A Phenomenological Exploration of Mathematical Engagement: Approaching an Old Metaphor anew

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> In doing research, we question the world's very secrets and intimacies which are constitutive of the world, and which bring the world as world into being for us and in us. Then research is a caring act: we want to know that which is most essential to being (van Manen, 1990, p 5)

Few are seemingly called and many, seemingly, are not - to the call of mathematics, that is. Perhaps, to be more fitting to most people's experiences and observations, I might instead write, "few are seemingly called and many are seemingly apathetic, panic-stricken and/or perplexed by mathematics" To the segment of the population that finds meaning in the engagement of mathematical activity - those who are 'called' - Witz (2000) suggests that mathematics as a discipline offers a path through which a self-opening, deepening or unfolding of self can occur. Or, as Heidegger (1927/1996) might say, it offers a domain through which the potential for unification of *Dasein* can occur through care. The activity of mathematics, then, can become imbued with personal meaning and significance beyond that of solely serving the functions of livelihood and/or egoistic achievement

In my own life, I can recall certain instances when 'doing mathematics' became for me more than just 'doing mathematics', becoming what Joseph Campbell (1991, p 182) has referred to as a "moving power of your life" or what Max van Manen (1990, p. 14) terms a "self-forming process". In some ways, the study of mathematics has brought certain hard lessons that for me have been particular to the discipline. On occasion, I have wondered whether it is in what I have perceived to be a certain, unrelentingly severe nature of mathematics that I have come away with these specific life-fulfillments.

All of which leads one to ask, how is it that some find in the study of mathematics such a road to one's 'life-fulfillment', while others find mostly bafflement, meaninglessness and frustration? Also, in what capacity has our educational system had a hand in such a polarization of the population? And how much of this dichotomy might be inherent within the subject matter itself?

Naturally, 'self-openings' likely occur for the latter ('uncalled') group through other subject matters or life practices, and ideally would occur for most in more than one discipline. Yet mathematics as taught holds, perhaps, the dubious distinction of being the subject matter that fosters the strongest feelings of antagonism toward its study or mastery and, for this reason among others, it may be worth delving into more deeply.

Perhaps one of many ways in which to approach such questions is to consider the nature of what doing mathematics is like, in particular, for those certain individuals who are able to find some meaning, or at least a modicum of satisfaction, in mathematical activity. Rather than looking for differences between types of people, those who like mathematics and those who do not, or even analyzing mathematics itself (as if one were to assume that it could exist without an experiencer involved in the engagement with it), my hope here is to seek insight through studying the *lived experience* of those who are actively engaged in it, regularly as well as irregularly In order to focus this inquiry, I pose the following question: what is at the heart of the experience of mathematical engagement? Or, what is the meaning derived from such activity?

A method (if there is one) and an entering into the phenomenon

> I give you the end of a golden string,
> Only wind it into a ball,
> It will lead you in at Heaven's gate
> Built in Jerusalem's wall.
> (Blake, 1820/1953, p 302)

In order to explore these questions, I talked with five people, two of whom are professional mathematicians, another two who are presently graduate students, one in engineering and the other in education, and one, my wife, who lacks advanced mathematical training but maintains a positive attitude toward the subject. The method I have chosen is to follow a few threads phenomenologically that have emerged from a series of informal conversations held with these five and to see where they lead As a complement to this, I also interweave these strands together to form a new and old way of looking at mathematical engagement.

I begin my inquiry with a very small portion of my conversation with my wife, Maya My dialogue with her allows me entry into a phenomenon that I found striking in its persistence (for I would not have expected it from this group) It was during our conversation that she professed to her experience of doing mathematics as being "fun, but frustrating" - simple words, but in particular the last word, 'frustrating', cropped up time and again without my prompting in the first four conversations (at which point I decided that I would specifically inquire of it in the fifth)

Sheila Tobias (1978) has written of 'math anxiety', how it afflicts a great many in the population, and in turn offers suggestions, as well as lessons, in gaining back control from feelings of frustration and helplessness Among those with 'math anxiety', we might expect frustration as part of the experience of mathematical activity, but its prevalence among those who are adept, or at least capable, perhaps indicates a quality inherent in the nature of doing mathematics

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itself. It could speak of the nature of the people drawn to mathematics or perhaps the manner in which we learn and become apprenticed into its practice?

