BRINGING FORTH WORLDS

LAURINDA BROWN

> We do not see things as they are, we see them as we are.
>
> Each friend represents a world in us, a world possibly not born until they arrive, and it is only by this meeting that a new world is born. —Anaïs Nin

My degree was in mathematics and I then taught mathematics in schools with 11 to 18 year old students before moving to work with prospective teachers at the university. My interest in learning, how it is done, from learning mathematics to professional learning, triggered the focus of the symposium papers and hence the articles in this monograph. I am a mathematician, teacher of mathematics, researcher, mathematics teacher educator and, as Tracy Helliwell and Julian Brown describe themselves in their article, a mathematics teacher educator researcher.

I am used to working and writing with people across a range of perspectives, new worlds being opened up to me as they become part of me through our conversations. These conversations, a word discussed by David Pimm, are represented in the pieces in this monograph. Reading the articles has been enlightening for me both for insight into what friends and colleagues have taken away from their interactions with me and my public writings and also the current reactions triggered as I read. These reactions were sometimes an awareness of change, difference over time, in my perception of the person writing; sometimes resonance with an idea; sometimes of an awareness of the other's different interpretation of an idea; sometimes the joy at remembering a conversation and where it has developed to and, mostly, pleasure at the professional and personal interactions with these individuals at the heart of my working life. For the rest of these reflections, I will structure the writing through themes related to learning that have arisen through reading this monograph.

Learning is new awarenesses linked to action

In 1979, Dick Tahta organised a first meeting of people to discuss forming a working group to study the *Subordination of Teaching to Learning* (SubTLe). The phrase is one of Caleb Gattegno's, an educator who had influenced Dick from the early days of the Association of Teachers of Mathematics (ATM). Weekend meetings were spent introspecting on our learning, followed by considering implications for our actions as teachers of mathematics. I remember, for instance, Dave Hewitt teaching us to juggle. In the current, now renamed, *Science of Education* working group of the ATM convened by Dave Hewitt, our practices are still the same, although now we focus on collaborating to produce ATM publications, such as, *Mathematical Imagery* and *On Teaching and Learning Mathematics with Awareness* [1]. To begin, individuals in the group offer an activity. After activity, we reflect together on our experiences, relating our new awarenesses, our learning, to our teaching.

Brent Davis gives an example of a new awareness, pedagogical impasses, related directly to experience and actions in his work with teachers. I remember us speaking after a seminar where he talked about the idea and, more formally, interviewing him for the ATM journal *Mathematics Teaching* (MT261, p. 43; David Wheeler, Dick Tahta and I have all edited MT). The joy comes from these remembered conversations whilst hearing Brent's voice as he discusses his "new favourite topic" (MT261, p. 43), his learning, in his article.

I find that I can introspect on my own learning through the context of acquiring a new skill. I am currently developing new awarenesses solving sandwich sudokus. A sandwich sudoku is a new type of sudoku, with the same grid, but at the end of each row and column, outside the grid, is written a number, which can be between and including 0 and 35. The number indicates the total between 1 and 9 in that row or column in the grid. So, if the external number for a row, say, is 0, then 1 and 9 are adjacent in that row inside the grid. Placing the 1s and 9s in the grid is an important step in solving the puzzle. I found myself being aware of where 1 or 9 could *not* go, and these awarenesses, for example, there is a 1 too close to the edge of the grid to fit a total of 24 and a 9 to the left of it, constraining where the 1s and 9s could go, led to effective action in completing the puzzle. Once I had this awareness of tracking where 1s and 9s cannot go, in starting a new puzzle I habitually focus on this action first.

Alf Coles has chosen to write about 'nots' in his piece. We have worked together since 1995. As a mathematics teacher educator, many of my awarenesses are also, like in the sandwich sudoku example, around 'nots', what is not said or done. For instance, in a first conversation with prospective teachers in a small group reflecting on their first experience of teaching a small group, some comment on themselves, for instance, how they were feeling, such as, being aware of going red (with embarrassment over some incident or simply feeling self-conscious standing in front of so many pairs of eyes). I am aware of what they are not talking about, such as, the children or the mathematics. Observing a prospective teacher in the early days of their practice, I am aware of what they are not doing, for example, if they are talking a lot, they are not listening to the children or hearing them, therefore there is no contingent interaction as they race through a prepared, detailed lesson plan. Reading Alf's writing makes me aware of what I was not aware of before, the idea of setting up a boundary. I am also aware of the potential of this idea to affect my future actions. However, I am also aware that, in working with mathematical situations such as Pick's Theorem, or setting up the culture of a working group, not all participants become aware of the 'nots', constraints on their

behaviours, quickly. In the case of prospective teachers, there are those who stay with, for instance, their own judgements and lesson plans rather than listening to what the children bring with them, for the whole year of the course, not being prepared to adapt. These individuals do not pass the course.

