Problems in the real world of mathematics education

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About 18 months ago, my spouse and I became very interested in New England contra dancing This is a kind of dance in which even klutzes like me can engage without undo embarrassment A lack of expectation of leading or following accompanies a tradition of inflicting oneself on a different partner for every dance Contra dances take place in pairs of lines, alternating men and women, with partners starting across from each other. After a sequence of figures within a group of four, alternate couples move in different directions to new groups of four, in which they repeat the sequence. In other words, the composite of a sequence of permutations on a string *abcd* results in the string *cdba*

Now many would find this description consistent with the cry I used to hear, often from middle-school mathematics teachers, that "Math is everywhere!" It's a phrase that has always intrigued me. What is mathematics, that it should be everywhere? Is this simply an oversimplification of the Pythagoreans' "All is number"? Rather, these teachers seemed to be referring to something else - to the fact that there were things all around us that could be counted or added or measured or described through geometric shapes or Fibonacci numbers or even fractals Is this really mathematics everywhere?

Indeed, over the past ten years or so, mathematics educators have tried to be more clear about this question. Encouraged by the U.S NCTM *Standards* document of 1989, we began to talk about 'modeling'. It is not the mathematics itself that is all around us; rather, we can use mathematics to *model* many aspects of our experience. We can, if we choose, describe the figures of a contra dance as rotations and reflections, resulting in a translation.

But why should we do so? For contemporary-educated mathematics educators, the term "real world" immediately leads to "problems" and then the synaptic leaps to "solutions" and "problem solving" and "mathematical models" are almost instantaneous "Motivation" and "engagement" and "inquiry" and "constructivism" may not be far behind

We even have nice, coherent diagrams: a real-world problem is represented by one corner of a rectangle; we mathematize the problem by setting up a model, in another corner of the rectangle. We move along a side of the rectangle as we solve the mathematical problem. And then we translate the solution back to the real world, thus placing a solution to the original problem in the fourth corner of the rectangle. And we very carefully explain to students that the process is not usually so clean; we have to cycle through the rectangle several times, re-modeling to get a solution that is reasonable.

But then where does contra dance fit in? The only problem in sight seems to be "How can you represent aspects of contra dancing mathematically?" Without a real problem to be solved through modeling, is there no mathematics?

In *Reconstructing School Mathematics: Problems with Problems and the Real World,* Stephen Brown [1] challenges us to look beyond our emphasis on mathematical modeling, stretching to see other, perhaps more valuable, connections between mathematics and our students' lives. Brown has been writing about these themes for over thirty years Now, as if to commemorate his retirement, he has compiled and reorganized them in some new and fascinating ways. The ideas include not only the philosophy and psychology one would expect from a leading mathematics educator, but also the connections with humor, theology, literature and feminism that have appeared in his many journal articles.

Those who have not read much of Brown's work may be asking, "Huh? Humor and theology in mathematics education?" Indeed

The message

It would be appropriate to expect that a book with the subtitle "Problems with Problems and the Real World" might discuss the real world. Actually, Brown reserves the question "What is the real world?" until fairly late on in the book. My own hesitance in using this term began years ago when I realized that the "real world" of most of my students was quite different from my real world Their concerns are not with mortgages and long-distance travel, but with relationships and cars and CDs and relationships and sports and relationships. I also noted how we and they commonly say such things as "that point drawn on the board is not a real point, because real points are infinitely small", indicating that the real world of geometry (and other mathematics) exists in the Platonic realm.

But Brown goes in a different direction. Along with Nozick (1990), he points out that some characters in literature, though fictional, are more real than the people we encounter daily Some works of art are intensely real. Some live people are more vibrant than most of us And some mathematical ideas - irrational numbers, infinity, paradoxes - have proved especially attractive to mathematicians over the centuries. Might we not use these ideas, then, to engage students, perhaps more successfully than we use "applications"?