Perhaps such frustration is a kind of initiation rite that one must pass through to enter into some apprenticeship of knowing. As this calls forth for a deeper exploration, I have chosen to enter this study through this particular aperture, through the experience of frustration in the activity of mathematics, written not so much in an effort to resolving the issue as if it were a problem-solving task in itself, but more to understand its nature and range of appearances and meanings. Through such an opening, I enter into the deeper and broader meaning of mathematical engagement.

Following a first thread: frustration as deception

> But what if pleasure and displeasure were so tied together that whoever *wanted* to have as much as possible of one *must* also have as much as possible of the other? (Nietzsche, 1887/1974, p. 85)

The verb 'to frustrate' derives from the Latin verb *frūstrāre* (to deceive or to disappoint – see Barnhart, 1995, p. 302), which offers a glimpse into the experience of frustration that is not always so apparent. To begin with the first derivation – when I am frustrated, I am most likely *deceived* in at least one of my approaches, my conceptual framing or understanding of the situation before me, with myself, with another, or with some aspect of reality, with which I am not reconciling myself.

This seems to apply whether I am thinking of a frustrating circumstance in personal matters or in a strictly mathematical problem-solving situation. This kind of *frustration as deception* was apparent in my talk with Pogo – a professional mathematician – who explained that at times he sat there:

> being extremely frustrated [...] thinking I had it, congratulating myself, going for long walks, and coming back and realizing I didn't have it.

My wife, Maya, also describes her own experiences of *frustration as deception* when she recounts, "You almost get the answer, but then it just doesn't go anywhere, and you go into a circle" and later, "There are times when I say, 'oh yeah, I get it,' but then you think you get it, and you don't get it, and you go into the circle". This notion of a circle, which she alludes to a number of times within the interview, seems particularly pregnant with relevant imagery.

The circle represents not only a perfection of symmetry when viewed from the outside or from above (Abbott, 1884), but when viewed from inside or within the ever-extending line itself, it can either be considered a hermetically-sealed container or cocoon of sorts or else a path without beginning or end. In other words, it is a kind of trap that also provides an uneasy sense of security. In referring to the mathematical process of discovery, Sinclair and Watson (2001) write, "sometimes 'getting it' can turn out to be misleading or, worse still, circular" (p. 40).

In our language, we also speak of 'vicious circles' of frustration, futility and failure. Again, Maya, the least mathematically accomplished of the interviewees, lends a certain lived-in insight worth probing for a deeper and broader understanding of the phenomenon when she says:

> It's hard to break [...] and you stick to this way, but then sometimes you just realize that 'this way' doesn't work, but you still want to stick with this way because you spend so much time getting into this trap, but sometimes you have to let go of that way, this way that you think is the way, and you have to switch to another way to solve it.

In her statement, Maya, who speaks from within this circle of frustration, reveals, equally, an almost paradoxical sense of security from being within her circle of futility and also of the endless nature and subsequent desire for 'breaking out' of such a predicament. Such a tension, no doubt, is part of the lure into being *deceived* into this kind of womb of frustration.

Yet, at the same time, the image of the circle recalls the notion of the hermeneutic circle – or what might be considered the inherent circularity of all understanding – in which learning is thought to be deeply influenced by one's implicit foreknowledge that dictates to some extent what features one will discern or otherwise overlook or ignore. However, there is also an understanding that every new cycling through the circle brings oneself past one's implicit assumptions and into a new territory of understanding – "There is thus a 'circle', but not a vicious circle" (Moran, 2000, p. 277).

If viewed in this manner, the circle can almost be considered an integral part of the process of coming to an understanding. Moran comments:

> This circle is not a contingent feature of understanding, but is essential to human being as being-in-the-world [and later in speaking again of this circular structure, he suggests] a dialectical process is in operation. (p. 277)

If one way to characterize deception is as a falling into a circularity of thinking, then one might ask what draws a person into such a place to begin with, and what mechanisms are involved in finding one's way out of or through it? Yet, if entering into such a circular structure could also be construed as an indication of an essential, dialectical process in action, what might that say of the nature of mathematical engagement?

