Pour Lire Brousseau [1]

PATRICIO HERBST, JEREMY KILPATRICK

> *On a boat, there are 26 sheep and 10 goats What is the age of the captain?*
>
> *Ana Sofia: One hundred and one dalmatians. (4 1/2 years)*

Academic discourse in education, certainly in North America, is commonly assigned to categories such as curriculum, instruction, learning, assessment, evaluation and policy These categories seem natural, allowing scholars to identify their fields of expertise and colleges of education to form departments named according to the services they provide As a by-product, these categories readily convert into academic discourse the natural discourses used in educational practice. For example, *instruction* refers to teachers and teaching, *curriculum* to textbooks and content, *psychology* to learners and learning, and *measurement* to testing. One can imagine the practice of mathematics education being approached differently, however, and Brousseau does take a different approach

A first question that arises when reading Guy Brousseau's *Theory of Didactical Situations in Mathematics* is: what is he talking about? The possibility of answering such a question sensibly requires that one discuss the frame within which it can be formulated Michel Foucault confessed that he learned this lesson from Jorge Luis Borges's (1952/1974) bizarre taxonomy of animals:

> In the wonderment of this taxonomy, the thing we apprehend in one great leap, the thing that, by means of the fable, is demonstrated as the exotic charm of another system of thought, is the limitation of our own, the stark impossibility of thinking *that*. (Foucault, 1970, p. xv)

The uneasiness with which ordinary academic discourse in education can accommodate Brousseau's theory is obvious: it is about learning, and about teaching, and about mathematics, and about their social conditions. Yet each time one adds an 'and', one would also like to add a 'but not ...'

Such uneasiness is actually symptomatic of a basic merit of Brousseau's work – the way it challenges the naturalness of the 'natural' correspondence between academic discourse and educational practice and, as a consequence, the way it provides a different frame to think *with* Still, the desirability of such a challenge can only be seen after the fact by asking, like Foucault: what is that *that* that one can now think *of*?

One could respond to this argument by saying that it is precisely the existence of a correspondence between sectors of educational practice and fields of academic discourse through which the existence of academic discourse in education is justified. In fact, the development and improvement of the social project of education is a common end that scholars and practitioners pursue. This common interest, along with the implicit belief that what is good for some is good for all, arguably guarantees that the combined efforts of all sectors participating in educational practice will yield both an efficient social practice and a coherent frame within which to talk about it

That way of thinking about the relationship between academic discourse in education and educational practice, however, is more akin to thinking about building a single house than to thinking about house building as an activity. It may provide a handy set of tools for solving educational problems yet leave out structural and functional educational problem posing. The discourse of mathematics education developed historically, however, at least in the United States, in a context that favors such thinking: the social imperative on the efficient action of each sector of educational practice has naturally led to a specialization of academic discourses in education by curriculum area.

The emergence of problems and theories in mathematics education has occurred as if the move were from genus to species – the adaptation of practical but general discourses to concerns related to a specific endeavor to which those discourses were assumed to apply. For example, one is more likely to speak of using *the* 'discovery method' in the teaching of mathematics than to question whether discovery in the mathematics class is the same as in the history class One is more inclined to ask what a learning theory says about the learning of mathematics than whether the verb *to learn* can take different objects without becoming ambiguous.

Those observations may be obvious, but it is important to make them explicit. As Brousseau sees it, the specific move made by *didactique* in the game of academic discourse takes it away from the assumption of a one-to-one correspondence between fields of discourse and sectors of practice. The distinction exists not at the level of declared means and ultimate uses: both mathematics education and *didactique* of mathematics aim at understanding and improving the process and outcomes of education Instead, it exists at the level of the construction of an object of study, which in particular influences what the verb *understand* means and what features can rationally be *improved*.