Becoming a mathematics teacher educator

Becoming a mathematics teacher educator, for me, is a journey related to awareness of awareness. I am working with the prospective teachers aware of their awarenesses, their learning, given shifts in their actions. A connection between many of the authors of the articles in this monograph is those who work with prospective teachers and research mathematics teacher education. At international conferences, like the yearly one of the *International Group for the Psychology of Mathematics Education* (PME), these are the people who can often be found attending the same sessions; co-organising working groups; or presenting in research forums related to such strands as *Teacher change; Teachers working with researchers; The learning of mathematics teacher educators.* Olive Chapman, Alf Coles, Merrilyn Goos, Peter Sullivan and Salvador Llinares are within this category, recently joined by Tracy Helliwell and Julian Brown, now teaching on the one-year course for secondary prospective teachers at Bristol. Olive and I share a background in narrative approaches, Olive having had a narrative supervisor for her doctorate and myself being part of a strong narrative community in Bristol, led by Jane Speedy. Alf talks about Bateson's take on story as being about relevance, for me, context and pattern through time. Bruner, in *Acts of Meaning,* talks about the role of narrativised folk psychology in organising experience, particularly framing as "a means of 'constructing' a world, of characterizing its flow" (p. 56). So, we are narrative beings who weave our stories of ourselves, 'segmenting events'. Self-study is the focus of both Olive's and Julian and Tracy's writing, where our learning as mathematics teacher educators comes under scrutiny. I am struck by the long conversations and a sense of history, another form of story, within this strand. Dick Tahta died quite a long time ago now, but he is still present for me. Dick was my tutor on the one-year prospective teacher course (PGCE) at the University of Exeter and I have taken that role with Tracy and Julian, who are now my doctoral students as were Alf and Richard. When Tracy and Julian, now colleagues, hear my voice in their heads from PGCE, I hear Dick's, saying similar things. Mentoring is an important aspect of learning journeys. However, from what we say as tutors, the prospective teachers take what they can connect to from inside themselves.

When I stopped working on the PGCE course, retiring, I continued to supervise my doctoral students and will do so until they complete. By then, both Tracy Helliwell and Julian Brown, doctoral students, were working at the university on the PGCE course with Alf Coles, who was also their second supervisor. As I dropped in for PhD tutorials, I was struck mainly by the culture continuing as Alf worked with Tracy and Julian reciprocally interviewing to bring awarenesses to the surface in developing as mathematics teacher educators. Although their length of experience in the role is different, the process of learning in these conversations as expert and novice mathematics teacher educators is the same. The word awareness, for me, comes from the work of Caleb Gattegno, for instance, the aphorism 'only awareness is educable'. Tracy is following in the footsteps of Olive and me, being a mathematics teacher educator who also uses narrative in self-study. Julian is focusing on awarenesses, recently bridging the worlds of classroom teacher and mathematics teacher educator.

Learning as seeing things as we are, not as they are

I read the first Anaïs Nin quotation at the beginning of this article after I had been reading through the articles in this monograph seeing patterns that connected. Here was another strand, each person's learning from the same stimulus is different.

As a mathematics teacher educator, one awareness that is highlighted clearly in Richard Barwell's writing is that learning is a different process for each of the individuals in a mathematics classroom and for prospective teachers. I resonate with Richard's first story, which happened before we started working together for first his masters and then doctorate. Richard writes of his "important realization" in teaching teachers in a non-English context,

> While we may be using the same (English) words, these words could mean very different things. […] I realized that the words we used to talk about teaching mathematics were informed by our experiences of learning mathematics, culture (in general, of education, and of mathematics), and languages. Since I and the teachers had very different experiences, our interpretations were also different.

Such an awareness is also true of children in an English-speaking context learning mathematics. A simple example would be of a teacher talking about volume and some of the children having an image of turning up the volume when listening to a song. How would the actions of teaching be affected by this awareness? Richard goes on to describe these changes, which were not leading "students or teachers to a particular outcome" but working together, followed by discussions of experience so that the participants came to insights that would lead to actions in their own classrooms. He no longer tried to force particular insights or connections.