Following a second thread: pushing us over the edge

> The first function of art is exactly that which I have already named as the first function of mythology; to transport the mind in experience past the guardians – desire and fear – of the paradisal gate to the tree within of illuminated life. (Campbell, 1991, p. 243)

In all of the interviews, there existed a pervasive assumption that I, myself, had overlooked. In fact, two of the interviewees had made explicit mention of it, but somehow I had glanced over them without thought. The assumption I speak of is that of an existent external force or pressure that partially drives the problem-solving task within the individual. This force might take the form of a teacher who assigns volumes of homework with a set deadline or it could be professional pressure through an academic culture that pushes for publication, mixed in with professional pride Whatever the case, that pressure exists As Yuan, the graduate engineering student said:

> There is a struggle in my mind about whether to learn [mathematics] quickly or deeply But I think how much a person struggles relates to how much time you have I mean, if I only have this homework during the whole week, then I will spend time on it.

I, too, can recall times as a student when I felt forced to make a choice between investing what seemed to me an inordinate amount of time into a single course for deeper understanding or not doing so.

Despite its sometimes productive role in pushing learning forward, these dynamics or external pressures we speak of can push too strongly or inflexibly at times. When the pushing is indeed too stiff, one more easily falls forward, whirling into a hole of confusion, a trap of sorts, a depressed attractor, into a circle of frustration. Whether external forces push us or we have internalized such pressures by becoming *driven* the results are likely to be similar.

Interestingly, being *driven* carries with it a spirit of not being authentically engaged with a task. The word derives from the verb, *drifan*, meaning "to push from behind" (Harper, 2000). Specifically, it denotes something distinct from the idea of being motivated from *within*. It possibly implies a mechanistic view of the individual, or for that matter an individual who is being manipulated by something other than the internal, or authentic self, for automobiles are *driven* (in particular, by people), as is other machinery. We also say, that "such and such is *driven* by greed" or by competition, or other such dehumanizing forces.

We might think of being *led* as its antithesis To lead finds its derivation in the Old English verb *lædan* ("cause to go with one") (Barnhart, 1995, p. 425), implying a presence of another. The word also carries another meaning, that of "a guide" A guide is different from someone pushing Both meanings of *lead* can be said to involve a *presencing* of the learner, or of the one who is *led*, for we are *presenced* by the caring *presence* of another.

We also often speak of "being *led* by an idea (or a hunch)", of *lead*-ership skills, not *driven*-ship or *push*-ingship skills. Many of us resist those who push us, but willingly follow those who will lead us (in productive directions), for in being led, there is an implicit choice given whether to follow the lead or not. With being pushed, one loses such a choice. In losing one's choice, one loses one's center or even selfhood on some level of one's being.

A third thread to follow: frustration as disappointment

> When you think you've wasted years in inquiry of any sort, you are thinking like a capitalist. (States, personal communication, 1996)

When we look at the other meaning of *frūstrāre* (to disappoint), we gain a glimpse into another facet of 'frustration' When I am disappointed, my wishes or expectations have been left unfulfilled, even thwarted. I have, so to speak, set myself up for a disappointment by certain expectations, which is not to say that one ought not have wishes and desires. What it does imply is that disappointment does not occur unless an expectation, or plan, has been set up. As Duane says, "Things don't always go the way you *planned*," even for the professional mathematician. A certain aspect of frustration involves disappointment in a plan that has been foiled, or *frustrated*. Maya describes her own *frustration as disappointment* when she explains, "Sometimes you feel like, 'how come if it's so easy I have to spend so much time on it,' and that's kind of frustrating too '" She finds out afterwards that somehow the effort she expends and the apparent level of the problem are not commensurate As with *frustration as deception*, there is a disparity, or irreconcilability, between oneself and the situation Similarly, Jin describes it as, "I might feel like, 'why was I so stupid?' and I might blame myself Sometimes, I might become confused, why [do] I spend so much time and still can't understand the problem."

