Historical Happenings in the Mathematical Classroom

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Many people have spoken and written about the need to integrate the history of mathematics into mathematics teaching J.A.L. Glaisher, for example, said in 1890, "I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history." And again G.A. Miller, in 1916, wrote "Probably the greatest usefulness of the history of mathematics is due to the fact that it puts more life into the study of this science." Our department has for some time been interested in trying to give substance to these claims. One effort in this direction was the creation of activities and materials on the history of mathematics, directly related to the curriculum, for teacher training courses.

In the last two and a half years we have been trying another direction, which consists of developing activities for classroom use. In this paper I shall present some examples of these activities, describe their rationale and structure, and conclude by suggesting some issues for discussion.

One of the principles that guides our work is the integration of history into selected curriculum topics. This principle seeks on the one hand to give relevance to the history, and on the other, to motivate and deepen students' understanding by exposing them to aspects of the evolution of selected concepts.

Historical number systems

In Israel, the opening unit in the seventh grade curriculum, that is, for pupils aged 12-13, is number systems and number bases. To accompany this we constructed an activity concerned with some ancient numerals and number systems. In this activity we compare the number systems of the Egyptians, the Babylonians and the Romans, and also compare these systems with ours The structure of the activity is as follows.

We began with a brief introduction to set the scene. We mention that Egyptian mathematics seems to have been developed mainly for practical purposes such as the calculation of areas of fields on the shores of the Nile, using measuring ropes. Equally briefly, we say something about Egyptian writing, and about its deciphering from the Rosetta Stone and the man who deciphered the hieroglyphics, Jean-Francois Champollion. After this introduction, we present the Egyptian numerals, and the principles of their number system, drawing attention to such features as the system having a base ten, having no special symbol for zero, and the interesting fact that the direction of writing can vary.

We turn to the Babylonians. Again, we say a few words about the development of their culture and their writing. Then we show the numerals for 1 and 10 and the way they wrote numbers up to 60.

There is, of course, a big difference between the Babylonian and the Egyptian systems and in order to clarify it further we take a look at our own system, explaining in particular the concept of *place-value*. In other words, the Egyptian and the Babylonian systems motivate us to review, in the light of other systems, our own place-value system. The following are some of the conclusions we arrive at with the students: the Egyptian had no place value; the Babylonian system was "halfway" between the Egyptian and our system.

An important understanding which the students work towards is the different ways of reading aggregates of symbols in a system where the same symbol can take different values. An array of cuneiform symbols, for example, can be read merely as a collection of symbols to be aggregated, or the relative groupings of the symbols can give them new meaning. Thus the "simplest" meaning of a collection of three ten-symbols and two unit-symbols is that is stands for 32. But if we separate the symbols into two groups, say two ten-symbols followed by one ten-symbol and two unit-symbols, we have a new meaning. Each of the first two ten-symbols can mean 10 times 60, and the three other symbols mean 10 plus 2, as before. On this reading we get altogether $2 \times 10 \times 60 = 1200$ plus 12, that is 1212. The same array can also be interpreted as 622, or as 1802, and there are many more possibilities.

Thus we discuss the problems of the Egyptian and Babylonian system and the advantages of ours. A question from the worksheet that the pupils do during this activity illustrates the approach: we ask the students to arrange according to size six numbers, given in Egyptian, Babylonian and Roman numerals, then ask them if other orderings are possible, and why. This innocent-looking (and not so difficult) question again shows the pupils, in a slightly different manner, that the Babylonian number system was "incomplete", giving rise to more than one possible meaning in the absence of a clearer indication of the value of the numeral in any place.

This activity exemplifies some of the features typical of such historical activity. By discussing different "man-made" symbol systems, their advantages and disadvantages, it can give the pupils some feeling for the dynamical development of mathematics. The students begin to appreciate that there is more than one way of representing mathematical concepts, and therefore more than one way of doing mathematics. Also, the review of our own numeration system, as mentioned, is of direct benefit to the regular curriculum work currently in progress, and the contrast with other systems can be more effective in communicating an understanding of its crucial features than a more conventional, non-comparative, review.

History and fractions

To complement this activity we developed for the children in the eighth grade (ages 13-14) an activity on calculations in ancient Egypt, dealing with the way Egyptians multiplied, and the way they wrote fractions. In the latest version of this activity (which is still being revised) we give the pupils an Egyptian source which they are led to decipher, and then to discover the way in which Egyptians multiplied by doubling and adding. The pupils learn this peculiar method of multiplying and, in our experience, many of them take to it. After doing some exercises which provide students with some confidence in the method, one of them said: "It's fun! Let's do some more." Another student pointed out how economical this method seemed to him: "All we have to know is doubling and adding."

The main part of the activity is devoted to fractions. We explain that the Egyptian approach to fraction was different from ours. This provides us with good opportunity to revise the several aspects of the concept of a fraction in a new light. For example, we discuss the fact that for us a fraction, say 1/10, may have several distinct but related meanings: as an operator (producing the tenth part of something), as a quotient (the result of dividing one by ten), as a rational number (a number object which one can order, operate upon), and so on. We hypothesize that the Egyptians recognized only the first meaning, for which they used what we call unit fractions (fractions with 1 as their numerator), and used any particular unit fraction only once. For example, they wrote 2/5, or the result of dividing two into five parts, not as 1/5 + 1/5, but as 1/3 (+) 1/15

They had, from the evidence of the Rhind Papyrus, a special 2/*n* table, apparently in much the same way that we use multiplication tables, in order to know how to express any fraction (other than unit fractions) as a sum of different unit fractions. The decomposition of a fraction decompositions. The worksheet that we give at this stage includes, for example, an activity around the decomposition

$$\frac{2}{m} = \frac{1}{\frac{m+1}{2}} + \frac{1}{\frac{m(m+1)}{2}}$$

for m odd.

