The Activity System of School-Teaching Mathematics and Mathematical Modelling

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There is a growing realisation that mathematical practices are context-specific This site-specificity of mathematical practices has given rise over long periods of time to particular activity systems which are highly enduring. In this article, my focus is on the activity system of school-teaching mathematics and the impact mathematical modelling has on it.

Varieties of mathematics

Over the past twenty years or so much has been written, debated and speculated about different kinds of mathematics prevalent in different spheres of the mathematical enterprise. These initiatives have different disciplinary bases such as sociology, psychology, anthropology, the philosophy of mathematics, political science and mathematics education. A consideration of the literature related to these varieties of mathematics indicates that they have quite a long history.

One of the earliest recorded separations can be traced to ancient Greece. Mathematics was divided into:

> This computational aspect [...] called *logistica* (a word related to our "logistics") [and] *arithmetica* [ ] a study of the abstract mathematical properties (Bunt, Jones and Bedient, 1976, p. 75)

This early partitioning of mathematics into practical and academic components was not only a simple split into a utilitarian way of dealing with mathematics and a scholarly way of so dealing. It was also a social stratification mechanism in which:

> *Arithmetica* was the concern of philosophers and gentleman of leisure [and] *logistica* was the concern of merchants and slaves. (p. 75)

Kline (1980, p. 22) takes this stratification thesis further when he argues that during this period a criterion used to exclude men from holding political office for ten years was their engagement with the utilitarian mathematics of the merchants and the slaves. These two forms of early Greek mathematics thus dealt with different objects, had different purposes, were practiced by different communities and had different consequences.

Concern about the poverty of a behaviourist view of learning to underpin the design of learning resources for learning mathematics and the ascendancy of the constructivist thesis as the desirable theory of knowledge acquisition saw the emergence of a partitioning of mathematics into institutional and teachable mathematics This derives essentially from Chevallard's theory of didactical transpositions (Artigue, 1994) which analysed the ways mathematical topics moved from their source to various arenas of teaching The way I understand this work of the French didactical school is that at the institutions where mathematics is developed it is not in a form teachable other than being explainable to peers. It is to be taught in institutions of learning such as schools.

> One questions the possible viability of the content one wishes to promote while considering the laws that governs the functioning of the teaching system. One tries to foresee the deformations it is likely to undergo; one tries to ensure that the object can live and therefore develop within the teaching system without too drastically changing its nature or becoming corrupted (Artigue, 1994, p. 28)

In schools, mathematics thus takes on a different form from that which it had in the originating institution. This is aptly the case when one considers earlier versions of introductory algebra at school level It is not difficult to see that the didacticised versions of such algebra only existed in the lower secondary school. One would be hard pressed to find things such as *Simplify* $x^7yz^{13} - x^7y - 14z^{13}yx^7$ in the institutions of antiquity where algebra originated. This teachable mathematics is not only different in appearance. It is an entirely different genre!

The thesis of the existence of varieties of mathematics with its own distinct characteristics is further evident in the ethnomathematical movement Ubiratan D'Ambrosio, the intellectual father of the ethnomathematics program, contrasts what he calls 'academic mathematics' with 'ethnomathematics', which is:

> the mathematics which is practised amongst identifiable cultural groups, such as national tribal societies, labor groups, children of a certain age bracket, professional classes, and so on Its identity depends largely on focuses of interest, on motivation, and on certain codes and jargons which do not belong to the realm of academic mathematics We may go even further in this concept of ethnomathematics to include much of the mathematics which is currently practised by engineers, mainly calculus, which does not respond to the concept of rigor and formalism developed in academic courses of calculus (1985, p 45)

In contrast to the previously mentioned separation of mathematics into two distinct types, D'Ambrosio paints a much broader picture and enters the heart of the academy by drawing attention to the *de facto* existence of different varieties of mathematics within the halls of the academy.

