Consciousness of the Unknown

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It almost looks as if analysis were the third of those impossible professions in which one can be sure beforehand of achieving unsatisfying results. The other two which have been known much longer are education and government. [1]

In the past I have avoided writing about the emotional side of students' development. This is not because I think it is unimportant. On the contrary, I believe it is a critical and integral part of all cognitive effort. But I have not been able to get to grips with theoretical explanations for what I observe about students when I work in the classroom. This paper is a starting point in rethinking my work concerned with students' learning of algebra. It is well known that many pupils find algebra difficult-it provokes extreme anxiety which is mainly focused on the use of symbols. In order to reduce this anxiety there is now much less of an emphasis on work with algebraic symbols in school. In the early days of my work with Logo I was also reluctant to introduce students to the idea of a symbol representing a variable, and when I did so was careful to use "friendly" names, and not what I thought would be anxiety-provoking single letters. But I discovered that students themselves shortened these "friendly" symbols to single letters. They were happily using the very same algebraic symbols which can cause so many problems in the traditional algebra situation. Having worked intensively with Logo and spreadsheets with pupils from 10 to 15 years old, I am now convinced that these computer-based algebra-like symbols are not approached in the same way as the paper-based variety.

I have always been attracted to psychoanalytic discussions of the unknown and the unconscious. "The unconscious is itself a kind of unknown knowledge that escapes intentionality and meaning, a knowledge spoken by the language of the subject (spoken, for instance, by his "slip" or by his dreams), but that the subject cannot recognise, assume as his, appropriate; a speaking knowledge nonetheless denied to the speaker's knowledge."[2, p72] The discovery of the unconscious implies that there can be no such thing as absolute knowledge, or total knowledge-the unconscious always remains unknown. Ignorance thus becomes a component of knowledge. Lacan suggests that ignorance is what will not be remembered and this itself is tied up with the idea of repression. "Ignorance, in other words, is not a passive state of essence, a simple lack of information: it is an active dynamic of negation, an active refusal of information."[2, p79] This suggests that teaching should be more about confronting resistance to knowledge than about lack of knowledge.

Symbolising in a computer environment

In searching for explanations about why computer-based symbols do not usually provoke anxiety in students we have to examine the nature and difference between the ways in which students interact with computer and paper symbols. In order to do this I shall draw on studies of Logo and spreadsheets in the mathematics classroom.[3, 4] Whether working with Logo or a spreadsheet students use a symbol to represent a general mathematical relationship, a relationship which almost always derives from a specific instance. So for example, in Logo students might generate a fixed Logo procedure and then use this fixed procedure to construct a general procedure. In a spreadsheet the symbolising of a general rule is also closely related to specific instances of the general (fig. 1).