The construction of an object of study

The age of the captain and the didactical contract

The rational construction of an object of study has some advantages over the generalization or the abstraction of such an object of study from practical experience (Bachelard, 1949, pp. 1-11; Herbst, 1998, pp. 15-24) Brousseau (1997, Ch. 6) illustrates the contrast in his analysis of the famous 'age of the captain' problem:

> On a boat, there are 26 sheep and 10 goats. What is the age of the captain?

Answers that students give, such as 36 years, might at first glance seem to warrant their teachers' censure, as well as Stella Baruk's (1985) and other educators' concerns about the teachers' failure to resolve such problems. Students' responses might be deemed meaningful by some but will obviously be interpreted differently by different people. Depending on one's role and position in the educational process, one might offer any of a number of conjectures as to what causes the students to respond as they do (e.g. if the students add sheep and goats to make years, it is because their teachers did not tell them that one adds homogeneous quantities only).

In contrast, the theoretical notion of *didactical contract* permits one to explain the reason for the existence (but not necessarily the material cause of the existence) of the students' responses and to account for their meaning, if any meaning should be attributed to them The students' responses are a plausible consequence of a break in the didactical contract associated with elementary word problems (in which students are supposed to use key words and the relative nature of the numbers given as a heuristic to separate relevant from irrelevant information - i.e. to find the operation 'hidden' in the problem). Aberrant responses are observed because the problem and the conditions under which it is posed violate the conditions students quite reasonably expect from a school problem In school, students are accountable not just for solving a problem but also for using what they have learned in school about solving word problems to solve it. Teachers are not supposed to ask for an answer that cannot be obtained with the information given, and students must somehow use all the numerical information given to obtain an answer (Chevallard, 1988).

The explanation in terms of a didactical contract does not assign blame, nor does it provide a ready-made way to remedy the situation. Rather, it treats the results of the age-of-the-captain problem as artifacts: no matter how "good" the teaching or the students, there will always be a way to produce such scandalous results from an assessment. If the age-of-the captain responses mean anything, they indicate that students' ways of knowing result from adaptations to a situation and its regulations. Any failure is not due to a psychological deficit on the students' part, an instructional problem on the teachers' part, nor a logical problem stemming from the content Instead, it is an epistemological problem that can be understood only by questioning the compartmentalization between curriculum, instruction and learning. As Brousseau writes:

> Students' reasoning is formed by a collection of constraints of didactical origin which modify the meanings of their responses and those of the knowledge they are taught. These constraints are not arbitrary conditions freely imposed by teachers; they exist because they play a certain role in the didactical relationship. (p 264)

A unitary and systemic approach to the practice of mathematics education

Do the learning and teaching of mathematics have to be placed within a special context in order to be studied? Are not general knowledge, common sense and innate pedagogical skill enough to produce a good mathematics teacher?

> It appears that a good epistemological theory [. .] is essential for answering these questions. *Didactique* studies the communication of knowledge and theorizes its object of study (Brousseau, p. 24)

Didactique attempts to furnish a *unitary* and *systemic* account of the phenomena associated with the production and circulation of mathematical knowledge (Brousseau, 1990; 1997, pp. 47-75). First, *didactique* represents a unitary approach because it constructs an object of study from a point of view that neither assumes nor seeks partial congruence with any views held by the (individual or institutional) sectors involved in the practice of mathematics education. Instead, it seeks to explain those views as they relate to the knowledge being managed and its phenomena.

Second, *didactique* represents a systemic approach because its discourse concerns the structure and function of a set of relations between subsystems (participants or institutions) and not the composition of those subsystems or the primary causes of their relations. Note that modeling the practice of mathematics education as a *system* does not require assumptions about an image of that reality (e.g. it does not say that the practice of mathematics education resembles a machine) Instead, the modeling refers more to products of the discourse that account for the reality (that show it to be plausible or rational; see Chevallard, 1992)