D'Ambrosio's assertion of a particular kind of mathematics "for children of a certain age group" in the above quotation is further bolstered by empirical evidence of different kinds of computational procedures utilised by children (Carraher, Carraher and Schliemann, 1985) and adults Lave (1988) inside and outside the school context. With regard to school-going children, Ernest (1998) makes the distinction between the practices of school mathematics and research mathematics, indicating some of the similarities and differences between these varieties. Whereas the prior-mentioned authors were making statements about the existence and, at times, about the differences between varieties of mathematics, Ernest accepts the notion of at least a bipartite partitioning but also draws attention to the existence of similarities among varieties of mathematics. What thus unfolds is that the varieties are not necessarily hermetically sealed with strict boundaries, but rather that there are instances of overlap.

As final instance of the varieties of mathematics there is a question on whether the mathematics of the mathematician and that of the mathematics educator is the same. Sfard (1998), for example, distinguishes between:

> the Typical-Mathematician's-mathematics and the mathematics-education-researcher's mathematics [that] came to be worlds apart [and] are two different kinds of discursive practices, specific to the two communities. (p. 505)

The mathematics occupying the minds of mathematics educators is not the same as that which occupies the mind of the mathematician and as Livingston (1986) states:

> at the work-site, the mathematician is not interested in a theory of mathematical discovery; he [*sic*] is interested in making mathematical discoveries (p 7)

This theory of mathematical discovery, in a wide sense, plays an important role in the mathematics of the mathematics educator. Furthermore, the mathematics educator deals mostly with elementarised versions of mathematics These versions that they deal with have undergone some transformation and are removed from their originating sources Mathematics educators seldom use original pieces of mathematics as the basis for their work And they do not only consider mathematics when engaging in their mathematical work. Insights from learning theories, pedagogy, philosophy of mathematics, history of mathematics and so forth also structure the mathematics of the mathematics educator. There is of course also the distinction between 'applied' and 'pure' mathematics which has been around for a long time. This will not be delved into in this paper and in order to get a glimpse of the essence and debate on this distinction the reader is referred to Davis and Hersh (1986. pp. 85-87)

So far, the case for overlapping varieties of mathematics has been made and demonstrated. The essence of the case is that mathematics has a different face in the wide variety of spheres where it is observable and that no one manifestation can lay claim to some form of superiority. In fact from the ethnomathematics movement it is suggested that mathematics should be considered as the union of the different ethnomathematics. In this regard Gerdes (1997) proposes that:

> ethnomathematics may be defined at another level, as a research field, that recognises the existence of many mathematics, particular in a way to certain sub-cultures (p 343)

It is this particularity that leads to the consideration that each variety is governed by its context and site of operation. In this sense, I will refer to site-specific mathematics with its own unique activity system. The notion of an activity system and its particularisation for mathematics that school teachers engaged in are discussed in the next section.

Activity systems

The idea of an activity system derives from the cultural-historical theory of activity and is an extension of the Russian activity theory articulated by Vygostsky and his followers (Engeström, 1987) An activity system evolves historically and "Activities [...] are systems that *produce* events." (Engeström, 1994, p. 45). The basic human activity system can be represented diagrammatically as in Figure 1 below

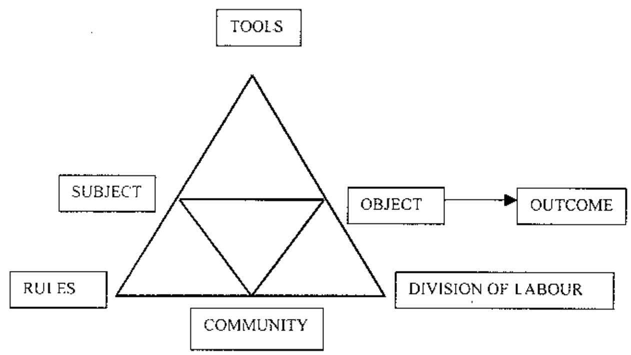

Figure 1: Basic activity system of human behaviour

In line with the proposition put forward in the previous section, the mathematics engaged by school teachers can be viewed as a variety of mathematics This variety is what will be called 'school-teaching mathematics'. School-teaching mathematics is not 'school mathematics teaching', which is what teachers do when they teach in schools With these considerations, the contents of the different components of the figure are given in Figure 2 below.

As is the case with all activity systems, the one for school-teaching mathematics has developed over a long period of time and is embedded within the types of school mathematics teaching as developed over the ages.

