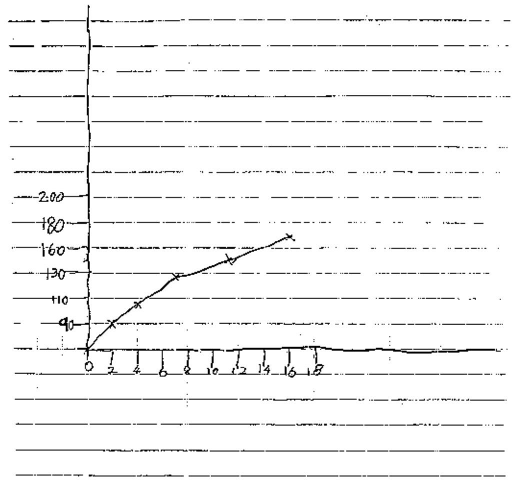


Figure 3
Sam's graph

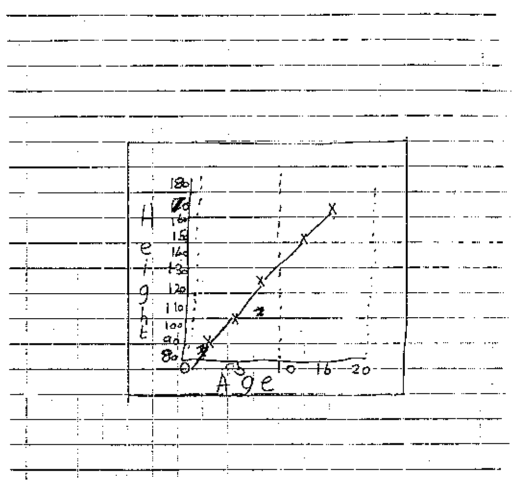


Figure 4
Emily's graph

All but two of the 10 children interviewed were prepared to attempt drawing a graph by hand These varied in their accuracy, but most showed a reasonable choice of scales on the axes, often copying those used by the computer. The children found more difficulty in plotting points efficiently The general strategy was to start drawing a line, often with a ruler, from the bottom left corner, and hope to arrive at the right place for the first point. We did intervene to suggest marking the point first, and then putting in the line, and all the children were able to accept this. If children seemed to be struggling, we offered them paper with suitable axes already drawn on it, and in general they used this effectively to produce a graph from which they could interpolate. Only one of the children, a boy with very low self-confidence, was not prepared to try drawing his own graph, even on the ready-made axes.

Examples of some of the most accurately drawn graphs are shown in Figures 3 and 4. Sam was inventive in his choice of axes, and used them confidently, even though he made one error in marking the vertical axis. Emily's graph is typical of the way in which a number of children copied the computer display, using the same scales for the axes and even writing the label for the vertical axis in the same way that the computer presents it.

Initial conjectures

From the evidence of these interviews, we conjectured that the computer had played a significant role in enabling children to work with line graphs, and furthermore in allowing children to build on their intuitive understanding to construct for themselves an understanding of what was required to draw such graphs by hand. It seemed possible that being able to produce graphs without needing to worry about the problems of scaling axes and plotting points freed children to focus attention on *using* the graph in a meaningful way. Reading values from the graph was generally easier than trying to interpolate from the data. Also, experiencing a number of examples of similar graphs may have enabled the children to assimilate some features of the use of scale, which they were then able to use in producing their own graphs. One feature of the software seemed to be potentially important here: if the size of the frame within which the graph is drawn is changed (something which the children would naturally do to get the graph looking as they wanted it) then the scale alters to fit the new frame. We felt that this might be implicitly drawing children's attention to significant features of the graph which did not change under these conditions.