The easy association of *systems* with simple mechanical or biological systems is a reification that Brousseau (p. 53) is quick to denounce To understand the roles that Brousseau's models and metaphors play in *didactique*, one must be aware of the dangers of identifying rational conclusions based on a model with empirical descriptions of a reality. The relations spelled out by a systemic discourse are not necessarily (and should not be identified with) the material causes of an observed phenomenon. In physics, for example, the equation for the movement of a falling body does not explain all aspects of that fall, including its causes. A model need not represent reality to produce results consistent with that reality, and thus prove helpful in understanding some aspects of it

This last point is especially difficult for mathematics educators to understand because of the roles they play (they are, at least occasionally, mathematicians or teacher educators or curriculum developers or evaluators). They act in environments whose common lore includes some cause–effect models that keep the educational project together: for example, teacher educators are needed to prepare teachers, teaching causes learning and curriculum reform is needed to improve practice A unitary and systemic account may thus run counter to educators' situated experience and reflective knowledge for the sector of practice in which they participate and the often-ideological visions of the whole that they develop

> Under the name *didactique* has emerged an attempt to construct a science of the communication of [mathematical] knowings and their transformations: an experimental epistemology that aims at theorizing the production and circulation of knowledge just as economics studies the production and distribution of material goods. [*Didactique*] is interested in what these phenomena [the production and circulation of knowledge] have that is specific to the knowings at stake, [. .] the essential operations of the diffusion of knowings, the conditions of this diffusion, and the transformations this diffusion produces either in those knowings or in their users. (Brousseau, 1990, p 260; our translation)

The question remains as to whether the pursuit of a unitary and systemic account of the practice of mathematics education is warranted: is it appropriate for mathematics educators to engage in this effort? The history of economics may provide an analogy It developed from the practical concerns of merchants into a science of the production and circulation of capital. Later, it was able to suggest, monitor and explain changes in national and international economies. This history shows that a theoretical move is not necessarily a luxury or a diversion of precious resources; instead, it can be the investment a civilization makes for its own survival Unfortunately, history also shows that the value of such a pursuit can only be shown after the fact Given its history thus far, it appears that Brousseau's *didactique* marks the birth of a basic (as opposed to applied) scientific perspective for understanding the generation and diffusion of mathematical knowledge Moreover, one can say that this perspective is autonomous, because unlike the work of most researchers in mathematics education, *didactique* does not borrow its object of study from psychology (or for that matter from any other discipline): *didactique* constructs its own object of study.

Basic elements in this change in perspective

Didactique aims to turn some empirical facts into phenomena: that is, to provide a rational explanation that can make those facts meaningful (Margolinas, 1998) *Didactique* looks into those social practices resulting in the production or diffusion of mathematical knowledge as the source of facts needing to be explained. It furnishes an *ad hoc* conceptual apparatus that can accommodate some of those facts and explain their plausibility or necessity in relation to the management of the knowledge at stake.

With respect to the facts associated with the age-of-the-captain problem, for example, the notion of didactical contract explains not only why a student's aberrant answer is predictable, but also, as Chevallard (1988) observed, why students seem to act using two conflicting logics. For example, given the problem "On a boat, there are 36 sheep; 10 fall into the water. What is the age of the captain?", a student may respond "26". The student, when asked to comment on the problem, says, "It's all right, but I don't see the relation between the sheep and the captain" (p. 16)

The notion of didactical contract allows one to restore rationality to students' behavior. Instead of asking who is responsible, one describes a phenomenon associated with those conditions in which school mathematics incorporates real-world contexts into the teaching of elementary arithmetic. Because word problems must be solved using the mathematics that has been officially learned, real-world contexts are chosen to contain indicators enabling students to ignore context and identify relevant mathematical information: for example, key words or the relations among the numerical data (e.g in the problem just quoted, the information that some sheep fall into the water is a clear indication for the student to 'take away').