Mathematical modelling and school-teaching mathematics

Reforms in school mathematics curricula place a lot of emphasis on the applications of and modelling in mathematics These calls are incorporated in phrases such as 'the relevance of mathematics', 'real-life situations' and the 'translating real-life situations into forms amenable for mathematical treatment, solving the mathematical problem and evaluating the mathematical solution in terms of the

School-Teaching Mathematics	
Tools	*Recording devices:* pens; pencils; erasers, paper and notebooks. *Recall devices:* textbooks; produced teaching resources. *Processing devices:* "remembered" manipulative techniques and heuristics; calculators (four-function and scientific). Resources containing school-type problems and their solutions. Situationally-constructed devices.
Object	Remembering of solution strategies for transorming them into personal teaching style. Identification of techniques of working with mathematics to improve examination success of learners.
Outcome	Improved pass rates of learners
Division of Labour	Individualistic.
Community	Other teachers at schools, relevant officials from education bureaucracy, mathematics teachers' organization
Rules	Single correct answer. Answers to be obtained in restricted time. Answers must be represented in format emulating that of the text in use.
Subject	Teacher of school mathematics

Figure 2: Contents of the school-teaching mathematics activity system

reality-based situation' Responses to these calls have resulted in curricula and in-service courses for teachers with each of these interpreting applications and modelling in their own particular way and utilising especially mathematical applications for their own particular purposes.

The Applications of and Modelling in School Mathematics Project (AMSMAP) at the University of the Western Cape is one such project and among its goals is the investigation of teacher mathematical modelling work and its relationship to a hypothesised school mathematical modelling activity system as depicted in Figure 3 below.

To pursue the goal mentioned above, various data were collected from a group of educational practitioners and a former college of education lecturer who attended a three-day camp on mathematical modelling, where they had to develop a mathematical model for the situation given in Figure 4 overleaf. The task is a reconstruction of a problem that appeared in various forms, such as for example in Bartholomew (1976) or Giordano (1996).

School-Teaching Mathematics	
Tools	*Recording devices:* pens; pencils; erasers, paper and notebooks. *Recall devices:* textbooks; classroom notes; mathematical cue-identifiers from request specification. *Processing devices:* "remembered" manipulative techniques and heuristics; calculators (four-function and scientific). Text related to the request statements. Situationally-constructed devices.
Object	Making of an artifact that can be termed mathematical according to the specifications of the model-requester.
Outcome	A defensible report on the mathematical model and its use. New patterns of school mathematical modelling work.
Division of Labour	Collaborative teams.
Community	Other teachers and sympathetic mathematics educators.
Rules	Different "non-equivalent" answers. "Unrestricted" time. Produced mathematical models have a provisional status and is open to improvement. Responses presented in user-defined format to "convince" specification requester.
Subject	Mathematics educators organising courses, in a variety of formats, on mathematical modelling production for teachers

Figure 3: Hypothesised activity system for school mathematical modelling

Thirteen educational practitioners were involved, four females and nine males, who worked on the situation in four pre-assigned groups – three groups of three participants and one group of four. The data collected consisted of the rough work the groups produced; observational notes of teacher work during the week-end; the final reports and an unstructured questionnaire. In addition, interview data were collected before their engagement in the task from some of the teachers on their perceptions on the incorporation of modelling and applications in the school mathematics curriculum. Given the particular way the data were analysed, these pieces are briefly discussed and exemplified below.

Examples of data

Rough work and a narrative on rough work

Rough notes are the notes and work produced during the construction process. It is fairly common that during most, if

A New Salary System for Teachers?

The Ministry of Education has given School Governing Bodies (SGB's) the option of determining and administering the salaries of the teaching staff at their schools. To do this, the SGB will receive an annual teacher salary amount based on the current salary scales. This amount will increase annually by the percentage increase negotiated by the teacher trade unions. SGB's are allowed to give the staff annual increases as they see fit. They are allowed to add to the staff salary amount funds they obtain from other sources such as carnivals which schools organise

The SGB of Xolana Yizo Zwetemba High School decided that their school will have a fair and reasonable salary system for the teaching staff of their school They hire your team as consultants to develop a salary system for the teaching staff that reflects the following circumstances and principles.

