

THE MENOUSA

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Have you ever wondered what the wives of participants in Plato's Academy got up to when their husbands were being used to display problems about knowledge, truth, and beauty? Have you ever wondered what Socrates' mother Phaenarete was doing when her son left his stone-masonry and went off to the Academy? What did Socrates' wife Xanthippe get up to when he was at the Academy, leaving her free of his interminable questions, [1] and what about his second wife Myrto?

Plato has Socrates describe himself as a midwife for ideas, by analogy to his mother Phaenarete who is a midwife (*Theaetetus* §148-9 in Jowett, 1871). It seems not unreasonable therefore that midwifery might have been a model for some of his practices. Xanthippe has generally received only one-sided bad press, and so it seems not unreasonable to suppose that Socrates' view of her as a difficult woman could perhaps be rephrased as being also, like him, a constant questioner.

Imagine therefore that when Meno went to Socrates to ask his question about whether virtue can be taught, his wife, who might be called Menousa, [2] had a friend and neighbour due to give birth. She might very well call upon Phaenarete to come and assist, only to find that Xanthippe has come as well. The birth is delayed, and so to pass the time, the three engage in discussion, Xanthippe asking questions and Phaenarete making notes. Menousa is curious about the theories she has heard Meno discussing concerning whether knowledge is something that is created or recollected, and so she asks the question which she knows is uppermost in her husband's mind: can virtue be taught? She challenges Xanthippe to elucidate, and in the course of the discussion the topic turns to the origins of knowledge and belief.

Xanthippe suggests that Menousa summon one of her slave girls in order to illustrate the point. The dialogue we present here sticks closely to the original Meno dialogue in the Jowett translation (1871). The difference lies in the mathematics, which is parallel to, but distinct and less well known. It suggests an extension or variation of what occurs in the Meno mathematically. We include the full dialogue to provide a slow emergence of the mathematics, [3] and to provide a suitable context for our later comments. Our purpose in doing this is (a) to explore some interesting mathematics, (b) to remind ourselves about some ancient thought about how knowledge might emerge in dialogue, and (c) for fun.

The Menousa

Xanthippe: It will be no easy matter, but I will try to please you to the utmost of my power. Suppose that you call one of your numerous attendants, that I may demonstrate on her

Menousa: Certainly. Come hither, girl.

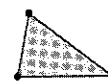
Xanthippe: She is Greek, and speaks Greek, does she not?

Menousa: Yes, indeed; she was born in this house.

Xanthippe: Attend now to the questions which I ask her, and observe whether she learns of me or only remembers.

Menousa: I will.

Xanthippe: Tell me, girl, do you know that a figure like this is a triangle?



Girl: I do.

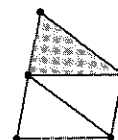
Xanthippe: And you know that a triangular figure has three edges, not necessarily equal?

Girl: Certainly.

Xanthippe: A triangle may be of any size or shape?

Girl: Certainly.

Xanthippe: And if the whole is repeated twice more, as here, then the whole is three times the original triangle?



Girl: Yes.

Xanthippe: And might there not be another triangle just thrice as large as the shaded triangle?

Girl: Yes.

Xanthippe: And now try and tell me the length of the lines which form the side of that treble triangle: what will the sides be?

Girl: Clearly, Xanthippe, they will be treble.

Xanthippe: Do you observe, Menousa, that I am not ~~teaching the girl anything, but only asking~~ her questions; and now she fancies that she knows how long a side is necessary in order to produce a triangle of three times the area; does she not?

Menousa: Yes.

Xanthippe: And does she really know?

Menousa: Certainly not.

Xanthippe: She only guesses that, because the triangle is treble, the sides are treble.

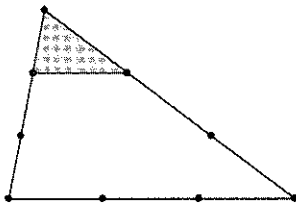
Comment: Xanthippe has led the girl to articulate a common assumption. We would probably not call this a 'guess' since it is likely to derive from experiences in which trebling one variable does depend on trebling another

Menousa: True.