The task of the student is not one of using mathematics to make sense of the situation, but rather one of suppressing the situation so as to find the mathematics (i e. the hidden computation, known to be there). The results relating to the age of the captain become meaningful within an epistemological explanation of school mathematics (as a system of public ways of knowing) and not within the usual psychological explanations of students' or teachers' personal knowledge

The example of the notion of didactical contract and the age-of-the-captain problem helps identify the elements needed for a theory of how mathematical knowledge functions in the social project of education A fundamental assumption entailed by the disturbing results of the age-of-the-captain problem is that school mathematics should equip students to function mathematically in non-school situations. The question of meaning becomes central: *studying* mathematics aims at more than being able to repeat a school text (whatever that text might look like) It also (and crucially) aims at students being able to function in situations where the meaning of the mathematics studied is called for or could be called for.

This point is not foreign to North Americans, who owe the existence of the National Council of Teachers of Mathematics to the pioneering efforts of those who advocated the need to "teach for transfer" and sustained (against complaints from general educators) the fundamental role of mathematics in understanding the world (see Fawcett, 1938; Judd, 1928; Kilpatrick, 1992; Stanic, 1986) The theorization of the practice of mathematics education proposed by Brousseau is based on the dialectical relationships between the production and reproduction of knowledge (as cultural capital) and the production and reproduction of meaning (as situated practice).

A first distinction to be made is between an item of mathematical knowledge and the circumstances of its production (as a meaningful solution to a problem, historically and personally situated; see Brousseau, pp. 21-24, 90, 100-107) Civilization today possesses a somewhat objectified mathematics that can be visualized in the form of a system of textual practices (with its symbolism, rhetoric, channels, positions, etc.). That system represents the items of knowledge that a culture can designate, generically or by listing them, to be taught and learned. What Brousseau calls the "work of the mathematician" (p. 21) – to *decontextualize, depersonalize,* and *detemporalize* his or her discoveries in order to make them part of 'mathematics' – points to the dialectic between production of knowledge and production of meaning. This dialectical opposition between knowledge and meaning marks the work of the mathematician, and it is well captured by Davis and Hersh's (1981) observation that: "the typical working mathematician is a Platonist on weekdays and a formalist on Sundays" (p. 321).

The metaphor that mathematics has a text of knowledge works only so long as one recognizes that its productive aspect is not to identify the mathematical text but the practices that produce and control the process of *textualization* (see Chevallard, 1991, p. 65-69). The point is not that there exists one text but that when mathematicians show their work, they are bound to "plug into" a mode of textualization (and a virtual text). They follow customary rules (e.g. search for the most general way to define or the most elegant way to prove) which seldom reconstruct or portray the meaning that a notion had for its discoverer. The distinction is well captured by the lexical distinction between the French nouns *connaissance* and *savoir* – a difficult distinction to make in English, where both words are ordinarily translated as *knowledge* (see Brousseau, 1997, pp. 23, 72, 235; Foucault, 1972, pp. 178-195; Herbst, 1998, pp. 34-38). In Brousseau's book, *connaissance* is translated as 'knowing'.

A second basic distinction is between society's demand for the transmission and acquisition of a piece of knowledge and the conditions for a meaningful accomplishment of this transmission and acquisition (see Brousseau, pp. 22, 30, 229). *Didactique* draws on Piaget's constructivist hypothesis which asserts that:

> pupils construct their own knowledge, their own meaning [...] as a necessary response to [their] environment (Balacheff, 1990, p. 259)

Didactique uses this constructivist hypothesis to problematize the relation between the knowledge to be transmitted and the possibilities for teachers and students to acknowledge (or monitor) the meaningfulness of this transmission. Unlike many educational outgrowths of constructivism, *didactique* does not turn the constructivist hypothesis (which concerns how personal knowing comes about) into a pair of handcuffs to shackle some forms of pedagogy (like direct instruction) or into an indictment of society's expectations for cultural reproduction.

The constructivist hypothesis is instead used as a tool to find the possible meanings that the learner may be attaching to a declared piece of knowledge being taught, given the characteristics of the situation in which the transmission takes place. In experimental research, the constructivist hypothesis is used to choose situations that are likely to produce perturbations in the knowing schemes of the learner and that afford accommodation of those schemes in ways compatible with the knowledge to be constructed.

