FUNCTIONS OF SYMBOLIZING ACTIVITY: A DISCUSSION

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Researchers who study mathematical learning have engaged in considerable debate about the role of social interaction in learning (Cobb, Jaworski, & Presmeg, 1996; Confrey, 1995; Davis, Sumara, & Kieren, 1996; Ernest, 1995; Lerman, 1996, 2000; Sfard, 2007; Steffe & Thompson, 2000a; van Oers 1996). In these debates, researchers using radical constructivism as a background theory have been criticized for not sufficiently attending to student-teacher communication (or student-student communication) (Davis, Sumara, & Kieren, 1996; Ernest, 1995; Lerman, 1996; Thompson, 2000; von Glasersfeld, 2000). Rather than rendering radical constructivism inadequate for studying student-teacher communication, these types of critiques can spur researchers who use this background theory to clarify and expand their positions (cf. Simon, 2009). It is in this spirit that I present a framework for studying the functions of students’ and teachers’ symbolizing activity.

The purpose of the article is to use the functions of students’ and teachers’ symbolizing activity to characterize student-teacher communication and to investigate how the functions contribute to acts of learning for both teacher and student. I lay the groundwork for this purpose by first describing key theoretical ideas – a scheme, symbolizing activity, communication, and learning. Then I suggest five ways that students’ and teachers’ symbolizing activity can function. Finally, I use these functions to analyze a sample piece of data from a three-year constructivist teaching experiment. The sample analysis illustrates how the functions of symbolizing activity help to characterize student-teacher communication, and how such communication contributes to a student’s and a teacher’s learning.

Schemes, symbolizing activity, communication, and learning

A scheme is a “package” of mental operations that are organized toward accomplishing a goal (Piaget, 1970). A scheme consists of three parts – an assimilatory mechanism, an activity, and a result (von Glasersfeld, 1995, 2001). When a person encounters an experiential situation, he or she uses current schemes to interpret it. Such interpretations may trigger records of prior operating, and the person may come to assimilate the situation as one that involves a particular type of activity. The activity of a scheme involves mental operations such as ordering, pairing, and uniting (joining), and a person uses these operations to produce a result.

When a person produces a material component during the functioning of his or her schemes (i.e., a person creates some form of perceptually available material), then a person is engaged in what I call symbolizing activity (cf. Ernest, 2006; Kaput, Blanton, & Moreno, 2008). Symbolizing activity includes verbalizing (e.g., a student saying the word “four” or providing a more extended verbal explanation), gesturing (e.g., a student using his fingers to measure the length of a segment), and notating (e.g., a student writing “13 × 5” or drawing a segment in a computer microworld). [1]

Although there are differences among these three forms of symbolizing activity, one reason to study them together is that when two or more people communicate, the different forms of symbolizing activity often function in concert with one another (Nemirovsky & Monk, 2000; Radford, Bardini, & Sabena, 2007; Kaput, Blanton, & Moreno, 2008). For example, when a student responds to a teacher’s question, the student might gesture, respond verbally, and produce some form of notation. Similarly, a teacher’s response to a student might involve more than a single form of symbolizing activity. For this reason, I situate my study of the functions of symbolizing activity within a model for communication – and view communication as an overarching function of symbolizing activity.

I consider communication to involve people in reciprocally assimilating what each person perceives to be other peoples’ symbolizing activity (Thompson, 1999, 2000). Figure 1 illustrates the process of communication between two people. In Figure 1 person A might begin communicating with person B by generating some type of symbolizing activity (e.g., a question, a verbal remark, a written problem). For communication to proceed, person B would need to assimilate person A’s symbolizing activity, using available schemes. If person B perceives that the assimilated situation requires some type of response, then he would determine what an appropriate response would be, and generate his own symbolizing activity (a response to a question, a verbal remark, etc.). Then a similar process for person A might take place: Person A assimilates person B’s symbolizing activity, determines an appropriate response, and generates further symbolizing activity. [2]

To illustrate this model of communication along with the ideas of scheme and symbolizing activity, I present a brief data excerpt between a teacher-researcher (me) and a student.

Excerpt 1: Carlos responds to the Outfits Problem [3]

1 T: You have four shirts and three pairs of pants. How many outfits could you make?
3 ....
4 C: [Carlos draws a diagram of a shirt and writes “×4” and a pair of pants and writes “×3” (fig. 2)].