Xanthippe: Observe her while she recalls the steps in regular order. [To the girl:] Tell me, girl, do you assert that a treble space comes from a treble line? Remember that I am not speaking of a trapezium, but of a triangular figure thrice the size of the original; and I want to know whether you still say that a treble triangle comes from a treble line?

Girl: Yes.

Xanthippe: But does not this line become trebled if we add another such line here?

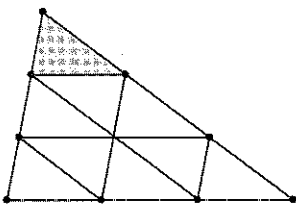


Girl: Certainly

Xanthippe: And three such lines will make a space containing the trebled triangle?

Girl: Yes

Xanthippe: Let us describe such a figure: Would you not say that this is the figure of trebled lengths?



Girl: Yes.

Xanthippe: And are there not these nine divisions in the figure, each of which is equal to the original figure?

Girl: True

Xanthippe: And is not that three times three?

Girl: Certainly.

Xanthippe: And nine times is not treble?

Girl: No, indeed

Xanthippe: But how much?

Girl: Nine times as much.

Xanthippe: Therefore the treble line, girl, has given a space, not treble, but nine times as much

Girl: True.

Comment: Xanthippe has used the girl's suggested construction and shown her that it leads to something else, not the trebling of the triangle.

Xanthippe: What line would give you a space of treble, as this gives one of nine times; do you see?

Girl: Yes.

Xanthippe: Such a space, then, will be made out of a line greater than this one, and less than that one?

Girl: Yes; I think so

Xanthippe: Very good; I like to hear you say what you think. And now tell me, is not this a line of treble that?

Comment: Either the girl is happy that more length means more area, and less length means less area, so between length must mean between area; or else she is agreeing out of obedience.

Girl: Yes.

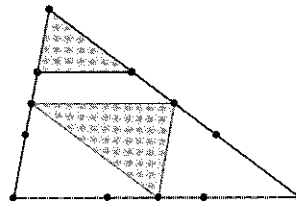
Xanthippe: Then the lines which form the sides of the treble triangle ought to be more than these original lines, and less than the others which we trebled?

Girl: They ought.

Xanthippe: Try and see if you can tell me how much it will be

Girl: Three halves.

Xanthippe: Then if we bisect each side of this triangle, we will end up with a new triangle and that makes the figure of which you speak?



Girl: Yes.

Xanthippe: But if this triangle is half as much on each side, there will be four of them to make up the whole triangle?

Girl: That is evident.

Xanthippe: And is nine fourths the same as thrice?

Girl: No. [4]

Xanthippe: Then the figure of treble is not made out of a half a line of three?

Girl: No

Xanthippe: But from what line? Tell me exactly; and if you would rather not reckon, try and show me the line.

Girl: Indeed, Xanthippe, I do not know.

Comment: In this article we have not explored the power relations in this dialogue. Some people choose to see it as taking place in a relation of oppression rather than expert-apprentice, with the only benefit for the slave being enforced loss of ignorance. Others see it as a depiction of how what is taken as knowledge can be challenged through guided enquiry

Xanthippe: Do you see, Menousa, what advances she has made in her power of recollection? She did not know at first, and she does not know now, what is the side of a figure treble the triangle: but then she thought that she knew, and answered confidently as if she knew, and had no difficulty; now she has a difficulty, and neither knows nor fancies that she knows

Menousa: True

Xanthippe: Is she not better off in knowing her ignorance?

Menousa: I think that she is.

Xanthippe: If we have made her doubt, and given her the 'torpedo's shock,' [5] have we done her any harm?

Menousa: I think not

Xanthippe: We have certainly, as would seem, assisted her in some degree to the discovery of the truth; and now she will wish to remedy her ignorance, but then she would have been ready to tell all the world again and again that the treble space should have a treble side

Menousa: True.

Xanthippe: But do you suppose that she would ever have enquired into or learned what she fancied that she knew, though she was really ignorant of it, until she had fallen into perplexity under the idea that she did not know, and had desired to know?

