

Students' Writing in Mathematics: Some Ideas and Experiences

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In this paper we will focus on a particular aspect of the relationship of mathematics and language: students' writing in mathematics.

Students' writing is generally used in school for the sole purpose of evaluation. That is, students write FOR the teacher to show what they have learned and so that what they produce will receive a grade. From our personal experience, as learners as well as teachers, we believe that writing could also be considered as a learning tool—and a powerful one, in fact.

Once we recognize the educational potential in writing, we must also realize that little of it has been exploited with regard to mathematics learning up to now. Students' writing in mathematics has been traditionally limited to produce answers to given problems, exercises, or test questions. Are these the only forms in which writing could take place in mathematics? We believe that several other combinations of mathematics-related content and writing formats are indeed available and could provide valuable learning experiences for our students.

For example, with regard to the content addressed, we might consider:

- 1) the "solution" of a problem, exercise, theorem;
- 2) a mathematical concept or topic;
- 3) "metamathematical" issues (for example: what is a proof or a definition, different points of view about the foundations of mathematics, conceptions of the nature of mathematics, and so on);
- 4) feelings and attitudes towards mathematics;
- 5) thought processes and strategies employed in the learning of mathematics.

With regard to the format employed, we could consider at least:

- i) schematic reports (as it might be the standard textbook exposition of a proof, or the answer to a problem);
- ii) "stories";
- iii) essays;
- iv) diaries and anecdotes;
- v) dialogues.

In the rest of the paper we will describe a few possible combinations of these contents and formats and we will try to analyze what specific contribution each can provide to the students' learning of mathematics. Whenever possible, we will refer to experiences we have had with students using

that specific form of writing. Most of these experiences involved students in regular mathematics classes, at high school and remedial college level. In a few cases we will also report writing experiences from mathematics education courses offered to prospective and/or in-service mathematics teachers. We believe, however, that similar experiences could be introduced into more content-oriented mathematics courses and at different levels of education.

Creating stories

The idea of asking students to create a story about some mathematical situations is not new. For example, Frederique Papy has often invited children to develop a story that could describe the relationship represented in a given graph. And the idea proved to be successful with respect to both mathematics and language learning.

A first alternative to students' simply writing the solution to given problems could be to ask them to create themselves the text of a word problem. The first author has conducted successful experiments in this sense with both high school and remedial college students. [1] The students were given simple tasks as, for example:

Write some sentences that include a question, such that the answer to the question should be 40×8 .

or:

You have a rectangle that is 14 by 5 and a square that is 5 by 5. Compose any word-problem or question that uses these two figures in a relationship to each other.

The products were then shared and discussed by the whole class. Some inventions were very imaginative, others contained errors and misinterpretations. Being exposed to a wide variety of similar problems, comparing and eventually revising them, was helpful for everybody. Since the student often chose situations from their own experience, they could see better how mathematics could apply to their own lives. The activity also gave students more confidence in reading and solving word-problems from the textbook.

Students could also be involved in the writing of "real" stories, in more open-ended contexts. For example, they could be asked to write a real-life story involving a certain number of mathematical questions. [2] Besides stimulating students' creativity, such activity could involve them in simple forms of problem posing and mathematization. Or the story could focus not so much on technical mathemati-

cal content, but rather on feelings, experiences and impressions about mathematics, its nature and its learning. Giving the opportunity to express such elements of the writer's experience with mathematics could also turn out to be important, for example, in helping to overcome some fears and blocks toward the learning of the subject.

Writing mathematical essays

Writing essays on mathematics-related content could possibly help students clarify and develop their conception of mathematics as well as their understanding of specific topics. Surprisingly, mathematical essays do not seem to have a place in mathematics classes, in spite of the fact that it is the form generally used by mathematicians to communicate results among themselves.

The first author has recently conducted some experiments with remedial college students involving the writing of essays on mathematical topics. The topics included both content that should have been familiar to the students—the commutative property—and content that had not previously been learned and had to be gathered through a brief bibliographic research—De Morgan's Laws. [3]

In the first case, for example, the students were presented with the following task:

The commutative law in mathematics is generally given very little depth in a mathematics class. But this property can be more interesting, and better understood, when extended to many operations in math, and in other fields as well.

Discuss the commutative law in mathematics by considering what the word "commutative" means as you look at various operations in mathematics—addition, subtraction, division, multiplication, exponents, matrix operations, possible others. That is, using examples of mathematical expressions containing a particular operation, does that operation "commute"?

The students were encouraged to look at mathematics textbooks and dictionary definitions, to think of examples and counterexamples in as many domains as possible. They were given a day or two to write a first draft. Following some rather extensive feedback from the instructor, and for some students, also from the writing instructor, the students had two days to write a final draft.

Students' attitudes to such an open-ended assignment and the resulting essays varied considerably in content, style and quality. There were those who kept insisting "tell me what you want me to write," while others understood better the spirit of the activity and were able to profit more from it.