There appeared to be a considerable difference between the confidence with which this group of children were able to work with line graphs—interpolating points, handling scales and axes, and drawing in points and lines—and the low levels of success reported by the researchers quoted earlier. However, we were aware of two factors which might have considerable significance in the children's performance. The class we were studying were introduced to line graphs in the context of a project they had been closely involved with for some weeks. The data they were working with was, although artificial in the sense that it referred to imaginary children, real and meaningful to them. This would not be the case for pupils in the other research studies.

Secondly, the line graphs the children produced were ones in which the appearance of the graph matched the phenomenon which was being graphed; the graph goes up as the child grows up. Kerslake [1981] suggests that graphs of this type are the easiest for children to interpret, and it is not clear whether Padilla *et al.*, or Swatton and Taylor, used test items involving graphs of this kind.

Analysing the role of the computer

Despite these reservations, it seemed that the use of a computer to generate line graphs might be having a significant influence on the children's ability to gain access to this area of mathematics. In order to get a better understanding of the role played by the computer, we undertook a comparative study with Y5 (9-10 year-old) children in the second project class. The use of line graphs was introduced in two ways, one using the computer, and the other relying on more traditional resources. Our conjecture was that the group of children who had used the computer would find it easier to produce their own graphs by hand, and to interpolate from them, than those who had worked only with a pencil-and-paper approach.

The comparative study was in three stages.

- As an initial activity we presented a table of data and a hand-drawn line graph of a child's growth, and asked the children to recognise specific points, and to interpolate. The results from this activity were used to establish a baseline of skills, and to divide the class (25 children) into matched pairs to form two groups.
- We then worked through teaching programmes aimed at providing parallel experiences for both groups, one using computers and the other using only paper-and-pencil methods, without explicitly teaching either group how to draw graphs.
- A final task ("Molly's graph") was given to both groups. This presented a new set of data and asked them to produce a line graph by hand and to interpolate from it the child's height at two different ages, and the age at which the child reached a particular height.

The teaching programmes were designed to give the two groups parallel experiences, without giving either group an advantage in terms of teaching skills of graph drawing. The **computer** group was given a series of activities, following closely the work done by the Y3 children. That is, they were given tables of data for various imaginary children, asked to produce graphs from them on the computer, and then to read off information which required them to interpolate. The **paper** group worked with the same data, but it was presented to them on duplicated sheets containing the table of data and a hand-drawn graph. They were asked to discuss and compare the graphs, and to read off information in the same way as the computer group. The hand-drawn graphs used the same scales as the default values on the computer, in order to keep the experiences of the two groups as similar as possible. The differences in the experiences of the two groups would be that the computer group would be active in creating the graphs, while the paper group were simply presented with them, and also that the computer-generated graphs could be manipulated on the screen, for example by altering the size (and hence the scale). This meant that the children could effectively see different versions of the same graph.

The final task of producing Molly's graph by hand was given to both groups after one session (about 45 minutes) of the teaching programme. The results at this stage were generally disappointing, with few children being able to produce hand-drawn graphs which matched the quality of those produced by the Y4 children. We felt that the groups had not had sufficient time to get to grips with the work, particularly as they had been introduced to the topic in rather an artificial way, without it being part of an on-going project. We wanted to focus the children's attention on the task of drawing their own graphs, but without explicitly teaching the skills of constructing axes and plotting points. In order to do this, we returned the children's initial attempts at drawing graphs without comment, and asked them to compare and discuss them in pairs. We then led general discussions in which the computer group compared their hand-drawn graphs with those produced on the screen, and the paper group looked back at the hand-drawn graphs they had been given. After this all the previous materials were put away, and both groups were given Molly's graph to draw again.

The outcomes of the comparative study

When we looked at the children's second attempts at producing their own hand-drawn graphs, one result was immediately clear: there was no difference between the computer group and the paper group. It looked as though our conjecture about the effect of the computer had not been supported. However, considering the results more carefully led us to rethink our ideas about the factors which were enabling the children to build on their intuitive responses to line graphs of this kind.