Most of the book, not unexpectedly, is about problems. Brown has written a great deal about problem posing, especially using the "What-If-Not?" format (Brown and Walter, 1990), and he reviews that But much of his writing is asking "What-If-Not?" about the current modeling *Zeitgeist*. How else might mathematics connect with the lives of our students? Here are a few of the alternatives he suggests:

1) Ask "What-If-Not?" not only after one has solved a problem but even before attempting to solve it

2) Teach "with the expectation that errors, questions from left field, and confusions have generative power [... recognizing that] even supposedly naïve students are capable of changing the way we view any aspect of knowledge or experience" (pp. 86-87).

3) Give students a problem and its solution and then ask questions such as:

   a) "Why might this solution have been difficult to arrive at even though it's fairly easy to follow?"

b) "Can you imagine doubting this result before you knew its derivation?"
c) "Can you create such doubt for yourself now?"
d) "What about this problem or solution might reflect the social structure in which it arose?"
e) "What about this solution leads some people to claim it's elegant?"
f) "Have you encountered other mathematical ideas that might be described that way?"
g) "Are these ideas similar to mathematical ideas you have seen in cleverness, emotionality, surprise or potential?"
h) "How do you feel about discussing the solver's thought process and elegance and social context in a mathematics course?"
i) "Are there similar phenomena in your (non-mathematical) life?"

Educational mathematics

The last few questions on that list might very well provoke the reaction, "But those are not mathematics questions!" Even granting that most of them might help students understand better the nature of mathematics or problem solving, one might point out that the last question is blatantly non-mathematical And therein lies a great deal of what Brown is trying to say

Common diagrams representing the process of mathematical modeling, such as the rectangle described above, have the "real world" on one part of a geometric figure and the "mathematical world" on a different part. Brown claims that the modeling concept represented in this kind of picture communicates to students that mathematics is something separate from their worlds. But not only was mathematics designed by human beings; not only does it contain controversies and emotions like the rest of the world; but, like other disciplines - from art to science to the social sciences - it can provide metaphors to help us make sense of our experience.

What metaphors? As examples, infinity can be a metaphor that brings insight to theology; isomorphism can be a metaphor for non-superficial similarities; distributivity a metaphor for love. Speaking meta-mathematically, recognition that some of the constraints of problems lie in the solvers and insights into mathematical existences and truth; can help all of us understand these same phenomena in other aspects of our lives Even in contra dancing, mathematics offers metaphors beyond permutation groups, isometries and Möbius strips. We can achieve insights about how we discern patterns, relate to other dancers and imagine ourselves in the place of our partner

By encouraging our students to think about such mathematical metaphors, we can go beyond deepening their understanding of mathematical ideas. We can even go beyond critical and creative thinking We can actually help them become more educated people - having a deeper conception of knowledge as a whole, being more tolerant of others' ideas, thinking in more sophisticated ways about moral issues and being more aware of their own thinking and selfhood

To provide these metaphors, we can choose mathematical problems that lend themselves to such discussions Brown gives several examples. Here are two:

1) A close relative of yours has been hit by an automobile He has been unconscious for one month. The doctors have told you that unless he is operated upon, he will live but will most likely be comatose for the rest of his life. They can perform an operation which, if successful, would restore his consciousness. They have performed ten such operations in the past and have been successful in only two cases In the other eight, the patient died within a week What counsel would you give the doctors? (p. 142)

As Brown says, to make a decision here, "mathematical consideration is one important dimension, but it is part of a larger cloth" (p. 142).

2) (paraphrased) A debonair census taker visits a home in which an attractive woman answers the door. In their initial conversation, they find that he's a mathematics teacher who's on his summer job, that she gave up mathematics teaching because she was unsuccessful in teaching contrived word problems and that she's recently divorced. A potential romantic liaison is suspended as the census taker gets down to business and asks the ages of the woman's three daughters. She answers, naturally enough, with a word problem: the product of their ages is 72 and the sum is the house number After working on the problem for a few minutes, the census taker says he needs more information, and the woman volunteers that her oldest daughter has a cat with a wooden leg. Then he says he's solved the problem. The question, then, is "What are the daughters' ages?" (pp. 156-158)

In the context of solving the problem, Brown points out that we have been taught to abstract - to ignore the fact that it is a census taker meeting a woman, that they are both interested in mathematics and each other, etc. But if we abstract too much, we miss the essential fact that because he is a census taker he knows the house number.