The fact that students were expected to develop their first drafts made the writing of these essays a much more powerful tool for learning than if they had produced only a single draft for evaluation. The task of writing a first draft prior to the discussion with the instructor established the direction the student was taking with the topic. By having written something first, the students had an idea of what to ask and what to discuss in the teacher-student conference.

In the case that a student had not written a first draft (negligence, "didn't know what to do," etc.), the interview consisted mainly of giving suggestions for the student to get started. And mostly what was written about later was a recital of what the instructor had said, with very little of the student's own development. The second and final draft was also an indicator of what the student did with the discussion in the interview. Some students still could not put together what took place in the discussion. Others "blossomed" a little more, and produced papers that were quite different from the original draft.

The activity of writing, consulting, and then rewriting served as a means by which the students could think about the commutative property or De Morgan's Laws. This format of student-teacher interaction, based on the student's ideas, presented to the students a conception of learning different from that which would place the teacher as someone who hands down knowledge.

Essays could also be used to have students think and talk "about" mathematics. In order to have a real, deep understanding of a discipline, it is not enough to know a list of results and facts. It is also important to know and discuss the methodology used to reach these results and the assumptions embedded in the discipline. We think that students could enrich their understanding of mathematics if asked to write essays—and consequently to reflect—on "meta-concepts" like definitions or proofs in mathematics. An experience of this kind was introduced into a number theory course addressed to graduate students in mathematics education. [4] The students were asked to write a short essay in answer to the question: "What characterizes a mathematical definition?" Their products, later compared and discussed in class, showed a wide variety of contrasting opinions about characteristics and criteria for acceptable mathematical definitions. The task consequently stimulated a series of interesting activities concerning definitions in number theory and in other areas of mathematics.

We believe in the opportunity of trying to initiate similar experiences with younger students, in regular mathematics classes.

Expressing feelings and beliefs about mathematics through diaries and anecdotes

We can also ask students to talk "about" mathematics in a different sense, and consequently using different formats. All of us hold feelings and beliefs about mathematics; we believe that these emotional reactions generally have a strong influence on our approach to the discipline and can even lead to misconceptions or hinder understanding and learning. [5] Therefore in school we should also recognize their importance and help students to become aware of them. Besides individual interviews and class discussions, the writing of diaries or anecdotes could also be successfully employed, and may present advantages especially with shy or emotional people. These formats seem especially suitable for this purpose as we need a very "free" format to let students talk of their experiences, opinions, and feelings, with as little formal constraint as possible.

Based on these assumptions, diaries have been used in an

extensive way in a methods course for prospective mathematics teachers for many years now. [6] An excerpt from the outline of the course presented to the students last year can best describe the format of such activity:

One of the activities during the semester will be to maintain a regular journal. Each week students will be expected to write an entry, and to react to what others write in their journals. Everybody (including the instructors) will receive a copy of everybody else's entry.

This will provide an opportunity to reflect on what happens in the class, what we are reading and thinking about, what happens in your student teaching experience. Partly, this is to facilitate communication among students and instructors, to share feelings about the course and about teaching in general.

The diaries provided the student-teachers with a forum for reflecting on and sharing their own feelings, experiences and beliefs about mathematics, teaching and learning. This activity provided multi-directional communication—as the format allowed every student and instructor to communicate directly with each other class member—and to extend the effects of the course beyond the class meetings. Through the diversity of opinions expressed by each individual, diaries helped the students to recognize and appreciate the existence of debate and controversy in the areas of mathematics and education, and to expand their conception of either discipline.

In a more circumscribed way, L. Feldman gave remedial college students the task of writing an anecdote about individual past experiences with mathematics. [7] This proved to be a device that helped some students to realize the origin or cause of their math anxiety, and constituted the first step in trying to overcome it.

Dialogues

Dialogues have often been used very effectively in the literature as a means to convey some content, even with regard to scientific topics. [8] There are in fact some characteristic elements of dialogues that make this format a very interesting didactical tool. First of all, the presence of different characters makes it natural to present and discuss different points of view, and may also help to show and analyze contradictions, paradoxes, difficult points in a topic. Dialogues have also been used to present more “naturally” the process of creating or developing an idea. Besides, dialogues may be considered one of the easiest forms of writing (they often constitute the first written exercise in a foreign language course), and may therefore present less “writing” difficulty for the students.

These positive characteristics suggest the opportunity of involving students in the writing of dialogues, especially in relations to problem solving activity and class discussions.

For example, writing a dialogue could be an appropriate way to report on a problem solving activity, as it may help the writer focus on the process of solving, and therefore identify and analyze different strategies, blocks, etc. Besides helping to reflect “after” the problem is solved on

the methodology employed and possible alternatives, we may argue that writing a dialogue to record our attempts “during the act” of solving the problem itself may also help reaching its solution. Recently, one of us has experienced with surprise the truth of this statement, by writing a dialogue as a means to solve as well as to report on the solution of a mathematical problem [Borasi, 1984].

The task of writing a dialogue could also be assigned to help the students realize the presence of different points of view even in a precise and rigorous discipline like mathematics. The content could again be chosen from philosophical or epistemological issues about mathematics as a discipline, as well as technical notions, methodologies of solution and approach, experiences and feelings about mathematics.