In each of the accounts, there lingers a certain underlying assumption involved. It is that time can be wasted, that a plan can go awry Naturally, plans can go awry. We take this for granted, but it serves some use to note that there are other views such as that offered by the *Tao Te Ching*:

> A good traveler has no fixed plans and is not intent upon arriving [ ] A good scientist has freed himself of concepts and keeps his mind open to what is. (Lao Tzu, in Mitchell, 1988, p. 27)

In a similar spirit, the biologist T. H. Huxley (1860) wrote:

> Sit down before fact like a little child, be prepared to give up every preconceived notion, follow humbly to wherever and to whatever abysses nature leads or you shall learn nothing. (cited in Huxley, 1900, p. 235)

Still, the prevalent and predominant view in education in the U.S is one that appears inextricably bound to the view that time can indeed be wasted in the pursuit of learning, that a direct route to an answer is preferable to a circuitous one. Such a view might closely be related to what Sfard (1998) terms the 'acquisition metaphor' of learning This is the framework within which most teaching and learning, as well as research, takes place It conceives of learning as a process of the individual learner *acquiring*, or gaining knowledge, concepts, meaning, constructs, and representations, as opposed to the view of learning where the individual becomes a participant in a process of community (of learners) building, which Sfard terms the 'participation metaphor' of learning As she proposes:

> If knowledge is conceived of as a commodity, it is only natural that attitudes toward learning reflect the way the given society thinks about material wealth. (p. 8)

As such, notions such as *efficiency*, *optimality* and competition, which one normally finds in the study of economics, become part of the norms and values in learning.

Consequently, it does not seem so unnatural to hear students bemoaning with frustration their perceived "waste" of time It also is not surprising to find students who content themselves with only a superficial understanding of a topic and deeply resist urgings from teachers to delve deeper. If time is not to be wasted, then learning to the degree of what suffices - i.e receiving a passing grade, getting the assignment done, etc. - seems only natural, particularly when considered in conjunction with the pacing of the traditional curriculum that stresses mastery of fact after fact, skill after skill, with hardly a pause. Again, we have arrived at a certain driven quality that is fostered into us as learners

A metaphor for gathering threads

> By way of metaphor, language can take us beyond the content of the metaphor toward the original region where language speaks through silence. This path of the metaphor is the speaking of thinking, of poetizing (van Manen, 1990, p. 49)

Using the imagery of the circle of frustration/deception as well as the 'good traveler' offered in the Tao Te Ching, one way in which we might metaphorically imagine a problem- solving task is that of journeying across a plane, or terrain, from one point to another, while also being cognizant of the surrounding whirlpools of frustration [1] Moreover, as with all journeys, the point becomes not solely to arrive at one's destination but to participate fully in the passage. [2]

Mathematical problems that are to be solved exist on some intellectual or conceptual landscape, and so I think that it is fairly natural to imagine mathematical problem solving as a negotiation of that particular terrain. Mark Johnson (1987) argues that humans have very general cognitive structures, called kinesthetic image schemas that are motivated by the structures of bodily experience and in turn give structure to a wide variety of cognitive domains. Among them is the source-path-goal schema, which matches our metaphor of mathematical activity as a navigation of a conceptual terrain (others include the container schema and the part-whole schema)

Burton (1999a) notes the prevalence of such a 'geographical' metaphor in the descriptions given by mathematicians when describing the ways in which knowing is understood A few excerpts follow:

> The geographical metaphor is an intellectually reasonable metaphor because you are faced with a problem so you are here and you solve the problem and you are there - it is a sequence of journeys in that sense - and along the way you have seen some new country.
>
> You have a mental model of the way in which things are working. It is a map with lots of holes in it, fragmented, and you put bridges between two bits or you fill in extra parts of the map . . Sometimes it means that there is a little alleyway, a shortcut between two things in my map that I haven't seen before. (p 133)

Through the embracing of such a metaphor for mathematical engagement, it becomes starkly apparent how in being *driven* and becoming less *mindful* or *present* to the task at hand - or trek before oneself - one can more easily fall, or be pushed, into these circles, or episodes, of frustration. On the other hand, if one genuinely is motivated from *within* or *led* in some manner or another as I have written of, there is likely a greater *presencing* effect, and in turn a prevailing mindfulness on the part of the traveler. Such *presence* of mind is perhaps the most valuable quality we can hope to carry with us when negotiating a terrain as treacherous as that offered by a challenging mathematical problem It is *how* we authentically engage ourselves.