A large number of the children (15 out of 25) were able to complete the task with Molly's graph successfully: that is, they could construct sensible axes using an appropriate scale, plot the points they had been given using these, and interpolate from their graph to read off information with reasonable accuracy. (We accepted as accurate any response which fell within the correct square on the grid.)

Three children did not manage to make a serious attempt at drawing Molly's graph, and the remainder of the class completed some aspects of the task with various degrees of success. After a number of attempts to present these results clearly, we adopted a system of scoring them separately on different aspects of the task. In order to get a graphical representation which reflected this analysis, we used a binary scoring system, awarding one point for the ability to interpolate, 2 points for plotting points correctly, and 4 points for constructing sensible axes. Those children who completed the task correctly thus scored 7. The scores used here do not reflect any values assigned to different aspects of the task: we did not, for example, feel that constructing axes was four times more difficult than interpolation. The scoring system was simply a device for separating out the different components of the task.

Figure 5 shows the results for both groups presented in this way, and makes a number of features immediately clear. For example, no child scored 6: in other words, there were no children who could plot points and construct sensible axes but who could not then interpolate successfully from their graphs. In fact, discounting the 3 children who did not score, only one child failed to interpolate from the graph she had drawn.

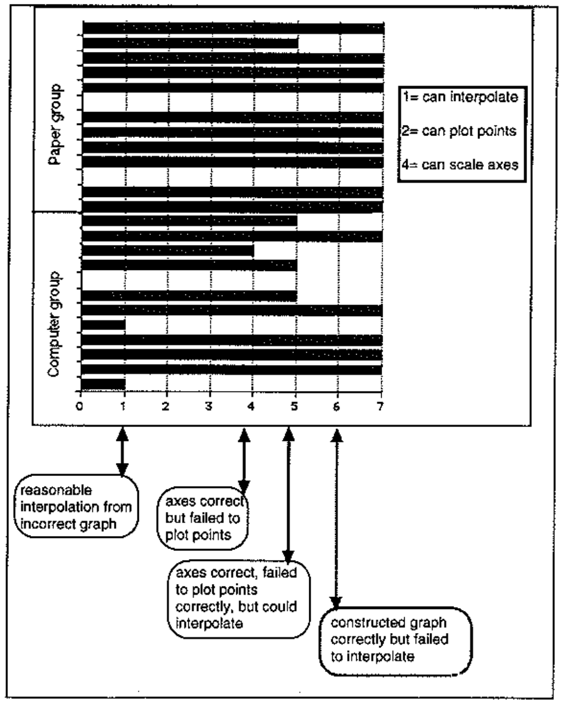


Figure 5
Analysis of results of final task

Two children made a reasonable attempt at interpolation even when their graphs were not "sensible" ones: their graphs illustrate features which we saw in a number of children's early attempts at producing line graphs.

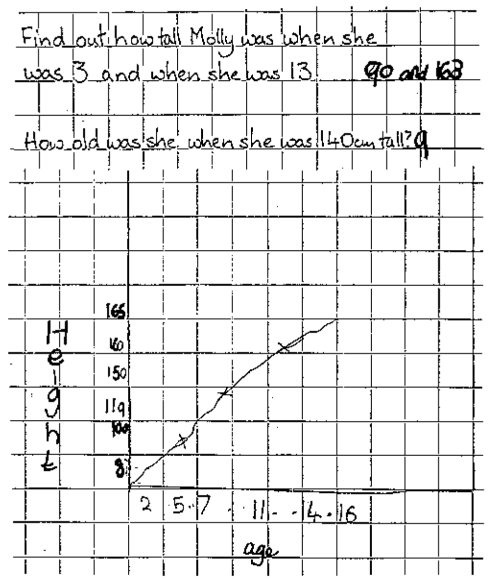


Figure 6
James' graph

James' graph (Figure 6) scored 1 in our analysis. James used the given values on the axes, with no attempt to relate the values to distances along the axes. He then drew a line across the graph, with no attempt to plot points. The crosses which appear on his graph were put there when James wanted to interpolate in order to answer the question.