Circumstances

Excluding the principal, there shall be four ranks in the school: teacher, senior teacher; head teacher and senior head teacher, in ascending order Staff memebers with a six-year qualification (Relative Education Qualification Value 16 (REQV16) are employed at the level of senior teacher. Staff members who are working on a six-year qualification are employed at the rank of teacher and promoted automatically to senior teacher upon completion of their qualifications. No new staff members with an REQV less than 16 will be appointed at the rank of head or senior head teacher Staff members may apply for promotion from head teacher to senior head teacher after serving in the rank for seven or more years The promotion decisions are made by a committee of the SGB and are not your concern

Staff salaries are for a twelve-month period July through June Salary increases are always effective from the beginning of July The Salary amount available for increases varies from year to year and is generally not known until January

The starting salary for a teacher is to be at the starting salary for a teacher with a Relative Education Qualification 13 (REQV13) and for a senior teacher it is the starting salary for someone with REQV14

Principles

- All teaching staff should get an increase any year that money is available
- Teaching staff should get a substantial benefit from promotion If one is promoted in the minimum possible time, the benefit should be roughly equal to the seven years of normal (non-promotion) increases
- Teaching staff who get promoted on time (after seven or eight years in a rank) and have careers of 25 years or more should make roughly twice as much at retirement as a new teaching staff member at REQV16.
- Teaching staff members in the same (new) rank with more experience should be paid more than others with less experience But the effect of an additional year of experience should diminish over time In other words, if two staff members stay in the same rank, their salaries should tend to get closer over time

The project

Design a new salary system You must also design a transition process, that will move all salaries towards your system without cutting anyone's salary, for existing staff members The existing staff salaries, ranks and years of service are given in table 1 The Chairperson of the SGB has asked for a detailed salary system plan that can be used for implementation, as well as a short executive summary in clear language, which she can present to the SGB and the teaching staff. The summary should outline the model, its assumptions, its strengths and weaknesses, and the expected results.

Figure 4: The social situation presented to the teachers

not all, mathematical work, the real work is the actual scribbles, marks, diagrams, doodles and so forth that normally land up in the dustbin The books, papers in journals and lecture notes are, in essence, the cleaned-up reports and contain minimal traces of the happenings during the actual construction process

This kind of data is deemed important, since it provides traces of the behaviour that participants were involved with at a particular stage They provide a sense of the occurrences at the work-site and workbench. Regarding qualitative research, in general, rough work falls generally within the realm of 'documents' and acording to the categorization provided by Merriam (1998, p 117) it falls with the 'physical materials' category

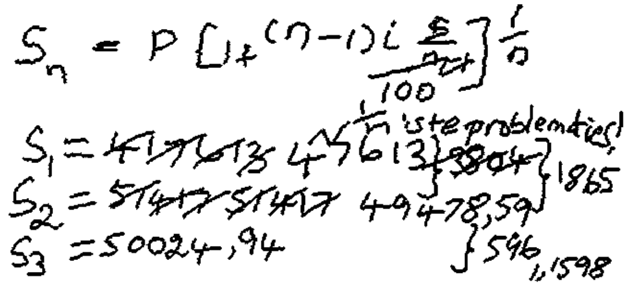

Figure 5: Example of rough notes

Figure 6 below shows an excerpt of the salary ranges and position tables (South African Government Gazette, 1999, p 43) which the participants were provided with

Salary Range	Salary Position	Salary Rands 1/7/1996	Salary Rands 1/7/1997	Salary Rands 1/7/1998
6	6 1	40836	44514	47613
	6 2	43344	47247	50442
	6 3	45852	49983	53361
	6 4	48360	52719	55176

Figure 6: Extract of salary range and position

Overall, the rough work indicated a high level of arithmetical calculations aiming towards finding a pattern between the subsequent salaries in the salary ranges and position Some of the initial patterns were: percentage increases and ratio increments.

Observation and reflective notes

Observation and reflective notes were not recorded on the spot at the moment when a particular group was observed The researcher would walk around and observe what the groups were doing After this reconnaissance, the notes were written, as soon as possible after the reconnaissance periods The reconnaissance periods varied from half an hour to one-and-a-half hours.

Examples:
Teacher 1 in group X suggested to colleagues that they should start looking for a formula and he started to write the formula for the sum of a geometric series.