Duane, himself a professional mathematician, offers a practical hint as to how we as learners might more clearly situate ourselves in the mathematical landscape into which we enter. He explains how in approaching a problem, he takes notes of almost a 'stream-of-consciousness' variety:

> These notes give a sense of what I'm thinking. Essentially, first lay out the plan of attack [ ] It happens in different ways. So what is it I'm looking for, what is it I need to show So, there will be things there such as 'so now it remains to show,' much like writing a proof. And then you just spit out the work.

This manner of explicitly establishing his *place* through this 'plan of attack', fits well the metaphor of traversing a terrain. Moreover, he spells out where he needs to go when he explains, "What is it I'm looking for, what is it I need to show." These statements could be read as Duane's way of organizing, as well as clearing out, his thoughts, which would not be wrong, but in line with our previous discussion we also could interpret them as his manner of *presencing* himself into the conceptual topography underlying the problem.

Related to Duane's comments, I would like to offer a revealing excerpt from my interview with Yuan:

> Yuan: I think when I keep on thinking the problem then suddenly the idea suddenly pops out
>
> Yuichi: Suddenly? You suddenly have the answer?
>
> Yuan: Maybe not the answer, but I think that when I keep on thinking the problem then I will find out some direction.

Here, Yuan distinguishes between receiving an *answer* and gaining a sense of *direction* through his efforts. Such a description of the problem-solving process, again, seems to fit the metaphor of traversing a terrain, or we might say that the metaphor fits Yuan's lived-in experience of the process.

Some theorists (e.g. Anderson *et al.*, 1996; Greeno, 1997) debate the merits of viewing learning either as a representation, or construction, of knowledge within the cognizing mind, or as a knowing that occurs within a community of learners solely within and as part of a situated context (the cognitive and situated views of learning). Perhaps it would serve some use to re-conceptualize the whole enterprise not in terms of whether the knowledge/knowing is here or there, but in terms of how the learner actually experiences mathematical engagement. As Davis (1996) puts it:

> Our mathematical knowledge, like our language, our literature, and our art, is neither 'out there' nor 'in here,' but exists and consists in our acting. (p 79)

From such an experiential point of view - considering its earlier-noted prevalence among mathematicians in conceptualizing their own mathematical activity - perhaps there is value, and an advantage, to saying that a learner engaged in mathematical thinking *enters into* a domain, or world, of thought If we embrace this conceptualization of mathematical activity, then the emphasis shifts to whether a learner can *enter into* and successfully navigate his way within a domain that perhaps is both individually and collectively defined.

Such an approach possibly embraces both views of learning, in that participation, or an entering into, is emphasized, while also allowing for the notion of the learner coming to know, or cognitively *map out*, the actual terrain that he is encountering Through the act of embracing the geographical metaphor, we find ourselves approaching what Lakoff (1988) terms the experientialist approach to cognition, which as I will touch upon briefly in a later section may be of benefit in better conceptualizing student motivation, and in turn achievement, in mathematics.

Deepening the work: how we enter into the terrain

> Get to work. Your work is to keep cranking the flywheel that turns the gears that spin the belt in the engine of belief that keeps you and your desk in midair (Dillard, 1989, p. 11)

What is perhaps even more revealing with the above excerpt involving Yuan is his notion of how he must "keep on thinking the problem" Another interviewee, Jin, spoke of "*staring* at the problem itself for a long time," as did Pogo and Maya In each of these accounts, staring at the problem could be interpreted as a behavioral description of an internal process, or even method, of immersing oneself into the conceptual terrain underlying the problem at hand. It could indicate a kind of hypnotic and transformative manner of entering into the landscape of some mathematical world, while at the same time, leaving behind the ordinary, natural world.

