tion with others unless forced to. My experience of getting to know my neighbour leads me to question the self-description of shyness, as I confidently hail him each day. Next time I get to know a neighbour there will be a change in the way I go about it and I could, perhaps, reach the stage of familiarity more quickly In the case of the dog, she may be subjected to a new experiment in which some event is always preceded by a signal. If she shows evidence of adapting to this new experiment more quickly than she learned to salivate then she shows evidence of change in the process of learning 1, *i.e.* she exhibits learning 2.

The hierarchy continues to learning 3 and 4, but these are beyond me at the moment. Bateson warns that learning 3 "is likely to be difficult and rare even in human beings" (Bateson, 2002, p 301), and equally so to imagine or describe.

Danielle, in the story above, showed evidence of learning 2 in that she changed the way she approached tasks of learning, *i.e* , she changed in her approach to learning 1.

In offering my classes the purpose of 'becoming a mathematician' I am trying to provoke learning 2. I am trying to direct students' attention towards a different relationship to messages within a mathematics classroom and into a way of learning 1 in which they develop their own criteria for when something makes sense, expect the feeling of new behaviours making sense and feel able to question until they have this feeling

There is a link between levels of learning and levels of communication. It is through the meta-communication about what it means to be 'becoming a mathematician' that I believe I can help create the context in my classroom in which learning 2 can occur.

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The following two communications are reflections after ICME-10, Copenhagen, Denmark, July 4-11, 2004. The task offered was to talk to issues raised from two highlights.

Mathematics knowing as fully embodied

TOM KIEREN

As usual I'll go astray from the request and mention three highlights that are inter-related

At the first ICME I attended in Karlsruhe in 1976, Micheal Otte made a presentation on the politics of mathematics education. As I remember it, he suggested that to the extent that our work in mathematics education at any level pertained to the service of mathematics knowing in schools at any level, then our work necessarily had a political dimension whether we noticed it or not - he thought we should and provided insights into such noticing

I was reminded of this by the opening session that was wonderful all around But I was particularly impressed by the greetings brought to us by the Danish Minister of Education; the Mayor of Lyngby-Taarbæk; as well as the dean from DTU (Technical University of Denmark). They had all thought carefully about what they had to say to the ICME community, at least as represented by those at ICME-10

Their message was that mathematics knowing was important to the lives of the various communities that they served and for the general public of Denmark. Thus, they suggested that our work individually and as a congress had the potential of serving them and those they served as well. I was reminded of the political significance of our work - deliberately 'small p political' - as well as the ethical responsibility inherent in offering one's ideas to be even considered by others and especially by broader publics beyond those we interact with, say the readers of FLM or our fellow teachers in schools.

This responsibility is evident in a second highlight This highlight is derived from the presentations of Daniel Ansari and Terezinha Nunes in a session on sources of theoretical (and research) ideas in mathematics education. While Daniel brought to our attention the ways in which the brain and its activity and operations might be seen to be underlying mathematics (at least number) knowing, Terezinha provided us with a view of mathematics knowing which necessarily goes beyond number and particularly beyond brain activity as such. As a community we are left with trying to learn and be aware of how the functioning of the brain might in many different ways impact on how we think about mathematics knowing and how such awareness and learning might help us help individuals engage in such knowing.

In particular, we need to understand such cognitive neuroscience work in order to value it appropriately in the mathematics education responsibilities suggested by my first highlight. On the other hand, Terezinha Nunes pointed to broader, personal mathematical knowing actions and interactions that we also need to understand and appropriately value in new ways. Thus, to responsibly and ethically serve the various communities in which we live we must somehow use theories and ideas that live in the tension of the awarenesses raised by both Daniel Ansari and Terezinha Nunes

One way to take forward such thinking comes from my third inter-related highlight - Rafael Núñez's presentation. This presentation was a *tour de force* in its use of technology as well as being rich in ideas including:

- the extension of his previous work with Lakoff (2000) and that of others such as Johnson (1987) on embodied metaphors and mathematics knowing to now define and illustrate how *fictive actions* penetrate mathematical thinking at all levels of sophistication from our formation and use of images to the way in which, in arguments in mathematics, for example, when we talk about functions gradually decreasing; monotonically increasing;

approaching a limit; or jumping from one value to another, we are engaging in fictive actions which lie at the heart of doing mathematics

- how physical gestures penetrate mathematical explanations and arguments at least in their living forms (illustrated with numerous quick-time movie examples)

- reports of research that showed synchronicity in brain activity related to such gesturing and the related uttering.