Menousa: I think not, Xanthippe.

Xanthippe: Then she was the better for the torpedo's touch?

Menousa: I think so.

Xanthippe: Mark now the farther development I shall only ask her, and not teach her, and she shall share the enquiry with me: and do you watch and see if you find me telling or explaining anything to her, instead of

eliciting her opinion. Tell me, girl, is not this a triangle which I have drawn?

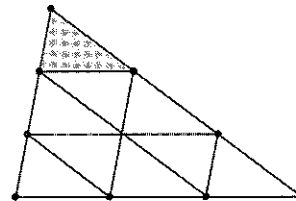
Girl: Yes

Xanthippe: And now I add three other triangles equal to the former one?

Girl: Yes

Xanthippe: And five more, which are equal to all the others?

Girl: Yes



Xanthippe: Here, then, there are nine equal spaces?

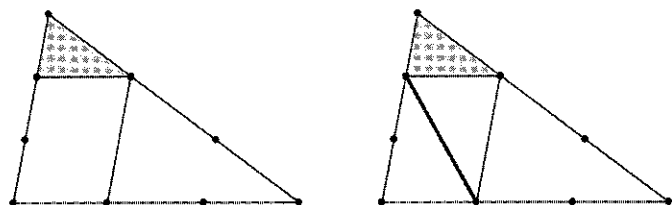
Girl: Yes

Xanthippe: And how many times larger is this space than this other?

Girl: Nine times.

Xanthippe: But it ought to have been thrice only, as you will remember

Girl: True



Xanthippe And does not this parallelogram region form four times the original triangle?

Girl: It does.

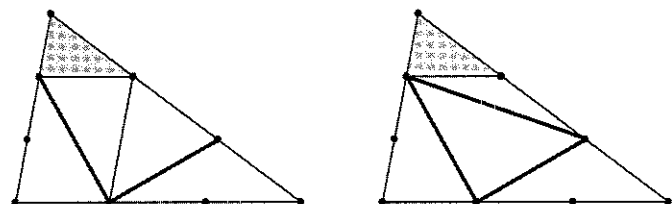
Xanthippe: And does not this line, reaching from third-point to third-point bisect that region?

Girl: So it seems.

Xanthippe: And so is each of these regions not then twice the original?

Girl: It is

Xanthippe And does not this line joining two of the third-points cut off a region twice the original triangle?



Girl: Indeed it is so, Xanthippe

Xanthippe: Finally, this third line, does it not also cut off twice the original triangle, leaving a new triangle in the centre?

Girl: Yes, Xanthippe, it is so.

Xanthippe: Look and see how much this space is in the middle.

Girl: I do not understand

Comment: The slave girl is having to think about harder questions than Meno's slave boy; it is possible that she has to retrace the construction herself in order to 'see' what is being indicated. And we notice that here she is not allowing herself to be bullied into acquiescence.

Xanthippe: Has not each interior line cut off two of the original triangles?

Girl: Yes.

Xanthippe: And how many spaces are there altogether?

Girl: Nine.

Xanthippe: How many spaces are there cut off here?

Girl: Two.

Xanthippe: And here?

Girl: Two.

Xanthippe: And here?

Girl: Two.

Xanthippe: Thrice two is how much?

Girl: Six, Xanthippe.

Xanthippe: Which leaves how many for the new triangle??

Girl: Three.

Xanthippe: That is, from the line which extends from third-point to third-point of the figure of nine triangles?

Girl: Yes.

Xanthippe: And that is the line which the learned call the trigonal. And if this is the proper name, then you, Menousa's slave, are prepared to affirm that the trigonal space is the treble of the original?

Girl: Certainly, Xanthippe

Xanthippe: What do you say of her, Menousa? Were not all these answers given out of her own head?

Menousa: Yes, they were all her own.

Xanthippe: And yet, as we were just now saying, she did not know?