The value of Brousseau's notion of the *adidactical situation* (see Brousseau, 1997, pp. 29-31; Kieran, 1998, pp. 596-597) – a situation in which the student takes a problem as his or her own and solves it on the basis of its internal logic and not in light of the teacher's guidance and direction – does not just lie in the ideological advantage of devolving to the student the responsibility for working actively to solve a problem that is meaningful to him or her. Adidactical situations are valuable because they signal the epistemological importance of the learner's *milieu* (which for the learner does not have didactical intentions). Within and against this *milieu*, the learner's activity may evolve so as to produce (by assimilation and accommodation) knowings that may eventually lead to a valid institutionalization of the target knowledge.

Adidactical situations do not provide a naturalistic paradise enabling children's 'free' knowing: rather, they offer conditions and constraints to ensure the meaningful production of knowings that entail a cost – they emerge as the solution to a problem. The basic postulate of the theory of didactical situations, which has guided the epistemological search for 'good problems' in Brousseau's (and others') experimental work, is that:

> each item of knowledge can be characterized by a (or some) adidactical situation(s) which preserve(s) meaning. [...] These adidactical situations arranged with didactical purpose determine the knowledge taught at a given moment and the particular meaning that this knowledge is going to have. (p. 30)

In fact, Brousseau warns against naturalistic simplifications when he comments on some educational applications of Piagetian theory.

> By attributing to "natural" learning what is attributed to the art of teaching according to dogmatism, Piagetian theory takes the risk of relieving the teacher of all didactical responsibility; this constitutes a paradoxical return to a sort of empiricism. But a *milieu* without didactical intentions is manifestly insufficient to induce in the student all the cultural knowledge that we wish her to acquire. (p. 30)

The object of study of *didactique* is constructed on a terrain marked by reciprocal tensions among three subsystems. They are:

(a) the knowledge to be transmitted and acquired;

(b) the knowings that emerge in the interactions between the subject and his or her *milieu*;

(c) the educational project that binds teacher and students into a relation demanding the reproduction of knowledge and simultaneously the production of meaningful knowings compatible with that knowledge.

Didactique can thus be understood as the study of the transformations of mathematical knowledge within the conditions and constraints of the social project of education. As Brousseau writes:

> The agreed effort of obtaining knowledge independently of the situations in which it is effective

(decontextualization) has as a price the loss of meaning and performance at the time of teaching. The restoration of intelligible situations (recontextualization) has as a price the shift of meaning (didactical transposition). The retransformation of the student's knowledge or of cultural knowledge takes up the process again and heightens the risk of side-slip. *Didactique* is the means of managing these transformations and, first, of understanding their laws. (p. 262)

As a program for research on the practice of mathematics education, *didactique* is thus characterized by an epistemological perspective focused on the knowledge that is at stake in that practice. The view taken of the knowledge at stake blurs the lines between the usual categories derived from practice, such as curriculum, pedagogy and learning.

The difference between *didactique* and the traditional division of academic labor can now be fully understood. As noted above, the traditional division is built on the reality of the various sectors of practice. Each field inherits a perspective that is fundamentally colored by its own role in the practice that it purports to study. As a result, the traditional division of labor is ill-equipped to deal with some of the problems that a perspective on the knowledge at stake can pose.

For instance:

- What is the implemented curriculum?
- What is effective mathematics teaching?
- What do student assessments mean?
- How do students learn mathematics (as opposed to how they learn to speak their mother tongue) – a distinction built into the etymology of the word *mathematics* – see Bochner, 1962, p. 22-28)?

Metaphors and models useful to didactique

Within the frame of applied rationalism (Bachelard, 1949), *didactique* aims to theorize its object of study and produce empirically verifiable propositions. The table of contents in Brousseau's book contains several metaphors which are used for that purpose: for example, the first two chapters contain 'the didactical contract', 'the paradox of the actor', 'the notion of "game"' and 'obstacles and didactic engineering'.