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In this data excerpt, I (person A) began communicating with Carlos (person B) by presenting him with a question (line 1) – a form of symbolizing activity. Carlos’s creation of Figure 2 was part of his establishment of an experiential situation, and so was part of how he assimilated the teacher’s symbolizing activity. Figure 2 suggests that Carlos assimilated my question using two composite units – the shirts and pairs of pants. He then solved the problem by ordering the shirts – a first shirt, a second shirt, a third shirt, and a fourth shirt – as well as ordering the pairs of pants – a first pair of pants, a second pair of pants, and a third pair of pants. His symbolizing activity for this process was assigning a color to each pair of pants and a color to each shirt. He then paired [5] the first shirt with the first pair of pants, the first shirt with the second pair of pants, and the first shirt with the third pair of pants, and repeated this process for each of the shirts.

Figure 1. The boxed text marks the beginning of communication between two people; the arrows can be followed from there (cf. Thompson, 1999, p. 2)

Figure 2. Carlos’s initial notation

Figure 3. Carlos’s notation for the shirts and pants (top) and his notation for the outfits (bottom)
the other shirts, symbolizing these actions with Figure 3. Finally, he united the outfits into four units of three, two units of six, and one unit of twelve, symbolizing these actions with his verbal statement and the numeral "12". My response, "okay", was intended to indicate to Carlos that I was able to assimilate his symbolizing activity to a way of operating that was sensible to me.

Although brief, this data excerpt situates schemes and symbolizing activity in the context of the model for communication that I presented above. In doing so, it illustrates the three parts of Carlos's scheme: He assimilated my question using two composite units – the shirts and pairs of pants; the activity of his scheme was ordering the shirts and pants, pairing shirts with pants to make outfits, and uniting the outfits together; and the result of his scheme was the total number of outfits he could make. The data excerpt also highlights the relationship between Carlos's symbolizing activity and mine. My symbolizing activity initially was the source for Carlos to establish an experiential situation. Then Carlos's symbolizing activity occurred during all three parts of his scheme, and it was sufficient for me to attribute a way of operating to him that was within the realm of my expectation for how a student might solve the problem, which led to my response, "Okay."

The above data excerpt, however, does not provide insight into significant learning on either Carlos's part or mine. That is, in the data excerpt, I found out that Carlos could solve the problem in a way that I could interpret, and so I did not need to modify my ways of operating (in the moment) to interpret what he had done. Similarly, Carlos's way of operating seemed solidly established, and so his response did not seem to involve learning on his part.

Here I use the term learning to mean a modification or adaptation of any of the three parts of a scheme or a reorganization of schemes in relation to each other (cf. Piaget, 1970; von Glasersfeld, 1995). Learning that occurs in the context of communication involves a person making adaptations to his ways of operating in relation to other peoples' symbolizing activity (cf. Thompson, 1999; 2000). In the case of student-teacher communication, a teacher's learning might involve the establishment of a new scheme that he or she attributes to a student, while a student's learning might involve a novel scheme that he or she creates in the process of problem solving activity (cf. Steffe & Thompson, 2000b).

Functions of symbolizing activity
To make more specific how students' and teachers' symbolizing activity functions, it is useful to specify the role and goals of the people engaged in communicating. For this reason, I situate my discussion of the functions of symbolizing activity within a discussion of the role and goals of teachers and students who are engaged in a constructivist teaching experiment. Specifically, I highlight one way that a teachers' symbolizing can function and four ways that a students' symbolizing activity can function. These five functions are not intended to be a comprehensive list of how symbolizing activity can function – rather, they grew out of the sample data that I present later. [6]

Teachers
Generally, one of the central goals of a constructivist teaching experiment is for the teacher to learn ways of operating from his or her students (Confrey & LaChance, 2000; Steffe & Thompson, 2000b). In order to achieve this goal, a teacher tries to act in harmony with students’ current ways of operating by de-centering from his or her own ways of operating in order to see his or her students’ ways of operating. In the process of harmonizing with a students’ current ways of operating, a teacher poses problems, asks clarifying questions, and chooses particular tools (e.g., a computer microworld or paper and pencil) aimed at making students' ways of operating “visible” (cf. Kaput, Blanton, & Moreno, 2008).

I use the term “visible” to mean that a student’s symbolizing activity can open the opportunity for a teacher to make a model of the student’s ways of operating. I do not intend the term to mean that a person’s symbolizing activity necessarily makes his or her ways of operating obvious or clear to other people. Rather, engaging in symbolizing activity is a pre-condition for seeing another person’s ways of operating. For example, in Data excerpt 1 Carlos’s assimilation of my question and his subsequent response made his way of operating in the Outfits Problem visible to me because I could assimilate this way of operating to my expectation of how a student might solve this problem. The process of making students’ current ways of operating visible constitutes one way that a teacher’s symbolizing activity functions for him or her in interaction with students.