> After receiving the problem the participants were silently individually reading through the problem. One of them looked at the person he was travelling with and commented that he told his partner that he was sure that this year's problem was to be one on salaries of teachers (OC: The teacher who made the comment did participate in a similar activity in 1998 and during one of our conversations at this competition I had mentioned that I was considering giving them a problem on teacher salary systems.)

> A form of work allocation, primarily something like everyone studying the problem on their own and coming with suggested ways of attacking the problem during the next meeting time (the next day) were the tasks given to each member of the group [27/08/1999]

One teacher was phoning a colleague and asked the colleague: "What is the formula that was used when we were busy with salary negotiations?" At some stages, the teachers were talking about the salaries they earned. There was no indication that there was talking about the principle of 'equal pay for equal work' which was part of the introductory presentation of the workshop. [28/08/1999]

Non-structured questionnaire

The non-structured questionnaire comprised of a single request stated as follows: Kindly write down how you experienced the competition over the weekend Restrict yourself to the 'academic' component the work pertaining the 'problem' and not to the 'social' component the food, the venue, and so forth of the weekend's activity

Examples:
Teacher RS:

> [This is a translation of the teacher's response which was written in Afrikaans] It was a struggle to understand the problem. The many principles, variables had your head spinning. We started by trying to get a formula from the table excited! Oh [...] the equation/formula does not satisfy all the conditions. The struggle starts again from the beginning or a different strategy is sought. Decide first to work with one post level only to simplify the problem point of departure gets a ceiling highest position and highest number of years in post level. Build other formulae around the norm. Process is much 'trial and error'. Again and again and doing things over and over. Fit and measure/test/verify. A possibility? OK. You get some confidence because there is no right or wrong tension of criticism is gone It was nice to prostitute your brain and test your limits. The end is sweet

Teacher SA:

> Problem challenging Frustrating not being able to immediately find a point of departure. Try and try again/trial and error. Considering different perspectives. Getting to know really what is to be done? Good working in groups Input/assistance/new insights into the problem gained by working in a group. 'Great' amount of data confusing - to distinguish between what is relevant/or not tricky. Sources/references/books should be available for consultation

Teacher PR:

> It was a daunting task. Much discussions went on in our group about the approach to the problem, I realised we were thinking at different levels. A lot of explanation had to be done in order to clarify different ideas Eventually discussions made sense and agreement on ideas were fine I got lost in the mathematics and the working out of logarithms X worked on a very high level of mathematics for (me!), but I understood our structure of the model I just felt that we should have been more open with the question our own ideas instead of sticking to the table that was given, because a lot of time went into *ontrafeling* (unraveling) of the table. We got to know one another fairly well and there was a good rapport between the group members.

Teacher JT:

> It was an academically stimulating experience. Engaging with a problem of this nature is a very taxing experience. For something like three hours we argued about what should be done We explored many dead ends. It was only towards the Saturday afternoon that we believed we had a grip on the problem. Of course, then we had to grapple with the representation. After looking at the graphical and algebraic representations, we settled for the algebraic one.

Final products

As the task indicates, the teachers had to come up with a final written report which included the mathematical model(s) they designed These final reports were submitted at an open meeting where the groups presented explanatory summaries of their reports. Excerpts of some of these final products are given below.

Interviews

As mentioned above, some of the teachers were interviewed on their views on the applications of and modelling and their teaching in school mathematics prior to their engagement with this activity An excerpt of such an interview is given.

I: If you are busy with the applications and modelling of school mathematics, what are you trying to achieve?

TX: I shall say currently the mathematics that we teach is very theoretical and there is no emphasis on the application of mathematics outside the mathematical setup. I am now speaking of mathematics as it is in the Western Cape syllabus

A New Salary System for Teachers by Rooibos Tea Consultants

Clarifications
The teachers at Xolana Ytage Z High School presently are divided into three levels and the circumstances require us to divide it into three 4 levels i.e. teacher, senior teacher, head teacher and senior head teacher. We then established a post ratio norm.

Assumptions
We accept a post ratio of 70% for post level 1, 18% for post level 2, 8% for post level 3 and 4% for post level 4 The percentage increase from year to year should decrease as the number of years of service increases Every teacher must get an increment by the end of the year We assumed that money is available.