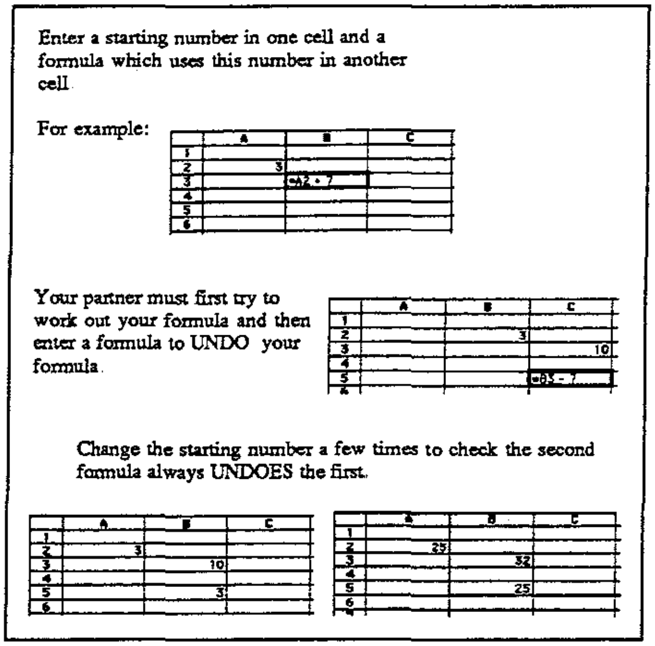

Figure 1: Undoing a formula

Although there are differences between a Logo and a spreadsheet environment, it is possible, in both, to negotiate a general rule from specific cases, and test out a general rule with specific cases. Our teaching emphasis has always been on the movement between the specific and the general. In this way the use of symbols to express a general relationship is supported by the computer environment. The symbolic expression derives from the student's own construction of generality. The authority for the rightness or wrongness of a symbolic expression does not come from the teacher. It comes from feedback from the computer. Whereas with paper-based algebra work the authority almost always comes from the teacher. Beginning algebra students have no resources for interrogating the appropriateness of their algebraic construction. So in most traditional algebra situations the student is exposing his or her ignorance to the teacher in a highly charged symbolic form. In the computer situation the student is exposing his or her ignorance to himself alone (and this may be why some students find it difficult to work in pairs at the computer). Interactive programming environments provide feedback which helps students make sense of their own constructions - they do not tell them whether they are right or wrong.

Dockar-Drysdale discusses the importance of the symbolic process in allowing individuals to internalise an experience. She suggests that disruptive "acting out" in the classroom can be a form of symbolising and communicating for children who are not able to symbolise internally.[5] She emphasises the importance of symbolising before intellectualising, which is what we encourage students to do when working in Logo or a spreadsheet pupils are initially taught to represent a relationship by pointing to the spreadsheet cells with a mouse. They do not have to type in the spreadsheet symbolism. But they see and make sense off this symbolism and begin to use it in their spoken language.

Jo: *Equals now what do you click on Sam.....so what will it be....B2 minus 4....*
Sam: *Yes B2 minus 4...no wait a minute....*

They soon learn (without explicit explanation) the ways in which the spreadsheet formulae change when being copied in a relative way.

A study with spreadsheets

As part of a research project [4] I recently worked with a class of 14-15 year olds who had experienced considerable difficulty with school mathematics-many of them were disaffected with mathematics and disaffected with school. Some of them were very disruptive in class. They were in the 4th out of 5 sets of their year group in a comprehensive school. The spreadsheet sessions took place once a week for one lesson (40 mins) over a period of 4 months, in a friendly and attractive computer room. The sessions centred around the idea of function and inverse function (for example figs. 1, 2) and solving algebra word problems. The class teacher and I were responsible for the spreadsheet sessions. At the beginning of a lesson pupils immediately started working in pairs on a spreadsheet problem-presented to them on a worksheet, and placed by the computers before they entered the room. The class teacher and I spent our time working with individual pairs, making suggestions, and supporting them as they solved the mathematical problems. Case studies of 10 of these students were carried out and this involved testing and interviewing them individually before and after the spreadsheet sessions and making notes on their work during the sessions.[6]

At the beginning of the study the case-study students carried out an algebra test and were then interviewed on their responses to the test questions. The majority of these students could not answer any of the test questions and I noticed how difficult it was for them to articulate their ideas in natural language. Most of the pupils had not used algebraic symbols before and gave responses like "nothing really" when asked what they meant. When they did give responses these clearly related to previous experiences

RS: *Does L have to be a larger number than A?*
Jo: *Yes because A starts off as 1 or something.*
RS: *What made you think that?*
Jo: *Because when we were little we used to do a code like that....in junior school....A would equal 1, B equals 2, C equals 3...there were possibilities of A being 5 and B being 10 and that lot....but it would come up too high a number to do it.......it was always in some order....*

Although all of these students had been relatively unsuccessful with school maths, most of them seemed unaware of their difficulties. So when asked "*what do you find difficult and what do you find easy*" a typical answer would be "*I find it all easy*". This type of response suggests a lack of awareness or possibly a blocking out of what they do not understand. Or it could be that they were never challenged by the relatively trivial mathematics which is characteristic of the mathematics texts used with low attaining students. Psychoanalytic theory suggests that all knowledge is acquired against a great resistance and it seems that many of the mathematics schemes for under-achieving students are feeding this resistance instead of working towards overcoming it.

When this class first started to work with spreadsheets their lack of confidence was very apparent. They minimised their engagement in the mathematical problems, continuously asking for help. They were reluctant to try out their ideas at the computer never producing more than had been asked for. It seems that they did not engage in the problems in order to avoid exposing what they did not know to themselves and the teachers. There was an unacceptable level of disruption in the class. Their behaviour was noticeably different from other classes with which I had worked. I felt concerned about the group, both as a teacher and a researcher.