An explication of how each metaphor helps *didactique* account for its object of study is beyond the scope of this article. We do, however, have some comments on how to understand them. It may be helpful to notice that these metaphors are not to be taken as passive and accurate portrayals of a reality, but as active tools to interpret that reality: metaphors or models are productive not because they represent something accurately (otherwise, why not just use the thing itself and forget about the metaphor? – see Eco, 1992), but because of their distance from the reality to which they refer. They afford plausible conjectures that it may not be possible to formulate from a look at plain reality alone (see Black, 1962, 1979; Chevallard, 1992)

Engineering what?

In his discussion of the object of study of *didactique*, Brousseau makes the following remark:

> It appears that a good epistemological theory accompanied by good didactical engineering is essential for answering these questions. (p. 24)

The French linkage between teaching and engineering disturbs many North American educators; especially those who connect the term 'engineering' with the connectionist-based factory model of learning. What are the productive aspects of engineering that are being called upon to inform the object of study of *didactique*? Do the basic premises of *didactique* actually allow one to think that it is a matter of engineering learners – in the sense of mass production of bonds, cognitive schemes or whatever general image of knowledge is current among educational psychologists?

The second question can be readily answered in the negative. *Didactique*'s focus on knowledge authorizes one to speak about engineering only the situations in which (an item of) knowledge is at stake rather than the psychological development of the actors. Reflection on the role of the learner in a didactic situation may help answer the first question and yield more light on our answer to the second.

The actions in which a student is involved will help the student give meaning to the knowledge he or she is said to have acquired. But that meaning is not necessarily legitimate for that piece of knowledge. As the age-of-the-captain problem shows, the meaning can be based entirely on the didactical association between questions and answers: students decide what operation to use according to the reciprocal relations between the numbers given, so if the two pieces of information are somewhat alike, "you ought to add". The observation of naturalistic situations affords didacticians the possibility of conjecturing what the meanings available to the student might be, but only experimental situations provide the opportunity to study whether it is possible to afford meanings that are more mathematical than didactical.

It is an engineering problem to design, regulate and make controlled observations of those experimental situations in which the publicly available meanings are to be optimized according to some criterion. But it is an engineering problem that does not deal with the finished learning product (if anything like that could be spoken about). Whatever an individual student actually learns (or demonstrates having learned) is, quite understandably, only partially accounted for by the situation in which he or she has had the opportunity to learn it. (And there is little more that the practice of education can do beyond improving the environments in which people learn. One can neither inject the knowledge into the learner nor mandate or seduce the learner into learning it.)

Therefore, didactical engineering does not deal with "producing a student", nor even with "producing the personal meanings that an individual student actually constructs". Didactical engineering deals with the production of the possible or available meanings of a student's activity – the actual opportunity to learn rather than the actual learning. In fact, didactical engineering's focus on the optimization of the meanings available in a situation helps put in its proper place the "successful" learning resulting from the "effective teaching" model (Brophy, 1986). It emphasizes that the criterion of success cannot be the adequate pairing of test questions and good answers if the nature of the available possibilities for the student to produce those answers is not a part of the equation (see Confrey, 1986).

As a consequence, one can also see that although *didactique* is not reducible to psychology, it does not aim to replace psychology either. That is an important point to emphasize, because, unlike situated cognition, *didactique* provides not a shift in focus on human learning but a perspective on a different problem.

Various situations, various games

Brousseau uses the metaphor of a game to model the functioning of a didactic situation:

> Modeling a teaching situation consists of producing a game specific to the target knowledge, among different subsystems: the educational system, the student system, the *milieu*, etc. [...] The game must be such that the knowledge appears in the chosen form as the solution, or as the means of establishing the optimal strategy. (p. 47)

The game metaphor portrays the learner as a player who is motivated to play the game (enter the adidactical situation, agree to solve the problem) for the game's sake (its mathematical interest). At the same time, the fact that games have rules allows one to see that any learner's actions must conform to constraints that determine which strategies are legal and may even result in a win (i.e. what knowings are well adapted to attacking the problem and possibly to solving it). Brousseau (pp. 48-54) presents an extremely interesting analysis of the extent and limits of this correspondence between situations and games.