Students might be asked to write dialogues as a means of reporting real debates or group activities that previously took place in class, or on the other hand they could be asked to create imaginary ones. Combinations of these could also be considered and evaluated.

We cannot report yet of experiences conducted in class with the writing of dialogues, although we are planning to do so in the near future.

Conclusions

The ideas and experiences reported in this paper show the possibility of engaging students in a variety of writing activities on mathematics-related content. Without having provided conclusive evidence, we would like at least to suggest the hypothesis that involving students in different writing experiences could contribute in a complementary way to their learning of mathematics. Further research is needed to determine more precisely why and in what sense writing is a learning tool, and what is the specific value and role of the variety of forms that students could employ in their writing of mathematics.

We would like to propose some observations suggested by our experiences about the role of writing for the learning of mathematics, mainly as material for the reader's reflections on the subject:

— Writing can give an opportunity to clarify our understanding of a concept or topic, as when putting down ideas on paper, we are faced with them and we are obliged to recognize their eventual shortcomings. This could provide a starting point for analyzing what we have not understood and for trying to work on it.

— Writing involves an organization of our ideas and notions about a topic. This may prove to be a difficult task, but with a worthwhile pay-off: the realization of relationships and connections existing among the bits and pieces that we have separately learned.

— The students' active involvement in the learning process could be seen as a merit of writing activities over reading a textbook and doing exercises. Advantages and disadvantages of writing versus oral discussion as tools of learning are not so immediately evident. A comparative analysis of these two activities would probably provide interesting insight on the educational value of both.

— Frequently in education, especially in mathematics,

there is a concern for a final draft, a final answer or a final solution: but thinking and learning are ongoing processes. By writing something in several drafts we might help or even "force" the thinking process, allowing us at the same time to take advantage of some form of feedback (from our own response, possibly other sources as well). Writing as a way of thinking and learning may sound almost like a paradox, closely resembling Meno's dilemma: [9] how can we write on something we do not know in advance, or, alternatively, how can we learn by writing something we already know?

Our experiences suggest the value of a stronger interaction between mathematics and writing in classroom instruction. In this paper we have mainly discussed how writing can have a positive role in the learning of mathematics. On the other hand, we might argue that choosing technical content, such as mathematics, could in some cases be of help in learning how to write. In other words, while writing can be a tool for learning mathematics, we should also consider the role that mathematical content can have in improving writing. Exploring the multiple ways in which mathematics and writing can interact with each other could provide valuable information about learning in both activities.

Notes

[1] This experiment of creating word problems from a given arithmetic problem was conducted primarily in an adult education setting, at the Educational Opportunity Center of the State University of New York at Buffalo, in Fall 1979 and Spring 1980 [Stempien, 1985]

[2] A task of this kind has been given as an optional assignment to college students in a remedial mathematics course taught by M Stempien at the State University College of New York at Buffalo in summer 1982. The number of responses was too limited to justify reporting the results of this experience.

[3] The writing of essays on the commutative property was done by students who were ready to enter a four year college but indicated weaknesses in reading, writing, and arithmetic (State University College of New York at Buffalo, summer program, 1983) The students who wrote essays on De Morgan's Laws were taking a course entitled "Math Survey" to fulfill a requirement for graduation in a two year college (Erie Community College, Buffalo N.Y., Fall 1983) Both experiences are currently being written about. [Stempien, 1984]

[4] R Borasi conducted the experiment as teaching assistant in the course "Seminar in Number Theory" taught by Stephen I Brown at the State University of New York at Buffalo in Spring 1984

[5] In this we agree with the rationale behind Thomas Cooney and Stephen I Brown's current project on the beliefs of mathematics teachers, and the relationship of these beliefs to their teaching, as described in the proposal. [Cooney, Thomas J and Stephen I Brown

"Conceptual Systems and Decision Making in Teaching Mathematics" Project Proposal presented to and accepted by the National Science Foundation, 1981]

[6] In 1973, Stephen I Brown introduced diaries in the course "Methods for Teaching Secondary School Mathematics" regularly offered at SUNY/Buffalo in the undergraduate program for obtaining teaching certification for New York State. Other doctoral students, including the authors, have used diaries in teaching the same course in following years.

[7] I. Feldman taught courses in the University Learning Center at SUNY/Buffalo in which students were given college credits to improve their writing, mathematical, and reading skills. The experience we are referring to here occurred in Spring 1983 and Spring 1984

[8] Think, for example, of Galileo's famous dialogues, Knuth's novelette entitled "Surreal Numbers," or the use of dialogues in Hofstadter's "Gödel, Escher and Bach."

[9] Plato, in his dialogue "Meno," stated the following dilemma: "A man cannot enquire about that which he knows or about that which he does not know; for if he knows, he has no need to enquire; and if not, he cannot; for he does not know the very subject about which he is to enquire." Petrie [1981] has discussed some interesting educational implications and possible solutions for this dilemma

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