> The task of the teacher may be thought of as resembling the parental function: that is, to act as temporary container for the excessive anxiety of his students at points of stress. It will mean that he will experience some of the mental pain connected with learning, and yet set an example of maintaining curiosity in the face of chaos, love of truth in the face of terror of the unknown, and hope in the face of despair. If he is able to do this he is providing the conditions which will foster in the student an ability to tolerate the uncertainties connected with learning. ..The pupil's ideas and thoughts are aided by a teacher who assists him in ordering them, particularly at such times when the learner becomes overwhelmed with too much undigested knowledge. The teacher's capacity to be reflective and thoughtful about data rather than producing ready answers enables the learner to internalise a thinking person.[7]

As teachers we responded to their anxieties by providing more structure to the written spreadsheet activities, so that pupils needed to ask for less help. The modified spreadsheet activity (fig. 2) placed the students in less of an unknown situation in comparison with the original activity (fig. 1). When students asked for help the class teacher and I emphasised that we wanted them to work out the mathematical rules themselves, but were otherwise forthcoming with our support. Our behaviour was critical during these first few sessions. If we had given them the type of support they were demanding (that is to solve the problems for them, thus providing them with the unknown answers) then this would have been feeding their resistance to knowledge. If we had not given them any support then their anxiety would have prevented them from engaging in the spreadsheet work. The feedback from the computer also provided considerable support for the students. They learned to try out their own solution to a problem and then modify this solution. These students were beginning to learn to face and work with what they did not know-a difficult and painful process.

Use the spreadsheet to help you construct the following tables of numbers:

X	Y	X
1	0.5	1
2	1	2
3	1.5	3
4	2	4
5	2.5	5
6	3	6
7	3.5	7

Write down the rule which tells you how to get the X number from the Y number

X = ...

Write down the rule which tells you how to get the Y number from the X number

Y = ...

Figure 2: Function and inverse function

By about the fifth computer session the students' behaviour had completely changed. They no longer turned to us as teachers for help, they engaged in the problems, constructing and reconstructing at the computer. Most of the pupils were now successful with the function problems. This was reflected in their responses to the post-test in which the majority of case-study students could answer the question correctly. The students used algebraic symbolism or spreadsheet symbolism to write down the function and inverse functions in the post-test, suggesting that this symbolic language had taken in an important mediating role.

Some of the students could use their experiences with spreadsheet code to make sense of traditional paper-based algebraic symbols. Jo and Sally, for example, were able to write down the inverse function for a simple function expressed in algebraic code (something which they had never been explicitly taught). When given the expression B = A÷3 and asked to write down A in terms of B they wrote down the formula A = B * 3 explaining that "*A equals B times 3 gets back to what ever you want to get back to*". When given p=m+n and asked to write down *n* in terms of *p* and *m* they correctly wrote down *n* = *p*–*m* saying "*you have to take that one from that one to know that one*". In her final interview I asked Jo why she thought she had improved so much in the test.

RS: *Why do you think you have learned from using the spreadsheet?*

Jo: *Because you've done different things and you've asked questions in a different way so you really think about what you are doing.*

RS: *Some people say that if you learn it on the computer you can only do it on the computer. Why is it that you can do it now when you are on paper?*

Jo: *Because you have to think before you type into the computer anyway. So it's just like thinking with your brain. You think of columns.*

RS: *Is computer work too easy sometimes?*

Jo: *No computer work makes you think about what you are doing.*

RS: *How do you know you are thinking?*

Jo: *Because your brain hurts ..I don't know ...you really have to think about it before you put it on the computer or you just much up all the spreadsheet.*

The students' attitude to the word problems was even more positive than their attitude towards the function problems. These problems are no longer much used within the UK curriculum. They are considered to be contrived and not as motivating as more realistic problems. But these students became successful at solving them with a spreadsheet and not surprisingly very much enjoyed them. When Dennis was asked what he liked and disliked about the computer work he said "*I like all of it now I know what to do*". In solving these problems the students had to represent the algebra word problem in the spreadsheet symbolic language. Although they may have been using a mouse to do this many of them were able to write down an algebraic representation when asked to do this in the final interview. In the final interview Jo was asked what *x* means and she said "*x is a column....a place where you can work these out....if you do something you couldn't work out you would just go onto the computer and type in different columns*". Jo was the pupil who in the initial interview said that a letter represented its position in the alphabet. The spreadsheet symbolism had taken on a mediating role in her thinking of the unknown.