Contrary to the modern-day image of the mathematician as a stiffly rationalistic, precise, and unimaginative man, such an interpretation lends credence to the ancient conceptualization of the mathematician as mystic. In ancient Greek culture, the deepest mysteries of the universe were thought to be described mathematically, and so the path of the mystic became synonymous with the study of this then secretive knowledge of mathematics. Chazan (1990) suggests that contrary to the general stereotypes:

> Uncertainty, irrationality, intuition and exploration [. .] characterize the everyday lives of mathematicians. (p. 14)

I think that the metaphor of a conceptual, or intellectual, landscape is not unique to mathematics, but what may be particular to it is the intensity of effort required to hold this conceptual environment before oneself without it collapsing into vagueness, or nothingness. From my own experience, I might say that it requires a certain patience and perseverance It requires, perhaps, a certain learned ability to approach seemingly small tasks with a certain precision. The poet William Blake (1808/1953) echoes a parallel sentiment:

> Without Minute Neatness of Execution The Sublime cannot Exist! Grandeur of Ideas is founded on Precision of Ideas. (p. 451)

Alan Schoenfeld (1994) describes the perseverance required as a:

> tolerance for hack work during the 'mucking around' phase of coming to grips with a problem. (p. 61)

In characterizing even the general act of reflective thinking (or problem solving), John Dewey (1910) points to the necessity of:

> overcoming the inertia that inclines one to accept suggestions at their face value [and a] willingness to endure a condition of mental unrest and disturbance. (p 13)

Here might be a different kind of *frustration* - one that is willingly entered into - different, perhaps, in nature from that which is based on deception and disappointment, more in line with the circularity of the hermeneutic circle, with uncertainty, with un-fixing one's views Or perhaps these are different facets of the same phenomenon of frustration If so, it may be that some can see, and in turn experience, only the deception and disappointment, while others are able to grasp the necessity or inevitability of it - even sense the potential joy in it - as part of the process of coming to know.

To the question of whether frustration can be subverted in his work, Pogo, the professional mathematician, replies:

> You just sort of live with it Most of it can't [. .] I think that it's part of the work, to be able to work your way through it It may be only one aspect, but I think that's important

Later, Pogo recounts how he stared at a problem for about three months:

> I'd sit and stare at it [.] and I would also try to calculate or write out arguments and so on [.] By the time I was done, I probably had several hundred pages of notes to myself, but a lot of it was just thinking about something that wasn't clear.

Maya also tells of the hardships involved:

> Sometimes you don't want to eat Your mom is like, 'time to eat - Lunchtime' But then you say to yourself, 'I just have to do this' and then sometimes you go to bed thinking of the problem and can't stop thinking

We might, then, ask what is it that calls forth from within these particular individuals such efforts of will, devotion, and concentration? How is it that they have not abandoned their interest or investment in the discipline, or relationship, if it is experienced as so demanding of their efforts to the point of frustration? Again, we might return to where we started, with Maya's comment, "fun, but frustrating" Somehow, she manages to hold onto the "fun" element of the experience in her recollection.

Pogo, even as he is describing an "extremely frustrating" experience, does not forget how it became "ultimately rewarding." Duane describes how in finishing a proof, he jotted in his notes, "Booyah!" to articulate his excitement and enthusiasm at a task accomplished. I, myself, have known and experienced moments of deep gratification and quiet joy in the process of immersing myself in mathematical activity in spite of enormous frustration faced along the way.

There exists, also, a body of research literature that more directly offers and describes in greater detail moments of insight - "'Aha!' experiences" - in mathematical engagement, and how they link to issues of motivation (Barnes, 2000; Burton, 1999a, 1999b; Davis and Hersh, 1981) In particular, Burton (1999a) suggests:

> So, coming to know, for my participants [all of whom were mathematicians], was represented by feelings, the powerful sense of Aha! which is what holds them in mathematics (p 135)

Perhaps in contrast to these sentiments, my conversation with Yuan, again, is most illuminating.