Analysis
The existing classification of three levels must be changed to the 4 level system The annual increase must be applied on all new levels to the extent that the average increase at that particular level equals the general agreed increase. All teaching staff will get an increase if there is money available. Our classification caters for the notion that a teacher with more experience should be paid more than a teacher with less experience

The Model
1 For any given level: Suppose the minimum salary or starting salary is A, for an annual increase of i (where $i > 0$)
Salary: $y_1 = A$ (y_1 is the existing year)
$y_2 = (1 + i_1)\, A$ i (increase)
$y_3 = (1 + i_2)\, y_2$
$y_4 = (1 + i_3)\, y_3$

$y_n = (1 + i_{n-1})\, y_{n-1}$

Condition $i_n < i_{n-1} < i_{n-2} < \cdots < i_1$

y_n is the annual salary year n.

2 Applying the principle of a teacher who is promoted on time and of careers of 25 years or more should make roughly twice as much at the time of retirement as a new teaching staff member at REQV16

max $A_4 = 2A_2$ — A_4 - annual salary on level 4; A_2 - annual salary on level 2

3 For promotion purposes
The minimum time to move from level 1 to level 2 is three years if that teacher meets the requirements for such a teacher.

$A_2 = (1 + i_6)\, y_6$ for A_1 where A_1 is the annual salary for level and A_2 the annual salary for level 2.

Strengths and Weaknesses
Strengths
The model is generally applied to any level It also fits the principles suggested by the SGB.

Weaknesses
The rate of increase is not having a specific pattern. It depends upon the availability of funds

Figure 7: Exerpt of final product by a group

Assumptions
1 Money will be available for a 10.5% across the board

2 The model will be based on a gliding scale to reduce the gap between levels.

3 We'll assume that all staff members are motivated and deliver the goods

4 Workload is spread evenly within a level

Solution for level 1

A Formula/symbols

1 Starting salaries

a)

REQV 13	R47613
REQV 14	R50442
REQV 15	R53361
REQV 16	R55176

b) A - starting salary
K - years experience (number of years)
L - last year i e 45
Y_k - salary in a particular year

$$Y_k = A(1.05)^k + [r_w - (L - K)c]k$$

r_w:

$$r_1 = \frac{REQV_{\max}16 - REQV_{\max}13}{45} = 1510$$

$$r_2 = \frac{REQV_{\max}16 - REQV_{\max}14}{45} = 945$$

$$r_3 = \frac{REQV_{\max}16 - REQV_{\max}15}{45} = 362$$

$$c = (r_2 - r_1) : (r_3 - r_2)$$

Figure 8: Excerpt of final product of a group

I: Now think about your own teaching Did you do something such as the applications of mathematics?

IX: The examinations To be honest, we are led by the examinations, particularly the matriculation examination. And our teaching was more geared to get the guy exam-ready in standard and whatever applications we taught in class was more determined by the matriculation examination. The teaching must be exam-oriented and whatever applications we teach are subjected to how prepared we want the guy to be for the examination

I: Am I correct to say that what you try to achieve is to prepare them for the type of applications that occur in the examination?

TX: To get them ready to handle them. If they basically get such a problem, that they can handle it

I: Something about modelling. Did you do it?

TX: No. To be honest, maybe you should define what you mean by modelling because this is something I don't really know.

I: What I mean essentially is what is described in the syllabus: a situation is described, a mathematical model must be constructed for the situation and the model is evaluated whether or not it is a correct or near-correct rendition of the reality situation.

TX: Well we do not actually [...] I cannot remember that we formally did modelling with such a situation If it did occur in my teaching it was by the way. [...] If I now specifically think where the applications occur more regularly [...] if I think of maxima and minima in differential calculus.

Then it is very important that the guy must understand that the gradient of the tangent at any point of the quadratic graph [...] the graph of a quadratic equation or of a cubic equation you get by getting the derivative of it. He must obviuosly understand where the derivative comes from and that at the turning point that the gradient is now [...] is the gradient at that point zero. He must understand why it is zero [...] it is because it is a horizontal line

And if you have that situation and you look at the applications of maxima and minima that you actually get an equation out of that applications problem where you eventually can take that equation and draw a graph and this is where he must understand why you must again take the derivative of it and equate it to zero to get the x-values for which you are going to get a maximum or minimum

I: If you have no obstacles, what will you try to achieve with the teaching of applications and modelling and how will you go about it?