Students
The role of a student in a constructivist teaching experiment is to be open to working on problems that the teacher poses, to communicate with the teacher and other students about his or her reasoning about a problem, and to listen to and interpret the solutions of other students involved in the experiment. In this process, one way that a students’ symbolizing activity functions is that it may be a necessary part of enacting a way of operating. I consider it to be a necessary part of enacting a way of operating when a student is unable to carry out his or her scheme without the use of some kind of perceptually created material; i.e., the student is unable to carry out a scheme in visualized imagination. Determining whether and when a student is constrained to
carrying out his or her scheme with the aid of some type of perceptual material requires more than a single observation. Nonetheless, a hypothetical response to the Password Problem is useful in illustrating this idea.

*The Password Problem:* Your school requires that you have a two-letter password to log in to your computer account. How many possible passwords could you create?

In working on the Password Problem, a student might write in rows “A—26 letters, B—26 letters, ..., Z—26 letters.” Then he might count the number of rows while pointing to each row and write “26 × 26.” A researcher might judge that the student’s symbolizing activity was necessary for the enactment of his way of operating, in part because the student did not curtail his way of operating prior to concluding that the result was “26 × 26.” Instead, the student wrote down all of the possible letters that could go with A, all of the possible letters that could go with B, and continued to do so all the way to Z, and then counted the number of rows. So, a researcher (or teacher) might judge that the student’s symbolizing activity functioned as a necessary component of enacting his scheme.

A second function of a student’s symbolizing activity is that it can involve monitoring and self-regulation. That is, students can use their symbolizing activity as a way to keep track of, or monitor, their ways of operating (cf. Thompson, 2000), which can in turn engender acts of self-regulation. By an act of self-regulation, I mean that a student re-initiates her activity in response to disequilibrium that she experiences while she is interacting with others (cf. Steffe’s auto-regulation, 1991, 1994; Pirie & Kieren, 1989). Acts of self-regulation and monitoring are intertwined in that when a student engages in an act of self-regulation, it often indicates that she is monitoring her ways of operating. In turn, once a student has engaged in an act of self-regulation, it is common for her to find a new way to monitor her scheme, which may include a change in her symbolizing activity (e.g., a student may use a new type of notation that better helps to monitor her operating).

Once again, a hypothetical response to the Password Problem can illustrate the ideas of monitoring and self-regulation. A student might respond immediately to the Password Problem by stating that the answer would be “twenty-six times twenty-six” or by writing down the corresponding notation. If asked why such a problem involves multiplication, a student might elaborate on his response with an explanation similar to the one in the previous paragraph. Because the student was able to provide an explanation, the student’s verbalization “twenty-six times twenty-six” pointed to a way of operating that he could regenerate. However, this way of operating did not need to be carried out in order for the student to experience the situation as involving multiplication. The student simply assimilated an experiential situation to the results of prior operating.

Table 1 provides a summary of the functions of symbolizing activity that I have discussed above.

**An illustrative example: functions of symbolizing activity**

Now that I have discussed some functions of teachers’ and students’ symbolizing activity, I provide a data sample from a constructivist teaching experiment. The teaching experiment was conducted with eight middle school students during the students’ sixth, seventh, and eighth grades. The

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Table 1. Functions of symbolizing activity
transcript presented here is taken from work with two eighth graders, Michael and Deborah, and illustrates the functions of Michael’s and my symbolizing activity in the interaction, and how these functions contributed to both of our learning.

Excerpt 2: Michael’s solution to the Outfits Problem

T: Let’s suppose in your closet you have three shirts and four pairs of pants. How many different outfits could you make?

M [Michael and Deborah respond immediately and at almost the same time]: Three.

D: Twelve.

T [responding to Deborah]: Twelve? [Looking at Michael] Let’s see, can you show how you are thinking about that?

M [under his breath]: Wait.

T [Hands each student a piece of paper and pencil]: Here go ahead.

….

M [Begins to draw a picture of one pair of pants and one shirt and writes 4 next to the pants and 3 next to the shirts (fig. 5a).]: You could take one and make one outfit. [Michael draws a tally mark to the right of the shirt and one to the right of the pants (fig. 5b).] Take another one, [Michael draws a second tally mark to the right of the shirt and one to the right of the pants], and take another one [Michael draws a third tally mark to the right of the shirt and one to the right of the pants]. And then you have one pants [left over]. Wait, do we do every possibility?

T: Every possibility you could possibly get.

M [Michael draws three tally marks followed by four tally marks, making two rows of tally marks (fig. 6).]: You could do that, that, that [Michael connects the first tally mark in the second row with each tally mark in the first row.] You could do a lot of possibilities of this one. [Michael is silent while he draws a line from the second tally mark in the second row to each tally mark in the first row. He repeats this process for the final two tally marks in the second row.] There is three for every pants, so basically twelve.