> We almost have to picture ourselves *in* the situation [...] Before pretending that you are the person actually taking the census, it seems perfectly reasonable to accept the conclusion that you do not have enough information. (p. 159)

And being able to put yourself in another's situation is essential for appreciating the ethical principle of impartiality:

> we cannot justify our own behavior as moral and that of someone else as immoral if the other person were in a position that is essentially the same as ours. (p. 154)

If we let explicit discussions of ethical principles like this flow out of good problems, we are using our mathematics classroom to help educate students.

In a conversation some twenty-five years ago, Brown speculated that what he was interested in might better be called "educational mathematics" than "mathematics education" [2] Although I did not notice this phrase in his book, it still describes his primary question: how can we teach mathematics with an aim toward educating students more broadly than helping them make meaning of mathematics?

How?

In addressing ways in which we might teach more educationally, Brown shows his tendency to apply "What-If-Not?" creatively to many dearly-held adages of mathematics education. For example:

- *Errors must be corrected immediately so that students won't learn the wrong thing.*

What if the concept of being wrong did not apply? Could our students become more critical and independent thinkers if they were expected to critique all claims made? What if the naïve conception of right and wrong answers were as irrelevant to the mathematics classroom as to the English classroom?

- *Teach from the simple to the complex and from the concrete to the abstract*

Does doing so often prevent students from seeing the human process of simplifying complex situations, remove what they might recognize as real and hide common structures?

- *Good teachers do not confuse their students.*

Can confusion from the interplay of intuition and rigor serve as a source of wonder?

- *The best assignments are completely unambiguous*

Does this message convey that mathematics is different from the rest of the world? What if we intentionally use ambiguity and ask students to analyze what is ambiguous?

- *Students need closure, resolution.*

Does resolution close off further thinking? Does it communicate an unrealistic conception of mathematics and knowledge?

Some further reactions

I have several reactions to this book besides the above.

1 Despite the questions above, the book has almost nothing to say about the real-world problem of what teachers should do tomorrow in their classrooms As Brown says:

> It is easier to advocate than to put in practice a view of mathematics education that supports:
>
> 1) the classroom as open dialogue;
> 2) the holding of resolution in abeyance;
> 3) the potential of errors not only to diagnose misperceptions but to generate new territory;
> 4) having students record the evolution of their thinking over a protracted period of time;
> 5) integration of mathematical experiences with other fields in a more global and pervasive manner than is suggested by "applications";
> 6) using the discipline to reflect upon an understanding of self and society. (p. 201)

It is up to others of us to think about how this might help teachers who face pay cuts or job losses if their students do not perform well enough on standardized tests Brown's suggestions of how materials might be written to have more "intrinsic interest" than our standard textbooks (novels or a Talmudic format), though fascinating, do not answer, "What script do I follow with my 150 students on Monday?"

2 Or, for that matter, what do I do with the parents of those 150 students? Too many of them do not really care; but many of those who do care have youngsters who are "good at math". Will these students be successful in a scientific or engineering or mathematical career? In describing studies of how child prodigies think while solving mathematics problems, Brown questions what might be jeopardized if this kind of approach replaced a curriculum focused on preparing youngsters for "thinking like a mathematician" (p 202). Although one may certainly join Hadamard (1945) and Poincaré (1908/1961) in arguing that metaphors are part of good mathematical thinking, turning the focus of the curriculum away from mathematics itself might diminish the population of future mathematicians

3. Can we find an intermediate stance, introducing the potential for educational mathematics into the daily life of teachers? Some efforts are being made in that direction. Here are two personal examples.

a) In recent summers, I have helped run a couple of mathematics workshops for para-professionals in elementary schools. In the brief time available, we challenge them to learn ways of teaching to promote inquiry. One para-professional came in one afternoon excited about what she had done while helping with a summer school class that morning The teacher had asked her to take aside two youngsters who were being disruptive while the other students worked on a set of drill problems in arithmetic The para-professional had simply added to the instructions at the top of the sheets of these two students: "Solve these problems in as many ways as you can". One of the students managed to stick with the sheet for 20 minutes, apparently a minor record; the other amazed the para-professional and teacher by staying "on task" for an hour and a half. More importantly to me, the revised task at least potentially communicated a conception of mathematical knowledge different from the one to which the students were accustomed From a practical point of view, it also potentially freed them from "freezing" on tests because they could not remember "the right way" to solve a problem.