Menousa: True

[They then, as in the conversation between Meno and Socrates, discuss further the provenance and sustainability of the slave-girl's knowledge]

Commentary

Although the Socratic dialogue is a much praised form of instruction, its educational function has been vigorously debated, probably ever since the dialogues were first made available through the medium of print. For example, the Society for the Furtherance of Critical Philosophy (SFCP, 2006) suggests that 'the Socratic Method encourages participants to reflect and think independently and critically' and that 'the endeavour of the group is to reach consensus, not as an aim in itself, but as a means to deepen the investigation'. Others (Higgins, 1994; Kerdemann, 1994) consider that during the whole dialogue Meno comes to realise that he does not know what he thought he knew, and begins to appreciate new possibilities, including Plato's theory of knowledge as recollection from previous existence.

When people point to 'the Meno' as a typical Socratic dialogue, are they pointing to the interaction with the slave about geometry, or to what Socrates does with Meno? Is the geometry lesson really a form worthy of being emulated? What assumptions are being made about learning? Max Wertheimer (1945/1968) asks similarly "Does he repeat what he has heard blindly, like a slave, or has he grasped, has he understood?" (p 80).

Perhaps an alternative formulation might be that having one's attention suitably directed can transform ignorance or mistaken conjecture into understanding. The slave girl knows how to make sense of Xanthippe's questions, and agrees that there is a contradiction (we could wonder if she felt she *had* to agree). Xanthippe is doing more than just offering a contradiction, however, because she has structured her questioning to build on the slave girl's own reasoning, midwifing new perceptions. Focusing on *movements* of attention requires more than simply directing attention to something, because there are different ways of attending to the same thing (Mason, 2003).

Plato believed that this successful shift of understanding was due to uncovering recollection, a view now less tenable, but what remains educationally insightful is the notion that learners are unlikely to be motivated to enquire unless they encounter a challenge to their thinking. This notion has been repeated many times in Western philosophy, in various forms. Martin Heidegger (1927), Edmund Husserl (1973), and Wertheimer (*op cit* p 49) all emphasise the importance of disturbance to initiating action. John Dewey made significant use of disturbance as that which initiates action, drawing on theories of John Locke:

Conflict is the gadfly of thought. It stirs us to observation and memory. It instigates to invention. It shocks us out of sheep-like passivity, and sets us at noting and contriving. Not that it always effects this result; but that conflict is a 'sine qua non' of reflection and ingenuity (Dewey, 1922, p. 207).

In cases of striking novelty or unusual perplexity, the difficulty, however, is likely to present itself at first as

a shock, as emotional disturbance, as a more or less vague feeling of the unexpected, of something queer, strange, funny, or disconcerting. In such instances, there are necessary observations deliberately calculated to bring to light just what is the trouble or to make clear the specific character of the problem. In large measure, the existence or non-existence of this step makes the difference between reflection proper, or safeguarded critical inference and uncontrolled thinking (Dewey, 1933, p. 74)

Leon Festinger (1957) coined the phrase *cognitive dissonance* for the same notion.

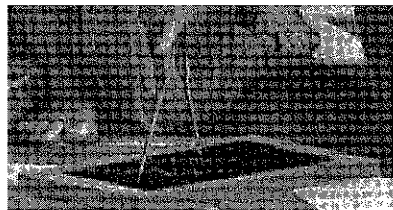
... dissonance, that is, the existence of nonfitting relations among cognitions, is a motivating factor in its own right ... Cognitive dissonance can be seen as an antecedent condition which leads to activity oriented toward dissonance reduction just as hunger leads to activity oriented toward hunger reduction. (p. 3)

While several learning theories assume that contradiction leads inevitably to a reorganisation of understanding, in 'the Menousa' above, as in the Meno, we recognise the linearity assumption (doubling sides doubles area, trebling sides trebles area) as a persistent naïve conceptualisation even when other relations are understood (De Bock, Verschaffel & Janssens, 1998) Fischbein (1987) suggested that we never displace intuitive pre-conceptions, but at best, through education, they can be overlaid. A more interesting question than 'did the slave girl remember this from somewhere?' is 'will she remember this in future similar situations without the supportive catechism?' We know she can answer these questions as posed, with a relevant diagram, but we do not know if she can reconstruct the argument, or its conclusions, for herself. The same applies to the Meno, of course, and our reconstruction here in a less obvious mathematical context has served to put us in the position of Menousa. What might Menousa be learning and how is she learning it? Plato thought that Meno was learning that knowledge is recollection; we think that Menousa is learning that what appears to be knowledge and insight can be constructed by a knowledgeable teacher creating a suitable environment and coaxing the obedient student, one-to-one, through it.