The 2/*n* table provides a wonderful context for investigation: for example, why were the particular expansions that we find in the papyrus chosen? Did the Egyptians prefer a particular type of unit fraction expression out of the many possible? We tackle this question by inviting pupils to raise hypotheses and test them against the table. One student's comment was: "Perhaps in another papyrus there are different resolutions, but we don't know about them." Many pupils observed that a combination of even denominators and economy in writing may have guided the choice of entries in the table. The different possible resolutions of 2/13 that we showed them supported these assumptions—but then we brought in the resolution of 2/95 as a counterexample.

At this stage, many of them said "So, what is the value of all this?". Their disappointment is not a serious matter, however. After all they have been engaged in quite a lot of meaningful mathematical activity. More importantly, we leave them with a good and real feeling that they have been engaged in an actual historical-mathematical investigation: thinking, exploring, applying their knowledge and logic to tackle a problem. They respond positively when we tell them that this is a research question on which many historians have worked. To conclude the activity we demonstrate a way of finding an expression for 2/*n* as the sum of unit fractions, and we also add, for advanced classes, the proof of this method.

Historical geometry in grade eight

A somewhat different activity is called *π and the circumference of the circle*, which discusses something of the nature of π and the attempts made to calculate its value. This is intended for early geometry in grade 8.

We begin by discussing intuitively the concept of shape, and figures of the same shape in which the ratio of the corresponding elements is preserved. The circle is an example. This brings us to the number π as the ratio of the circumference of any circle to its diameter, and we discuss how it appears in four ancient and mediaeval sources: Babylonian, Chinese, Greek and Jewish.

We show that the Babylonians knew a value for π, which we would write as 3 1/8 or 3.125, and used it in their calculations. The Chinese Li-Hui got better results using polygonal approximations to the circle, and we show some pictures explaining how he got his results. Archimedes, who lived about 600 years before Li-Hui, used essentially the same method, but refined it by taking circumscribed polygons in addition to inscribed polygons, and thus made it possible to set bounds to the value of π that are as close as one wishes. The activity ends by showing a Jewish source mentioning π. Here we have the opportunity to relate mathematics to the pupils' own culture. Maimonides in the 12th century, known to a wide audience as a philosopher and physician, was also one of the greatest codifiers of Jewish religious law. His assertion of what amounts to the irrationality of π is interesting in that it is probably one of the earliest sources of its kind: "You should know that the ratio of the diagonal [diameter] of the circle to that which surrounds it [circumference] is unknown and we cannot speak of it at all in truth; and the inability to conceive it is not due to us. ..but it is unknowable by its very nature, and its existence is not attainable, but it can be known approximately."

If these and similar activities are adapted or developed in other countries, we would suggest, wherever possible, making use of local sources such as the above for motivational reasons, even when the significance of the source is not strictly relevant to the central mathematical story of the concept. For example, in the U.S.A. we could bring in some of the entertaining stories about circle-squarers which appear in Beckman's *A history of π*.

Summary

Perhaps the best way to summarize this paper is to share with readers some of the questions and doubts we expressed to ourselves during the development of these activities and their implementation so far. We intend to continue developing such activities and to continue questioning, doubting, testing and modifying. Questions such as: What are the principles that guide us? What are the main things we want the pupils to take away from such an activity? Do we want them to achieve mastery of the ancient techniques, and be able to solve any question that relates to the subject of the activity? Or, maybe, do we just want them to get an impression as to how mathematics was done in the past? Could this activity replace some present materials in the curriculum? and so on.

We don't have any definite answers, but here are some of our ideas and how the development of the materials evolved. At first we thought of building these activities as a series of lectures, accompanied by a worksheet at the end. After some trial runs we concluded that the materials should be presented in a more active and engaging way—students either got bored, or didn't carry over information from the end of one lecture to another. Thus we changed the format to a dialogue between the teacher and the pupils, and between the pupils themselves, with questions given strategically *during* the activity.

We also feel that an activity that deals with the history of mathematics should attempt some kind of integration between the history and the mathematics from which mathematics education can benefit. There are two complementary strands to this. We should do mathematics to understand its history—that is, while the pupils solve problems with numerals and the methods of the Egyptians and the Babylonians, etc., they can see the development of methods and tools through history. On the other hand, we should do history to understand the evolving nature of mathematics. The historical perspective shows pupils that mathematics is a creative human activity which often started for practical reasons. We also want to show that mathematics is not rigid and that there have been many changes over the centuries. At the same time, through history the pupils are *doing* mathematics. While learning about unit fractions, for example, they are solving a lot of exercises on fractions. The activity on π integrates the use of the Pythagorean Theorem and irrational numbers, approximations, etc. Also, students are reviewing in a new light many things usually taken for granted, such as our place value system, our "small x" for solving algebra problems, several meanings of fractions, etc.

All the above is guided by a golden rule: pupils should *enjoy* doing mathematics. From the informal reactions of pupils so far, we feel we may be on the right track.

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My aim is to exhibit a parallelism between the actual mode of evolution of geometrical knowledge in the race from the earliest times of which we have authentic historical information, and that by which school youth can most readily and efficiently assimilate this experience. It is to be specially remarked that I make no attempt to prove the existence of a necessary parallelism between the racial and the individual development of geometrical knowledge (...) What I hope to do is something quite different, viz. to show that for educational purposes, the most effective presentation of geometry to youth both as regards geometry and spirit, is that which in main outlines follows the order of the historical evolution of the science.

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