An aspect of this awareness became triggered for me during another event marking my retirement, which was organised by an already retired colleague, who worked with me for a long time on what was the core of my working life, a one-year post-graduate course (PGCE) to prepare students to become mathematics teachers in the age range 11-18. Jan Winter had invited local teachers spanning the thirty years of my involvement with the course, ex-PGCE students, mentors, heads of mathematics and colleagues within mathematics education at the university. Some of these individuals had had multiple roles, having been prospective teachers who then mentored and became heads of department and senior managers in local schools and even colleagues at the university. One individual, who had been in my first ever PGCE group, was still a local head of mathematics and had been a mentor. There was a limit, thirty, on

the number who could fit in the chosen venue, so the people who came were invited representatives. Jan had asked people to bring a story of working with me. We sat in a circle and the stories were read out. There was some discussion after each story was read so the atmosphere felt relaxed. A number of people commented, as with the symposium articles, on what they attributed to be my influence on them. The following interchange triggered learning.

> S described how the year had been distressing for him because he had separated from his partner and he recounted various statements I had made that supported him. I reported not having any memory of saying those things nor, indeed, some thirty years later, of him having separated during the year. What I remembered was that that cohort also experienced my own separation from my partner in the final weeks of their course, in June. In the following October, after the new PGCE course had recently started, I was sitting in my study reflecting on whether I could continue as a university academic, particularly supporting prospective teachers. S came through the door, asking me how I was. I replied, "I'm not sure I can do this job anymore." What he then said that, in fact, kept me going through a difficult and emotionally draining year of divorcing, was, "Just remember, they don't know who you were before." S then reported not remembering having said that. Neither of us had a memory of saying what was important for the other.

There were, of course, many other things said in our conversation that neither of us remember. Alf and I led the PGCE course for many years, co-teaching and co-learning after teaching through reflective narrative conversations where we have described ourselves as narratively interviewing each other to support our developing awarenesses. Since Alf was there in the circle, it was natural for us to talk in this way about our experiences of the evening, considering samenesses and differences in the stories. We got to the awareness that we cannot set out to influence others. Many points are made in any interchange but the important ones to the other emerge for them linked to their history, what is important to them at the time. What seems key as a teacher educator is to articulate awarenesses as they arise. Dick Tahta said that to me, talking about his experience of supervision when he was a relate counsellor. If the other is not ready for that awareness they will simply not hear it. In mathematics classrooms, this awareness seems central. From your perspective as a teacher, a child gives an answer to a problem in a whole-class discussion. You recognise the comment as the answer from your experience of doing the problem previously, however, as the discussion in the classroom continues, it becomes apparent that that comment was one of many and was not recognised as an endpoint by the students. Frustrated prospective teachers telling a class, "We did this yesterday", do not have the awareness of the difference between their own perceptions of what happened and the myriad variations of what was taken away by the children. Learning is contingent on connections so is a process. As a mathematics teacher, what would it look like to be working on the culture of a classroom so that the process of doing mathematics by the children is central?

Learning as new worlds born through friendship

As an international academic mathematics educator, one aspect of the role that leads to rapid learning is visiting the universities, homes and often schools in other countries. My first trip as a consultant was in 1993 to Indonesia, when I had not travelled on a long-haul flight before. I literally could not recognise or see the world when I walked out of the airport into heat and humidity. I needed to act, to find a taxi. Knowing is doing, so, in acting, I started to make sense of this new world, where I was to work with a friend who had completed his Master's in Mathematics Education at Bristol. My first trip to visit David Reid, now editor of FLM, was in 1997 on a British Academy funded trip to Montreal, giving a seminar at Concordia University (where David Wheeler first taught in Canada) and writing with David Reid and Vicki Zack in her house in Montreal. In this section, I want to compare and contrast two visits to friends in contrasting parts of the world to illustrate long conversations and the difference in interpretation of ideas given the context in which they are embedded.

Reading Peter Sullivan *et al.*'s contribution took me back to spending nearly three months at Monash University, Melbourne, working on research projects through visiting schools; sharing the teaching of a master's curriculum course with Helen Forgasz; giving seminars; attending reading groups; and being a colleague. Here was an example of a place that was, of course, different but was also, in many ways, so similar to my own context. Helen and I co-taught the master's curriculum course comfortably. My awarenesses from home led to effective action. Visiting primary schools and interacting with teachers led to learning through reflective discussions. This trip was the first time I had accepted such a long-term invitation, my commitment being to the one-year PGCE course in the UK. I was wondering whether the length of time would be a problem for me, being away from home, familiar places and people. However, my time in Melbourne, Australia was busy and passed quickly.