Currently in the UK and elsewhere the latest remedy for teaching is 'questioning'. Socratic dialogue is one of the 'methods' on offer and often alluded to if not actually called upon. We offer our dialogue as a counterweight to superficial adoption without serious thought about task design.

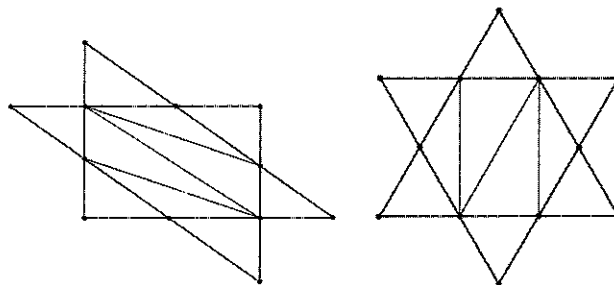
Origins of tripling the triangle

The idea of the transposed dialogue arose when Anne Watson returned from viewing Raphael's *School of Athens*, in the Vatican. [6] In the bottom right hand corner is a figure,



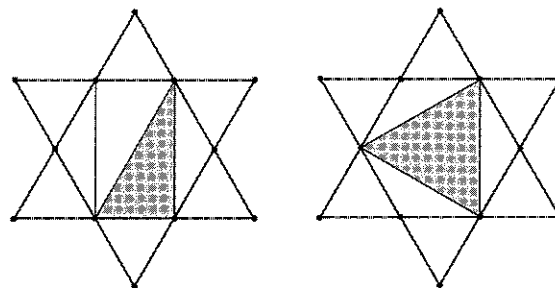
usually described as Euclid, with a pair of dividers applied to a diagram on a slate. Anne wondered what Euclid was working on and developed the tripling argument as follows:

The diagram appears to be as follows (seen on the left in perspective, and on the right as a Euclid-eye-view):



The diagram on the slate is probably two equilateral triangles overlapping, but of course presented in perspective by Raphael. What significance can be found in the two right-angled triangles in the centre of the figure?

The exploration went this way: the triangles intersect each other in such a way that their sides are divided into three equal lengths, and we thus have equilateral triangles of sides 1, 2 and 3 units. The shaded triangle is right-angled, as can be seen by angle-chasing with parallel lines and/or equilateral triangles. This shaded triangle has equilateral triangles supported on two of its sides whose areas are in the ratio of 1:4. Since Pythagoras' theorem is a central concern of Euclid, this suggests seeking an equilateral triangle on the third side.



A clue can be found in the symmetry of the diagram: the two other 'diagonals' through the centre of the figure (in the dialogue we chose to call it a 'trigonal') provide the required equilateral triangle. We have constructed a triangle that is three times the area of the smallest triangle in the diagram, and one-third the area of the largest.

Why did Xanthippe not offer this approach to the slave girl? It seems to us that if the Meno dialogue has something to offer mathematics pedagogy, it is not so much in the chains of questioning, but in the structuring of those questions. They depend on the teacher knowing about intuitive understandings and constructing situations that not only expose contradictions but also offer new ways to 'see'. They depend on assumptions about how people apply intuitive knowledge in unfamiliar situations. In both dialogues, the teacher is not merely providing stock 'What if ...?' questions, but has a plan with a knowledgeable endpoint. Our own exploration of the diagram started from knowledge about the Pythagorean connection between equilateral triangles on the sides of right-angled triangles, and a few other

theorems; the slave's starting place is to assume linear scaling will work. Thus Xanthippe's approach uses, as well as confronts, current assumptions. There is not only cognitive conflict at work here, but also constructive conceptual development. It is not only the pedagogy that has been designed but also the task, as a sequence of questions.