b) A research project underway in Minnesota is examining the primary influences on beginning mathematics and science teachers. What affects their teaching style the most: how they themselves were taught, what the instructors of their methods courses said they should do, advice they get from colleagues in their first job, etc ? Researchers at one college are beginning to see that a main influence on beginning mathematics teachers is the textbook, along with materials they borrow from colleagues In these, they find a "script" to follow so do not sink in the morass of unfamiliar responsibilities

If they are going to be following a script, can we at least give them a good script to follow? Over the past two years, I have been writing and editing teacher's editions for the *Discovering Mathematics* curriculum series (Murdock *et al*, 2002; Serra, 2003). I am trying to help teachers grow toward a new view of mathematics, to be passed on to their students. I cannot claim to have progressed as far as I want, much less as far as Brown would advocate, but there is one more volume to go!

4 Many of Brown's ideas might be considered to be about transfer of mathematical experience to other fields. How do these unusual ideas fit into the research on those questions?

5 Then there is the problem of boys Brown describes how his ideas are consistent with feminism (p. 166). But, for better or worse, by nature or by nurture, research such as that of Pollack (1998) shows that many boys do not learn well in that kind of classroom. They work best alone. They do not think best out loud. They want to compete, not cooperate They want right answers, not more questions. They would much rather do something than make words about it. They rebel at talking about their feelings, even if those feelings are of low self-esteem induced by the academy's claims that they are inadequate In years of making schools more girl-friendly, we may have alienated another half of the population. Indeed, males among readers of this journal may be different, just as readers of this journal were different from most students in mathematics classrooms. And yes, boys will thrive better in the world outside of school if they can acquire these skills and adapt these attitudes. But it is not their mode of operation now. What do we do on Monday with a bunch of boys? I am still groping for answers to this one.

Opensure

There is much more to say about the book. For example, Brown also discusses constructivism "gone awry"; the value of resisting changing our intuition in light of proof; the learning that can take place from explicitly examining relationships between mathematical and other uses of the same words. The book also contains enough errors to embarrass a professional copy editor.

At least since Thomas Kuhn (1962), the word *paradigm* has been overused Much that is merely new has been touted as a "paradigm shift", as if it offered a radically new way of looking at part of the world An internet search turned up "about 161,000" entries, claiming as new paradigms everything from the millennium to cars and recipes. Within the field of education, the list includes small groups, on-line courses and a focus on learning instead of teaching.

So I am very hesitant to describe this book as a new paradigm. But Brown's ideas are consistently different enough from those of most others in mathematics education that I suspect they will be ignored or resisted by many as if they were in a new paradigm. At the minimum, it is appropriate to ask whether or not we have here wholly new lenses through which to view our worlds

Brown calls these lenses "humanistic", but they have a different focal point from those most easily identified with the Humanistic Mathematics movement (White, 1993): mathematics as a human activity and teaching mathematics with consideration for how humans think and feel and learn Rather, Brown's lenses concentrate on teaching mathematics as a tool for education, rather than on what we teach about mathematics or how we teach it

The ideas here are so unorthodox that, in comparison, many current discussions in mathematics education - about "profound understanding", correcting errors, inquiry-based learning, constructivism, problem solving, "math wars" or a new way to teach quadratic equations for deeper understanding - seem pale. The fact that Brown's responses to the NCTM Standards are to the 1989-95 versions, not the latest, seems immaterial. These conversations are on different axes altogether.

Just as unconventional as Brown's ideas, though, is his playfulness. Through word play, self-reference and ceaseless questioning, he communicates the fun of the intellectual dance. Can we hope to do more for our students?

Notes

[1] Brown, S. (2001) *Reconstructing School Mathematics: Problems with Problems and the Real World*, New York, NY, Peter Lang. (ISBN: 0-8204-5103-7)

[2] The name of the institute I direct came from that discussion Brown was my doctoral advisor from 1970 to 1972.

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