For me the two ideas of fictive and gestural actions illustrate well what is meant by mathematics knowing as fully embodied. In particular, they show that mathematics is a communicating form of knowing and that in both forms of activity we are trying to engage others with and in our knowing actions in ways that seem to me to embody both the actor and the other (such as, listener, watcher, evaluator) in a larger body of mathematics. Further, both these two features, fictive and gestural actions and their detailed study, may provide us with a means for looking at and using the tension of awarenesses, raised by Terezinha Nunes and Daniel Ansari, in our theories and practices as we try to serve the communities represented by those who so cordially and thoughtfully greeted us as ICME-10 participants.

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Lost and found in ICME-10

FULVIA FURINGHETTI

Traditionally ICME conferences are places where you meet people or lose people - the definition varies if you are an optimist or a pessimist. At an ICME conference you meet a lot of interesting people, but they are not necessarily those that you planned to meet! The number of participants, after the heights of ICME-8, Spain (2762 participants) is decreasing (2074 in Japan, ICME-9; 2161 in Denmark, ICME-10), but there are still a lot of people

For instance, as was the same for many participants, my accommodation was a long way away from the conference venue. After an initial moment of dejection (when I realised that breakfast would be at six thirty all week), I saw the happy side - having a long, relaxed trip with time for talking with my colleagues, at the same time as enjoying the wonderful Danish public net of transportation. Whenever else have I had, or ever will have again, a half hour of conversation with David Tall, without interruptions from other colleagues asking him something?

As chair of the group *History and Pedagogy of Mathematics* (HPM), affiliated to ICMI, my main interest has been to go to events in the programme that combine history and mathematics education. The tradition of having a substantial activity in this field, which dates back to the working group on *History and pedagogy of mathematics* held at ICME-2, was continued at ICME-10 through Topic Study Group 17 (TSG 17): *The role of the history of mathematics in mathematics education*, chaired by Man-Keung Siu and Constantinos Tzanakis.

TSG 17 was the place where the three souls of the HPM Group (mathematics, history and education) met together. Thus we attended presentations by mathematicians looking at history to find materials for their teaching, historians providing suitable excerpts and historical information, researchers in education who attempt to investigate if and how the cultural artefact 'history' might become a mediator of mathematical knowledge in the classroom.

The programme offers a wide range of events that can be confusing. As well as TSG17 there were other forms of activity that explored the role of history in mathematics education for students and teachers:

- a session in the form of 'poster round tables' labelled *History of mathematics and mathematics education* allowed a better understanding of the concise content of posters through discussion in an informal, fruitful atmosphere

- lectures from Victor Katz, Luis Puig and Evgeny Shchepin

- workshops from Victor Katz with Karen Michalowicz and from Avikam Gazit.

All these activities evidenced that that spirit of the HPM group is much more than the use of history in mathematics education - it is the conception of mathematics as a living science, a science with a long history, a vivid present and an as yet unforeseen future, together with the conviction that this conception of mathematics should not only be the core of the teaching of mathematics, but it should also be the image of mathematics spread to the outside world. This spirit of the group was evidenced by the telling of its history, since its foundation in 1976, told by Florence Fasanelli in one of the meetings of the group during ICME-10.

This brings me to another issue in which I'm particularly involved, that is the history of mathematics education itself. I applaud the scientific committee, which for the first time has established a new topic group (TSG 29 chaired by Gert Schubring and Yasuhiro Sekiguchi): *The history of the teaching and the learning of mathematics*. I'm strongly convinced that researchers in mathematics education and teachers need to know about this history to feel themselves to be part of the big project of modern civilization. This topic was also developed through regular lectures, such as that of Anosov on the development of Russian mathematics education in the past 300 years and in National presentations, such as those from Romania and Russia.

In his regular lecture, Geoffrey Howson illustrated the role of two main characters of the past (Klein and Freudenthal) in the development of mathematics education. His talk was a first-hand witness, by a person who lived through years of the ferment of initiatives after the second world war.