TX: He must see that the mathematics does not stand loose That which he learned is something which he does learn in isolation You use it. Somewhere in society it has an applications possibility. Come we say most of it, most mathematics has an applications possibility [...] I will not expect it to become an obsession that every situation he gets there that he should try to solve it but I at least want him to reflect and how it shall fit in a mathematical situation

The data described and exemplified above were subjected to analysis within a structured framework

Data analysis

The unit of analysis was the complete data set of all the groups It comprised of all five pieces of data for the four groups. To analyse the data, an instrument based on mathematical modelling types and the sociological imperative of mathematical work was constructed. For the mathematical modelling types, the classic modelling type problems as articulated by Pollak (1979) was conscripted. According to Pollak's typology, mathematical modelling and applications problems vary from whimsical formula-seeking ones to detailed situational-analysis and problem-specification ones. The sociological imperative of mathematical work derives from Restivo, van Bendegem and Fischer (1993), who assert that within the production of mathematics there is a social and a technical side that receive attention. Consideration of the modelling and applications types of problems and the social imperative of mathematical work resulted in the analysis grid in Figure 9.

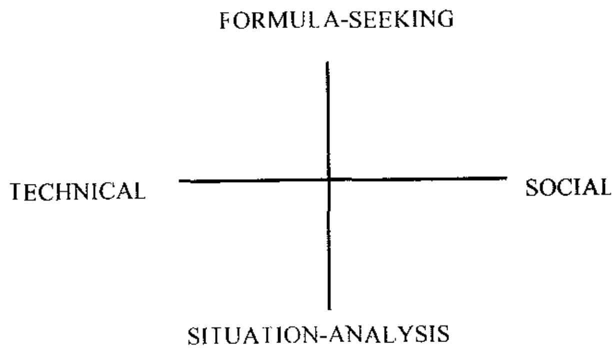

Figure 9: Analysis grid

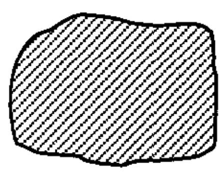

Figure 10: Fitting measure

The data were read and re-read, studied and re-studied. After each of these actions, the fitting measure given in Figure 10 was laid over the analysis grid to indicate what according to the reader was the best representation of the data set in terms of the analytic framework In order to ensure accuracy of fit, a colleague was presented with the data set of one group, the analysis grid (with the articles which were basis for the constructed grid) and the measure and requested to analyse the data set accordingly. His analysis differed slightly but the difference was considered negligible since the overall pattern of coverage of the analysis grid was not different. This process of analysis resulted in the analysis grid coverage of teachers' mathematical modelling behaviour given in Figure 11.

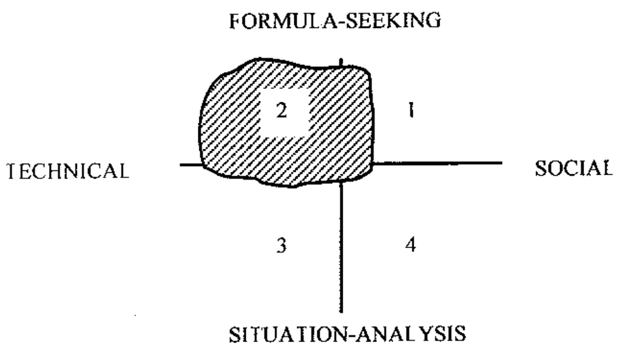

Figure 11: Pattern of teachers' mathematical modelling work

From this analysis two things stand out:

1 the high level of work in quadrant 2;

2. the low level of work in other quadrants

The low level of work in quadrants 1, 3 and 4 is surprising given that the workshop occurred just after there was a strike by teachers related to salary increments. Furthermore, the majority of the teachers involved in study were part of a design and development team of a module dealing with retirement planning where issues that falling in quadrant 4 were attended to, as is obvious from the excerpt from the module given in Figure 12 below

From Module:
Interestingly, in developing the model we wondered about the selling of clothing on credit. In these cases shops such as Edgars, Milady's and Markhams do not give people any discount who pay cash. These people have to pay the same price for an item as people who buy on credit. In essence these shops are saying that the item will cost the amount these shops will charge in six month's time. The cash client thus pays the future price of the item but really he or she should pay the current price. With our model the current price can be worked out and we recommend that people who are cash payers should use our argument to bargain for a discount.