What then happens when the diagram is viewed in perspective, as in the original picture? Because of the perspective, the shaded triangle in our right hand figure is always three times the area of the smallest triangle and one third of the largest hence the inspiration for the dialogue.

Our exploration continues however, because it is very natural for mathematicians to generalise. If you divide the edges of a triangle in the ratios of $l:1$, $m:1$ and $n:1$ respectively, then the ratio of the areas of the inner and outer triangles turns out to be

$$\frac{1 + \lambda\mu\nu}{(1 + \lambda)(1 + \mu)(1 + \nu)}$$

For example, if the ratios are each 1:1, then the area ratio is 1:4 as is well known (and essentially, in terms of squares rather than triangles, the content of the original Meno dialogue). If the ratios along the edges are each 2:1 then the area ratio is 1:3. Inspired, you might try division ratios of 3:1 only to find an area ratio of 7:16

An alternative formulation of the area ratio is as follows. If you treat the division points as dividing the edges in the ratio of $a:(1 - a)$, $b:(1 - b)$ and $c:(1 - c)$ respectively, then the ratio of the edges is

$$abc + (1 - a)(1 - b)(1 - c),$$

which has a certain symmetry about it.

The question then arises as to whether there are any other simple fractional ratios that give rise to a simple fractional area ratio, akin to the two simplest ones (essentially, the Meno and the Menousa). If the same ratio is repeated on each edge, say $a:(1 - a)$ then the area ratio is $3a^2 - 3a + 1$. There are some other intriguing quadruples (a , b , c , and the area ratio) to be found:

$$\frac{1}{2}, \frac{1}{2}, \frac{1}{n}, \frac{1}{4} \quad \frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{2} \quad \frac{2}{7}, \frac{3}{7}, \frac{3}{7}, \frac{2}{7} \quad \frac{1}{7}, \frac{1}{7}, \frac{3}{7}, \frac{3}{7}$$

Clearly any rational value for the area ratio can be achieved, but are there simple ratios, which give, say, an area ratio of $1:n$ for $n > 4$?

An alternative interpretation

An interpretation of Euclid's figure on the slate offered by Leo Rogers is that Raphael was very likely to have presented a diagram pertinent to him as an artist. Thus the double triangle with the three interior segments could be an indication of the method Raphael used to draw, not just in point perspective, but in line perspective, so as to depict a floor tiled by squares, but in perspective.

It is not too far fetched to believe that such geometrical knowledge was available to the Greeks, at least in the imagination of Raphael who could be seen as acknowledging the roots of his own skills

Acknowledgments

Our thanks to Leo Rogers and Benedict Heal for their participation in the discussion, and to Eric Love for useful comments, having not been able to be present at the original exploration.

Notes

- [1] In the Phaedo, Xanthippe is described as being very difficult to live with: "Xanthippe, you know her, she was there, crying and weeping in her womanly way, holding his child on her knee and saying to Socrates 'this is the last time that you will speak with your friends or they with you.'" [Phaedo] and in the Xenophon symposium Socrates claims to have married her because "if I can tolerate her spirit, I can with ease attach myself to every human being else."
- [2] In Greek, μένο is a masculine name with no feminine version. It means literally 'remain, stop, stay', while μένος means 'fervour, zeal' and αἰμενος means 'with joy'. One Greek colleague suggested Menousa as a possible feminine version.
- [3] Indeed, mathematics may not emerge at all unless you engage with the reasoning!
- [4] It seems that this slave girl is used to having to do elementary calculations, perhaps in relation to her work in the kitchen.
- [5] Although anachronistic to us now, the term torpedo is original Greek for a stingray. It was first used for a weapon in 1806, 11 years before the publication of Jowett's translation. The term 'torpedo' was taken up by Davis (1984) to describe tasks which challenge learners' implicit assumptions, concept images, and conceptions (see final comments). The translation by Guthrie (1961) has 'numbing him like a sting ray' where Jowett uses 'torpedo'.
- [6] A reviewer reminds us that Fowler (1999) uses this segment as a frontispiece, and also contains an imaginary interaction with the slave boy doing somewhat harder mathematics.

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