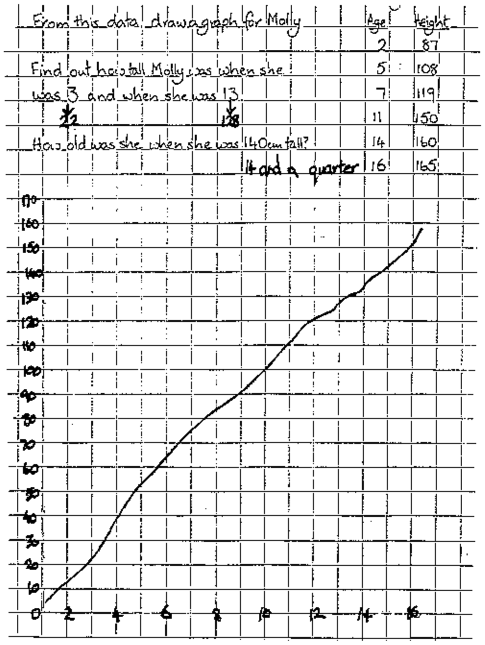


Figure 7
Sophie's graph

Sophie's graph (Figure 7) scored 5. She did scale the axes for her graph, although the scales she chose were inappropriate for the data she had to enter. This may not have been apparent to Sophie, because like James she drew a line on her graph with no attempt to plot points. On reflection, this is an understandable response, since on the graphs the children had been presented with during the teaching programmes, the line was a much more significant feature than the points it passed through. Despite the problems with her graph, which make it appear nonsensical to us, Sophie went on to interpolate accurately to answer the questions about Molly's growth.

Re-assessing the outcomes

The results of this study suggest that these 9-10 year-olds were able to respond intuitively to line graphs of children's growth. Most of them seemed to be able to cope with interpolating points not actually marked on the graph, and reading information off axes with differing scales. Furthermore, in the process of working with graphs, many of the children seemed to assimilate skills which enabled them to draw simple line graphs by hand. What the results don't show is the difference we had expected to see between those children who worked with the computer, and those who worked only with hand-drawn graphs on paper.

On reflection we feel that it was not our initial conjecture (i.e. that the computer played a significant role in allowing children to take an intuitive approach to the graphing tasks) that was at fault, but rather our experimental design. We were so concerned to make the experiences of the two groups as similar as possible, and not to give the computer group unfair advantages, that we built into the experiences of the paper group many of the features that we now see as significant in supporting the children's understanding.

We have identified a number of related features common to the experiences of both groups which we feel have contributed to the children's high levels of success, in contrast to the much lower success rates recorded by other research studies.

The work was set in a *meaningful and familiar* context. This enabled the children to make sense of the activity, and relate it to personal experiences. Indeed, physical growth is not only something children know about intellectually; it is emotionally significant as well.

The presentation of a *complete image* allowed children to focus on a holistic understanding rather than on understanding the separate components. In contrast to the way in which line graphs are traditionally introduced, we did not deconstruct the task into constituent skills—constructing suitably scaled axes and plotting points. If attention is focused on these, it may be difficult for children to keep in mind the context and purpose for which the graph is being drawn. Indeed, the skills of constructing graphs are often taught in isolation from any meaningful context, and so appear to children to be an end in themselves.

The children in both groups worked with *a number of similar graphs*, representing the growth of different children. This provoked discussion which focused attention on both differences between the graphs, and their common features. This in turn encouraged the children to discriminate between those features which change and those which stay the same, and so gain some feel for the conventions of graphing. This would be very hard to do if, as normally happens in a classroom situation, children see only one example of each of a number of completely different graphs. In this situation, the child has no clues as to what to attend to in terms of these conventions. It is impossible to judge from one example only, what is particular to *this* graph, and what is common to all graphs. To use John Mason's terminology, the children in our study were able to begin to move from looking at the particular graph to looking *through* the graphs to see the underlying structure.