Yuan:	I think that this is the main problem with asking others to help me because I'm not sure, but I think the only way to learn something is to keep on thinking about this problem or read some materials in this subject. Yes, I admit that when I do ask my friends instead of thinking these problems, this will save me a lot of time, but I don't think it's good for me.
Yuichi:	But you still do it.
Yuan:	[...] when I try to get my knowledge by talking to people and discussing about these problems I always feel - how do you say? - it's a little bit [...] there is always a feeling of uncomfortable[ness] for me, because I will think, "Oh this knowledge that I get from people doesn't really belong to me"
Yuichi:	Because they just gave it to you?
Yuan:	Yes Maybe it's because I didn't really struggle to get deep into the parts.
Yuichi:	So struggle is important?
Yuan:	Yes.

For Yuan, there is a sense that something is missing. There is not personal fulfillment as with the others He has not *struggled*. He has not infused his activity, perhaps, with enough *care* The conversation with Yuan does not imply necessarily that learning from others in itself is faulty, but perhaps if it is not preceded by some initial struggle, or at the very least care, then what follows does not carry personal, or existential, meaning and instead remains a kind of abstraction.

I return to Nietzsche's (1887/1974) observation:

> But what if pleasure and displeasure were so tied together that whoever *wanted* to have as much as possible of one *must* also have as much as possible of the other? (p. 85)

Even if such a view were to be an over-simplification of the phenomenon of mathematical engagement, it perhaps carries in it some truth of the meaning of the experience. It points to some tension that exists in the lived-in experience for the practitioner. It tells us, also, that it may not necessarily be the purely cognitive facets [3] of learning mathematics that become the primary stumbling blocks for learners, but that the affective, embodied, and experiential aspects of the learning process may in fact be more fundamental in the experience for the learner and hence primary in deciding whether a student will find meaning, and even success, in the study of mathematics or not. In support of such a notion, Gattegno writes:

> This requires that we recognize that problem solving is not essentially an intellectual activity [...] the finding of the solution is a much more complex travail in which affectivity is the force that keeps us working on the problem and gives us the stamina to continue working on it. 'Staying with the problem' is the most important feature of problem solving, and yet one that is almost never discussed. (Gattegno *et al.*, 1981, p 43)

In approaching what is no doubt an 'old and used' metaphor in the *mathematical landscape,* we might ask, what is to be gained through embracing such a metaphor as a model of mathematical engagement? Likewise, what implications might the preceding comments, in particular on the nature and role of frustration and struggle in mathematics learning, have for educators and for researchers in the field?

Pedagogical implications made explicit

> One learns to know only what one loves, and the deeper and fuller the knowledge is to be, the more powerful and vivid must be the love, indeed the passion. (Goethe, 1963, p 83)

When it comes to the phenomenon of *frustration as deception/disappointment,* if we begin with an acknowledgement and acceptance that struggle, and possibly even frustration, may ultimately be an intrinsic part of the experience of mathematical activity [4], then it becomes clearer to see that what sets apart those who are successful from those who are not may not necessarily be that the former persevere through their frustrations, for as we have seen even the most accomplished will pull away from problems What does seem to separate them is that they *return to the problem*. Put in another way, we could say that those students who do not return to the scene of their struggle are likely those who do not find success with mathematics.

As educators, we will likely not find success ourselves by creating a culture that avoids struggle, or even frustration, when it arises. What will be a more productive direction to take will be to acknowledge its likely inextricable link to mathematical engagement, prepare the student for it, convey to the student the potential excitement and power in accomplishment that lie on the other side of these episodes of frustration, and somehow find ways to invite the discouraged student back into the task of engagement, again, through our lead, or caring presence

On the issue of time raised by the discussion on *frustration as disappointment,* I find it telling that most students are given few, if any, opportunities for enjoying the fruits of their labor in mathematics education. As an aspiring/recreational pianist, I have noticed that my development as a musician involves an alternating dance between periods of work, toil, and growth, and interludes where my abilities seemingly plateau while I simply revel in the enjoyment of playing. Though my lived-in experience in these latter phases is primarily one of enjoyment, I am aware that there is some consolidation of learning taking place also.