From Actuary's Commentary:
Another aspect which could be used in the Conclusion is that of using a credit card to buy goods, but to pay at the latest possible date B but without using the credit facility. In doing so, one scores in paying the "current" price some four weeks later, suing "future money".

Figure 12: Excerpt from the module on retirement planning

Relating this finding given in Figure 11 to the various components of the hypothesised activity system for mathematical modelling it can be concluded that for

Tools: The existing tools were utilised as mediators in the hypothesised way. There was hinting at using texts other than only explanation-exercise-declaration texts that need to be mastered. Engeström (1987) posits that texts are normally seen as the objects in the sense that they are to be mastered and not as resources assisting in the pursuance of the object. There is thus slight evidence that this inversion might be reversed. It is indeed so that this happens in situations where participants are allowed to work in close proximity to resources such as the conditions prevalent in the Mathematical Contest in Modeling of the Consortium for Mathematics and Its Applications (Giordano, 1996)

Object: This was defined in the problem specification. However, the initial major pursuit of the teachers was to find a formula via percentage and ratio increments. It can be postulated that, even given that an artifact according to specifications of the requester was produced and submitted, the objective striven for did not vary much from that of the activity system of school-teaching mathematics given in Figure 2.

Outcome: The outcome is fashioned by the object. The minimal variation of the object from the school-teaching mathematics activity system also resulted in minimal variation from the outcome of the activity system of school-teaching mathematics.

Division of labour: Through the set-up, the participants were required to do collaborative work. The collaborative work was interspersed with individual work on the same sub-task and collectively-decided different sub-tasks. For the latter sub-task, the division was made on the basis of the identification *in situ* of the expertise of the individual group members.

Community: Mathematics teachers, mathematics educators and school administrators interested in the implementation of personally constructed versions of the applications of and modelling in mathematics in the school-teaching mathematics were the community participating in the project. Given that the participants taught or had experience of teaching different grades, had different levels of post-school mathematics training, were diverse in terms of their mother tongue, were from previously racially-defined school environments, were employed in various sectors in the education system and outside the system (one participant was a mathematics educator working in a bank) and were even from two different countries (two of the participants were from Eritrea), awareness about cross-cultural, cross-national, cross-employment domain and cross-qualification collaboration was emerging.

Rules: There was partial acceptance of different 'non-equivalent' answers with the proviso that some models were more 'acceptable' in terms of their mathematical elegance. A growing acceptance that more than the school-type time and structure organisation must operate emerged. The provisional nature of the produced model was only minimally accepted

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The emerging activity system indicates that the 'movement' towards an activity system for mathematical modelling is mixed. Certain components of the school-teaching mathematics activity system were disturbed but others remained intransigent. Bearing in mind that the school-teaching mathematics activity system had developed over a substantial period and that its current manifestation is embedded in layers of previous ones, the minimal disturbance can to a certain extent be expected

Another issue is also whether the points of disturbance are, in a sense, permanent. This is much more difficult. Change is a complex somewhat evolutionary process and not only the result of the doings of individuals as agents of change. Nor can it be expected that a one-off or even a series of events embedded in the actions of individuals would result in some different form of production. It is the individual's embeddedness within an existing activity system which on the one hand maintains the reproduction of the existing and on the other hand contributes towards the transformation of the existing. This 'transformed', however, is always unknown and can only be identified *a posteriori*

Conclusion

Reforms in school mathematics call for the applications and modelling of mathematics. Mathematics teachers, in South Africa at least, are not conversant with the way of working intrinsic to mathematical modelling. Their engagement in mathematical work, in a context near-similar to that in which mathematical modellers undertake their work, suggests that the activity system of school-teaching mathematics governs, guides and structures their way of working. This activity system does not only point to the individual but to persons acting in a historically-developed structure with all its visible and tacit codes of conduct and ways of doing things

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