At stake in this game between the learner and his or her *milieu* is an item of mathematical knowledge. The learner can win that knowledge by developing mathematical knowings that will give meaning to it. But what does losing mean in this case? In what sense can mathematics be *lost*? The game metaphor raises this challenging question that otherwise would rarely be posed.

The fact that a student may agree to play the game is no guarantee that he or she will win it. But it is the teacher's responsibility that the student should nevertheless acquire the item of knowledge he or she is supposed to learn. What can be lost is the validity of the moves the player is allowed to make in order to win the stakes (as though a gambler were allowed to arrange the deck of cards before dealing), in this case the meaning of the knowledge transmitted and acquired. The Topaze and Jourdain effects described by Brousseau (pp. 25-26) show how this "losing of the mathematics" can occur. For example, in the Jourdain effect, named after the character in Molière's play *The Bourgeois Gentleman* whose teacher lets him believe that he is speaking prose, the teacher takes an innocent observation by the student and interprets it as a meaningful mathematical text. About the Jourdain effect, Brousseau says:

> In order to avoid debating knowledge with the student and possibly acknowledging failure, the teacher agrees to recognize the indication of an item of scientific knowledge in the student's behaviour or answers, even though these are in fact motivated by ordinary causes and meanings. (p. 26)

In as much as the student may win a game that is no longer a mathematical game, these effects show that the mathematics at stake can indeed be lost.

The teacher, an actor on the stage

Some advocates of reform have suggested that to promote the authenticity of the student's efforts to solve a problem, the teacher should not know the solution. Within the theory of didactical situations, Brousseau poses a related question:

> Can the teacher escape [...] the direct intention of teaching some particular knowledge? Can she escape the didactical situation? After all, it would perhaps be enough for her to be a mathematician and to behave as such in front of and with the student. [...] Can the didactical system be envisaged without teachers? (p. 45)

Societal demands on the teacher require that such original activity be severely limited in time and quantity. (Imagine how parents and administrators would react to hearing that "not even the teacher knows the answer".) Moreover, in a broad sense such originality is impossible: the teacher knows which questions are interesting to answer and which ones can reasonably be attacked – the teacher knows at least half of what there is to know. In fact, as Brousseau remarks: "the teacher is an actor – with or without text" (p. 46).

Brousseau draws on Diderot's 'paradox of the actor' to state something similar about the teacher, as follows:

> The more the actor feels emotions he wants to display, the less he is able to allow the audience to share the feeling because, being a "continuous observer of the effects that he produces, the actor becomes a sort of spectator of spectators as well as being what he is himself and can thus perfect his game." [...] If [the teacher] produces her mathematical questions and answers, she deprives the student of the possibility of acting. She must therefore ignore time, leave questions without answers, use those which the student gives her and integrate them into her own process by giving them a larger and larger place. [...] If the knowledge is determined in advance, this "liberty" becomes nothing but an actor's performance, and the student is invited to be another actor, restricted to a script, or at the very leas an outline which she is not supposed to know about. (p. 46)

It is therefore impossible to think of the practice of mathematics education as devoid of any intention to reproduce knowledge. An important outcome of the actor metaphor is that it uncovers the epistemological deception (and political conservatism) underlying educational forms of radical naturalism (see also Brousseau, pp. 267-269).

The metaphor of the teacher as an actor enables one to think of the didactical situation as (no more than) an actor's script. Moreover, it enables one to see the role of the teacher as impossible to replace by any sort of carefully developed instructional tutorial or teacher-proof curriculum. As Brousseau indicates:

> In theatre, the reciprocal responsibilities of the actor and the author have been regulated since Shakespeare, Molière and Diderot. (p. 268)

One of the advantages of *didactique* is that it provides the scientific means to recognize the substantive and irreplaceable role of the teacher in students' learning of mathematics.