I was thinking a lot about Fullan's secrets of change just before and during the trip to Australia, particularly about relentless consistency. In reading Fullan's *Six Secrets of Change,* I was struck by his Secret 4, Learning is the work, a discussion of the importance of relentless consistency within the system, not to dampen creativity but to allow the rethinking and redoing cycle that seems to be so important. At the time, I was trying to explore the difference between those colleagues who looked for new activities to use in sessions in order to be stimulated and stimulating and what was so different in my own experience. I had taught, effectively, the same sessions for over 20 years, even though they were structured to support the developing insights of the learners so what actually happened was never the same as the previous year. Similarly, in my mathematics classroom, doing the same problem with different groups in the same year and with the same year groups over time [2] led to my being able to share awarenesses as a mathematics teacher with my students. Relentless consistency seemed to describe my practice. I saw in it what I had been doing over many years. As a teacher and teacher educator, I do something that I am comfortable with, that I have done many times before, allowing me to adapt, set up the rules of the community I like to

work with and begin to learn my students. I always keep attention on the process and comment about routines as they emerge. I gave a seminar on relentless consistency at Monash University. Reading the work of the research group including Peter Sullivan in the article in this monograph gave me insight into the way this group has taken the idea of relentless consistency on in their context, making their own connections, to support the teaching and learning of mathematics.

The experience of working in Australia contrasts with time spent at the University of Alicante, Spain. The Universities of Bristol and Alicante are a short flight away from each other. I first met Salvador Llinares in 2010 as a work colleague when we travelled to Ouro Preto, from Belo Horizonte, PME34, Brazil. With others, we had volunteered our time to work with doctoral students. It became apparent, talking with Salvador, that our contexts and theoretical positions were different and yet there seemed to be many similarities in what we were trying to do. It can be hard to identify just what the other means when the contexts are so different. It took me quite a while to realise that my assumption, supported by time at Monash, that university teachers visited their prospective teachers in school, is simply not the case in Alicante. This different context leads to a completely different set of actions with prospective teachers in the university and yet the premise of having activities for the prospective teachers to do remained the same. Academically we work on similarities and differences between the noticing paradigm used by Alicante and the enactivist perspectives of the Bristol group.

Enactivism, learning about learning

In 1995, I attended a discussion group at the PME conference in Recife and met David Reid. Our meeting is documented, rather mysteriously, in the *Enactivist theories* entry of Springer's *Encyclopedia of Mathematics Education* (first edition). This meeting and the friendship that followed is another example of "a world possibly not born until they arrive, and it is only by this meeting that a new world is born". My memories of that meeting are of being opened up to the worlds of Maturana and Varela, which already felt familiar. At one point someone asked a question, "Isn't this the same as Bateson's blind man and stick?" I had read Bateson's *Steps to an Ecology of Mind* when I was an undergraduate at Oxford in 1972, reading mathematics. I underwent a paradigm shift, replacing the need for certainty, the right answers that felt comfortable for me through learning mathematics at school, with process.

I made my own connections with the content of the seminar in Recife and had many discussions with David during the rest of the conference. Having initially recommended that I read *Tree of Knowledge,* having got to know me better, he suggested I began my reading with *The Embodied Mind.* Our e-mail conversations after the conference became an FLM article, *Fork in the road,* originally entitled, *Anti-dichotomies* (edited by David Pimm, then editor of FLM). David Reid and I have continued to be friends, writing together, often about enactivism and implications for the teaching and learning of mathematics, since then. A rich new world of possibilities, connected to who I already was, opened up for me at our meeting. With the aphorism knowing is doing, enactivism, learning about learning, fits well with my early experiences of the science of education group, introspecting on learning. In a contribution to the hundredth issue of FLM, David asked whether FLM was an enactivist journal. There were certainly members of an enactivist grouping, whose sessions choose themselves for me to go to at conferences, at the symposium. However, FLM is still recognisably David Wheeler, the founding editor's, journal, with a broad mix of perspectives not leading readers to a particular outcome but opening up discussion, in Brent's words "opening the space of the possible" (see FLM's aims on the inside front cover). Even though this is not an issue of FLM, I am sure the editor would be happy to receive your communications that share your engagement with authors in this monograph.

Notes

[1] These publications can be purchased from the Association of Teachers of Mathematics website, https://www.atm.org.uk. Follow the link to 'Store'.

[2] Brown, L., Reid, D.A. & Zack, V. (1997) On doing the same problem. *Mathematics Teaching* **163**, 50–55.