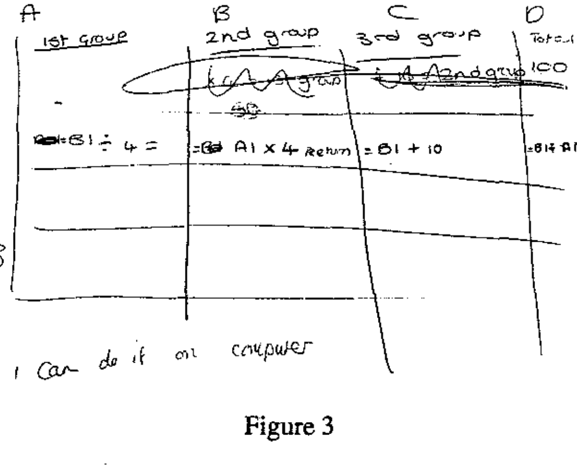

Figure 3

At the end of the study Jo was asked to solve the following algebra story problem away from the computer.

> 100 chocolates were distributed between three groups of children. The second group received 4 times the chocolates given to the first group. The third group received 10 chocolates more than the second group. How many chocolates did the first, the second and the third group receive?

Jo's solution (with no computer present) illustrates the way in which the spreadsheet code was beginning to *play a role in her thinking* processes. She had drawn a spreadsheet on paper to support her solution and had correctly written down all the rules represented in the problem. She had not specified the unknown and if she had been working at the computer the "circular reference" error message would have provided feedback on this error. When interviewing Jo I asked her *"If we call this cell X what could you write down for the number of chocolates in the other groups?"* and she wrote down:

Jo, who had always been unsuccessful with school mathematics, had learned to use symbolic code to represent mathematical relationships.

James was another student. He finds it very difficult to express his mathematical ideas in words and in the pre-test and interview answered very few questions correctly. There is some evidence that he sees a mathematical relationship in a visual way and maybe he has never learned how to express this in natural language. In the pre-interview he initially said that x represented *"nothing really"* but when pushed said that it could be *"a number.....one number"*. James is very slow at responding in conversation and also when working on paper. This causes him considerable anxiety and he said in the final interview that the reason he likes working on the computer is that he can then work more quickly. I suggest that the reason he works so cautiously is that he does not like to produce a wrong answer. When working on the word problems at the computer he would firstly enter all the formulae, leaving the cell representing the unknown number blank. After entering the formulae he returned to the cell representing the unknown number and tried out a range of numbers in this cell. In this way he did not have to face incorrect numeric solutions as he constructed the formulae. When asked the following question in the post-test *"We know the length of this rectangle is double its width. The width is called X. Can you write a formula to calculate its length. Can you write a formula to calculate its perimeter?"*, James gave the following response.

Ja: *The length is X times 2.*
RS: *That's right.........do you know what perimeter is?*
Ja: *All the way round the outside.*
RS: *Could you write a formula for the perimeter?*
Ja: *X times 6.....*
RS: *To get that and that it would be X times 2 and then that's X times 2 and that's X times 2.....*
RS: *When you write a formula what do you think of the x as?*
Ja: *Just a number....it can be any number because you don't know what any of the others are.*

Contrast this with his response in the pre-interview.

RS: *Do you know what perimeter means?*
Ja: *All the way round.*
RS: *Could you write down what the perimeter would be using x?*
Ja: *No.*
RS: *Could you tell me how you would do it?*
Ja: *Not really.*

A concluding remark

I suggest that for James it was the experience of symbolising mathematical experiences with a spreadsheet language which helped him to begin to express his ideas in natural language. Students who are unsuccessful with mathematics often work with text books which minimise the use of language, and very rarely discuss spontaneously in the classroom. These students need, even more than successful students, to learn to express their mathematical ideas in language.

> Knowledge, in other words is not a substance but a structural dynamic; it is not contained by any individual but comes about of the mutual apprenticeship between two partially unconscious speeches that both say more than they know. Dialogue is thus the radical condition of learning and of knowledge, the analytically constitutive condition through which ignorance becomes structurally informative; knowledge is essentially, irreducibly dialogic [2, p86]

When symbolising in a computer environment students are confronted with what they do not know, and this is an important step in breaking down their resistance to learning. After a gap of six moths I recently returned to interview some of the students who took part in the study described in this paper. I wanted to ask them some more question, but I also wondered it they would have forgotten (or repressed) much of what they had learned at the computer. They had not forgotten–by symbolising their new knowledge. Sadly they have very little opportunity of using this knowledge in their normal mathematics classroom, because, of course, students who are likely to obtain unclassified grades in their public examinations cannot be expected to use and understand algebra.

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