When working with computer-generated graphs, children can easily produce a range of graphs from the same data. For example, the children could just as easily have made bar charts and scattergraphs as well as line graphs from the growth data. They could, of course, also have made other, less appropriate types of graph as well (see Pratt [1994]). Furthermore, the graphs they produce are dynamic, in the sense that their size and proportions can be altered by dragging the corners of the frame. Unless the user has specified otherwise, the scales shown on the axes will change as the graph is distorted. The appearance of the graphs can be changed through menus which control the scales on the axes, the orientation, the style of markings and labels, and so on.

The children were set *purposeful tasks* to perform which involved them in using the graphs. These tasks were relatively complex, since they involved interpolating for values which were not plotted directly on the graph. The tasks were meaningful to the children in two senses: finding out and comparing children's heights at particular ages was an activity they could understand, and the fact that the answers they needed could not be predicted easily gave purpose to working from the graph.

The role of the computer

In our research study, the features described in the previous section applied to both the computer group and the group who worked only with hand-drawn images. In the normal classroom situation, computers offer the only possible means of providing children with access to immediate graphical images drawn from what we have come to call *hot data* (Ainley & Pratt [1993]). This is data which has been collected or researched for a clear purpose, as part of some wider exploration, rather than as an end in itself. Using computers can allow children to have control: to select the data which is appropriate for their work, and to produce graphical images of that data quickly and easily. Thus, despite the apparently inconclusive results of the comparative study, we would still want to claim a vital role for computers in an intuitive approach to graphing

Some tentative conjectures

While it would obviously be inappropriate to attempt to draw firm conclusions from such a small study, our experience does suggest that young children's ability to work with line graphs is greater than is generally recognised When presented with purposeful tasks in a familiar context, the children seemed able to act intuitively to use line graphs This leads us to conjecture that the low levels of success recorded in other research may be due to pedagogic rather than cognitive obstacles, and that the intrinsic cognitive demands of reading and interpolating points on line graphs, and of scaling axes, may be lower than generally supposed. Some support for this view is given by Bryant and Somerville [1986], who claim that even six to eight year olds do not find the spatial demands of plotting and reading points on ready made line graphs difficult.

Furthermore, whilst the children's attention was focused on carrying out the tasks, other skills were used and assimilated *without* direct teaching This is a possibility which cannot easily be reconciled with traditional pedagogic approaches We see this as an example of what Hewitt [1994] refers to as *functionalisation*: the process by which skills reach a level at which we are able to function with them automatically Hewitt quotes Rousseau as saying *Before you can practise an art you must first get your tools* [1911], but responds to this by suggesting that *In order to practise your art, you acquire the tools* I am reminded by this of discussions with teachers who feel that children would need to learn keyboard skills before they could use Logo or a word processor, so that they don't become frustrated by their slow typing. I point out that I developed my (quite creditable) typing ability mainly through programming and writing at the keyboard.

Hewitt claims that attention focused on a higher level task within the overall activity can drive the functionalisation of skills at levels subordinate to this. So while the children's attention was focused on interpolating to find heights at particular ages from their graphs, they were also developing skills such as scaling axes, which in this context are subordinate to interpolation

Whilst we would not claim that the children we worked with would necessarily be able to transfer their understanding of interpolation and handling scaled axes to *any* graphing situation, we did feel that they had gained a sense of the conventions of graphing They would be receptive to future teaching, and relatively little effort would be needed in order to consolidate and extend their understanding We conjecture that current pedagogies which focus on deconstructing the process of producing a graph into constituent skills as a prerequisite for children being able to understand and use graphs are likely to be less effective than putting children "in at the deep end" with meaningful tasks to perform Indeed such traditional pedagogies seem likely to perpetuate the perception of graphing as a difficult topic

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