While in such a stage of enjoyment, I eventually become aware of the limitations of my abilities, and thus the urge for further growth incites within me the discipline required to continue to build upon my abilities. In mathematics education, students rarely get to "take a breather" in their accumulation of knowledge, or acculturation into the discipline. I do not urge for allowing students to disengage entirely from mathematical studies. Instead, I am proposing something akin to this notion of allowing for a period of enjoyment and consolidation of abilities, in place of what is more commonly found in our schools, embodied by a constant pushing forward of the learner's amassing of knowledge. As it is, the average student will likely never come to feel any sense of mastery, much less enjoyment, but will instead, perhaps, come to feel a certain inadequacy or deficiency in their abilities

Taking a slightly different approach to this notion of not rushing one's learning (particularly in the acquisition of skills), Hiebert *et al.* (1996) have proposed a curriculum that 'problematizes' mathematics. The heart of this approach resides in Dewey's (1929) philosophy on problem solving, in which *doing*, or *engagement*, is held central in coming to know and to understand. Thus, rather than having students mastering skills and then applying them, students are encouraged to be engaged in formulating and resolving problems.

Implicit in this approach is Davis' (1992) notion of mathematical understanding as the 'residue' that remains following mathematical activity, which strikes me as being harmonious with the geographical metaphor of learning. Through active engagement, students are thought to gain and develop insights into mathematical structures, strategies for solving problems, and even dispositions toward mathematics as a kind of by-product of their engagement. Also implied within this approach is a valuing of the *process* of question posing and the subsequent activity involved in reaching a solution, as opposed to the *end product*, or acquiring of skills.

To return to our original question, we might say -without intending to be clever or redundant - that the meaning in mathematical engagement is to be found in the engagement itself. It is, thus, in the act of immersion - through a process of *involved care* and *embodied struggle* - that one finds meaning, for through it, one enters into a relationship and connects with and into a world of knowing. From a slightly different perspective, one might say that by becoming connected to another, one transcends the self. As Frankl (1959) suggests, personal meaning, or "self-actualization is possible only as a side-effect of self-transcendence" (p. 133) The meaning is found, I believe, through a kind of self-forgetting - or as Davis (1996) puts it "a dissolution of static notions of the self [that permits] a re-membering of intersubjective awarenesses" (p. 38) - which can occur through a committed engagement with mathematical activity.

Notes

[1] How the landscape comes to be, whether it exists external to oneself as fixed and determinate, or whether its form and structure are altered and even constructed by one's knowing is not an issue I have taken up here in this paper. If the reader is interested in pursuing the notion of the landscape, or pathway, as being formed in the walking, rather than being pre-structured, Davis (1996, pp 83-100) presents an interesting and insightful perspective on this through viewing mathematics curriculum as *currere* ("the running of the course").

[2] The notion of problem solving as a metaphorical path leading from one state to another might be considered one of the oldest clichés around (Lakoff and Turner, 1989; Holyoak, 1990), yet most have mined this metaphor for efficiency and effectiveness in getting from a starting point to an ending point (i e , solution), namely the cognitive features of problem solving. The emphasis that I have taken up here, as will be seen, is around the usefulness of the metaphor in reconciling the affective, embodied, and even quasi-mystical aspects of problem solving with the cognitive For an in-depth discussion of this same 'environmental' metaphor in the context of number sense, and its reconcilability with situated cognition, see Greeno (1991)

[3] I mean here, cognitive facets as traditionally conceived Some theorists (Maturana and Varela, 1987; Varela, Thompson and Rosch, 1991) conceive of cognition as including the affective and embodied facets of one's being as well

[4] Of course, in anticipation of such difficulties, a prepared teacher can gauge the level of the problems he poses so that they remain challenging without being overly frustrating, but he will not always so precisely assess the actual abilities of the students, nor will he be cognizant of how a problem relates to that level of development within the student on every occasion Yet, he cannot completely refrain from posing problems for fear of stirring up frustration in his students Instead by becoming familiar with the range of struggles and frustrations, perhaps, as inherent within the discipline and learning better how to address them as they arise, he will be a more complete teacher. As van Manen (1990) writes, a critical pedagogical competence involves "knowing how to act tactfully in pedagogic situations on the basis of a carefully edified thoughtfulness" (p 8)

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