The legal force of the didactical contract

As noted above, the responses to the age-of-the-captain problem occurred because the problem posed to the student broke the didactical contract established around the practice of solving arithmetic word problems. The metaphor of a contract leads one to postulate that the reciprocal relations between teacher and student with respect to the mathematics to be learned are regulated as though a legal contract were in force. The metaphor is useful in explaining the 'rationality' of some observations of apparently aberrant student or teacher behavior (see examples in Brousseau, pp. 25-29), in contrast to the rather censorious explanations common in education (that blame either teachers, students, textbooks or even society). The metaphor allows one to see that the imperative to maintain the didactic relation against all odds imposes on teacher and students alike the need to negotiate their responsibilities permanently regarding the knowledge at stake. As in a legal contract, the actual value of the goods and services being contracted may well differ from their nominal value.

Yet the contract is always implicit. In a sense, as Pimm (1988) says: "The last thing that a *contrat didactique* is is a contract (because it is completely tacit and unspoken)" (p. 33). In fact, a reification of the theoretical notion seems to have led some to think that technical sophistication could help a teacher establish a *good* didactical contract. But making the didactical contract explicit would break the didactic relation. It would explicitly rule out the existence of two distinct "values" for the goods and services contracted (that is, it would either disable any negotiation about what is to be done, keeping its nominal value constant, or would explicitly change the value of what is to be done). As Brousseau indicates:

> The theoretical concept in *didactique* is therefore not the contract (the good, the bad, the true, or the false contract), but the hypothetical *process of finding a contract*. It is this process which represents the observations and must model and explain them. (p. 32)

The existence of a contract permits the teacher to ask that the student engage his or her own meanings and demands that the teacher identify knowledge in the students' actions. Conversely, the contract enables the student to negotiate the task and to receive an acknowledgment of the status of his or her activity. Thus, the contract is a contract in the sense that it regulates what can be done (from a legal point of view), even if the clauses themselves are implicit. The contract becomes visible when transgressed, as in the age-of-the-captain problem. Therefore, although the notion of didactical contract may help the teacher understand his or her practice, it is not a technical tool for acting on that practice. Instead, it is a technical tool enabling the researcher to study practice.

Conclusion

Didactique as elaborated by Brousseau forces a radical shift in both our perspective and our work as mathematics educators. Historically, the field of mathematics education developed as psychological approaches to research were applied to the teaching and learning of mathematics. The phenomena of study were delineated according to what were understood to be the natural components of educational practice. These components provided the objects of mathematics educators' study. *Didactique* constructs a political economy of the field. It compels us to see mathematics education as dealing not with a collection of 'obvious' components, but rather with phenomena best studied by analyzing the knowledge at stake in a given situation. Students come to know in situations where any knowledge that society in general and their teacher in particular wants them to acquire must undergo transformation so as to become meaningful.

Meanings emerge in situations that can be engineered by analyzing a situation and developing an array of possible meanings for students to take from it. Knowledge is won in a game learners play against the *milieu* set out by the didactic situation. The teacher as actor must give the student freedom to act, but cannot forget his or her own role and script in the play. Both student and teacher work within an implicit contract they continuously negotiate to regulate their actions. No metaphor from economics, engineering, game theory, dramatics or the law can represent the full complexity of a teaching and learning situation. Instead, such metaphors together help in producing a model – a theoretical object of study – for mathematics educators to explore as they attempt to ground their field.

Note

[1] This article is based on a conversation between the two authors about the book *Theory of Didactical Situations in Mathematics*, the first collection of articles by Guy Brousseau (1997) to appear in English. All quotations from Brousseau, unless otherwise indicated, come from this book. Our text here is intended to provide not so much a review of the book as some scaffolding for unfamiliar anglophone readers who might want to read it.

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