Michael said “wait” (line __), indicating that my question and Deborah’s response created a state of disequilibrium for him. At that moment, however, he proceeded with his solution that led to the result three. During his solution, he used written (figs. 5a & 5b) and verbal symbols (lines __–__) in order to explain to me why he concluded the result should be three. His explanation indicated that he paired the first pair of pants with the first shirt, the second pair of pants with the second shirt, and the third pair of pants with the third shirt, leaving one pair of pants left over.

At this point, I was able to assimilate Michael’s symbolizing activity to a way of operating that made sense to me – so his symbolizing activity allowed me to see why he initially concluded the result should be three. For Michael, producing a left over pair of pants created a perturbation, and he asked the question, “Wait, do we do every possibility?” (line __–__). Michael’s question suggests he monitored how he had paired shirts with pants to make outfits during his explanation (lines __–__). It seems likely that he monitored his initial solution because Deborah had responded differently than he had. By monitoring his activity he became aware that he should be able to use the left over pair of pants to make an outfit, just as he had used the first three pairs of pants to make outfits. Michael’s monitoring, and my confirmation that he should do “every possibility [he] could possibly get” (line __), occasioned an act of self-regulation – he reinitiated his activity in response to his interaction with Deborah and me. In doing so, he produced new tally marks, drew lines from each of the tally marks in the first row to the first tally mark in the second row (fig. 6), and once again verbalized part of his solution (lines __–__). After Michael had drawn lines from each of the tally marks in the first row to the first tally mark in the second row, (lines

Figure 5. Michael’s initial drawing and tally marks

Figure 6. Michael’s tally marks
to-one correspondence scheme (i.e., his solution of the Outfits Problem) was occasioned by my question and Deborah’s different response. My question occasioned an explanation of his initial solution, and Deborah’s different response occasioned him monitoring his initial solution, and occasioned his subsequent act of self-regulation.

Attending to the functions of symbolizing activity
I have proposed a framework for studying the functions of teachers’ and students’ symbolizing activity, built out of scheme theory and a conception of communication based on reciprocal assimilation of symbolizing activity. I have demonstrated how this framework helps to characterize student-teacher communication in mathematical interaction, and how it enables an analysis of how the functions of symbolizing activity contribute to students’ and teachers’ learning. However, the framework I have proposed raises a significant question that is worthy of consideration in further theoretical discussions and research studies: How can the functions of symbolizing activity be delimited? This question arises because the definition I have given of symbolizing activity is broad, and the number of ways that such activity might function in student-teacher communication is wide and varied. One approach for addressing this question is to tightly link the study of the functions of symbolizing activity to situations where a researcher deems learning has occurred for teacher and student. Tightly linking functions of symbolizing activity to moments of learning may help to delimit the functions under consideration. It also makes a central goal of developing the framework providing accounts of learning that are situated in the context of characterizing student-teacher communication. Therefore, this article, and the development of the framework, responds to von Glasersfeld’s (2000) call for researchers who use a radical constructivist perspective to highlight the role of student-teacher communication in their research on learning.

Notes
[1] Portions of the research reported in this manuscript were part of my doctoral dissertation, Students’ construction of algebraic symbol systems, completed in 2007 at the University of Georgia, Athens under the direction of Leslie P. Steffe. All research reported in the manuscript was conducted at the University of Georgia.
[2] I use the word notation broadly for any type of graphic record. Other researchers have differentiated among different kinds of graphic records, but I do not go to this level of detail here (e.g., Kaput, Blanton, & Moreno, 2008, p. 29; Lehrer, Schauble, Carpenter, & Penner, 2000).
[3] Thompson (1999, 2000) notes that this definition of communication is compatible with symbolic interactionists who take as problematic the issue of how two physically disconnected systems can communicate with one another (e.g., see, Bauersfeld, 1980; Cobb, Jaworski, Presmeg, & Moreno, 1996).
[4] In the data excerpts, T stands for the teacher-researcher (me), C stands for Carlos, M stands for Michael, and D stands for Deborah. Comments enclosed in brackets describe students’ nonverbal action or interaction from the teacher-researcher’s perspective, and four ellipses indicate omitted dialogue.
[5] Pairing is used to imply that a student has established an outfit as a new type of unit that has been created from putting a shirt with a pair of pants.
[6] I structure what follows by identifying that certain functions of symbolizing activity pertain to teachers and other functions pertain to students. However, it is not my intent to identify these functions exclusively with a student or teacher (i.e., a teacher might use their symbolizing activity in a
way that I identify as a function of students' symbolizing activity. I simply structure my discussion this way because it fits with the sample data analysis I provide. For other examples of how symbolizing activity functions for teachers and students see the dissertation mentioned in note [1].

References


Thompson, P. W. (1999) 'Remarks on representations, conventions, and common meanings', paper presented at the Panel for the PME-NA XXI working group on representations, Cuernavaca, MX.


