MATHEMATICAL TEXTS AS NARRATIVE: RETHINKING CURRICULUM

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Narrative is a persistent metaphor in discussions about mathematical texts, particularly with regard to mathematics curriculum, and yet little theoretical foundation has been offered for its support. The use of this metaphor offers insight into the aesthetic differences between ancient Greek mathematical texts (Netz, 2005) and reveals the potential aesthetic opportunities of mathematics textbooks (Sinclair, 2005). It is also present in the framing of mathematical word problems as a literary genre (Gerofsky, 1996) and in discussions about the narration, or "author's voice", of mathematics textbooks (Herbel-Eisenmann & Wagner, 2007; Love & Pimm, 1996). In addition to the analysis of written texts, the narrative metaphor reveals new information about spoken mathematical texts. For example, Stigler and Hiebert (1999) propose that viewing an enacted mathematics lesson as a story exposes the need for building connections between its sequential parts.

This body of work raises the question *how is mathematical text to be understood as narrative?* Netz (2005) notes the strong similarities between mathematical text and narrative, explaining that "The concept of 'narrative' applies almost directly to mathematics, in that mathematical works—just like many other works of verbal art—tell a story: they have characters, and our information about the characters gradually evolves" (p. 262). Solomon and O'Neill (1998), however, disagree, arguing that although mathematics content may be embedded in a narrative structure, it is not narrative in nature since it has no time referent. Solomon and O'Neill therefore introduce a dichotomy: texts *about* mathematics that are narrative in that they describe chronological events (such as "First I did this, then I did this") and non-narrative texts *of* mathematics (such as a proof). Sinclair (2005) also notes some challenges with her own framing of mathematical texts as narrative:

> Now I've substituted the word "text" for "story," a move I am not quite comfortable making, since it is not at all clear that all [mathematics textbooks] tell stories. In fact, story-telling depends on the text as well as the audience, the reader/writer, as well as the teller (is a story still a story when it is separated from its telling?). I am also not comfortable about substituting text with story since I find it challenging, in mathematics, to identify just what a story is. Must it have a beginning, middle and end? Must it have setting and plot and characters? What kinds of genres are there? (p. 4)

These conflicting perspectives demonstrate the need to address Sinclair's challenge to articulate what is (and is not) a mathematical story. Therefore, in this article, I offer a third option to Solomon and O'Neill's dichotomy: a way to conceive of texts *of* mathematics *as narrative*. To do this, I draw from narrative theory to articulate a framework that can support mathematics teachers in their curricular design work and inspire new possibilities for designers of mathematics curriculum.

To start, I introduce the narrative framework of Meike Bal (2009), which offers insight into the interpretation of narrative texts. I then use this framework to propose a way the content of mathematical texts can be interpreted as narrative. Although this framework addresses mathematical texts in general, the goals for my work are educational and relate directly to work with mathematics curriculum (both written and enacted). In this article, therefore, I use a portion of a textbook to illustrate how mathematical texts can be read as narrative. The remaining sections elaborate this framework further by discussing some important differences between the narrative reading of mathematical texts and that of literary narratives. I also propose examples of mathematics text that cannot be read as narrative. The final discussion raises some educational implications of this mathematical story framework.

Narrative system of layers

Building on the tradition in literary theory of Russian Formalism [1], Bal (2009) recognizes narrative as a system of three layers: (a) the media (including its narration) in which a story is told, referred to as *text;* (b) the sequence of events temporally encountered and perceived by a reader, referred to as *story;* and (c) the logical sequence of events as constructed by a reader through interpreting the story, referred to as *fabula*. These layers, while not independent, help to explain how different tellings of the same events affect the story. For example, the distinction of *story* and *fabula* articulates the difference between the "truths" reconstructed by a reader (the fabula) and how the content is temporally revealed (the story).

For Bal, an *event* is a transition or change, or as she explains, "the transition from one state to another state, [...] a process, an alteration" (p. 189). Although it may seem that there is only one sequence of events in fixed literary texts, Bal identifies two: (a) the sequence of events encountered by a reader temporally while reading and (b) the chronological sequence deduced by the reader. The fabula is the reader's determination of the order, "A happened, then B happened, then C happened," even though the story might instead reveal B first, then C, then A, for dramatic effect. Thus, a *fabula* is the residue across multiple events; it is the reader's logical reconstruction of the relationships between the events of the story.

The same information can therefore be organized differently with different effects for a reader. A story can be recognized as a manipulation of the fabula, which "focalizes" (directs the attention of a reader) and otherwise "colours the fabula" (Bal, 2009, p. 18). For example, in *Romeo and Juliet,* consider the effect when a reader is informed at the beginning that the lovers will take their own lives at the end (in Shakespeare's script) compared with when this information is withheld (*e.g.,* in the 1996 movie). When a reader knows in advance that the couple will die, he or she may wonder throughout the story, "what will cause them to die?", whereas without this information, the same reader might instead wonder, "will they live happily ever after?" In both cases, a reader may construct the same fabula including not only the death of the lovers but the events leading to their tragic end.

How might mathematical texts be considered in terms of these layers, particularly when it comes to addressing Sinclair's concerns? Although this question can be addressed broadly in terms of mathematical texts in general, I will focus my discussion on mathematics textbooks, to further the goals of developing a framework to aid curriculum design.

Interpreting mathematical texts as narrative

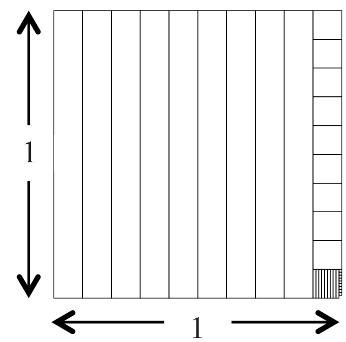
Of the literary layers defined by Bal (2009), the *text* layer is perhaps the easiest to connect with mathematics textbooks as they have easily distinguishable media (bound texts) with verbal (*e.g.,* expository, questions, tasks) and diagrammatic signs (*e.g.,* illustrations, figures, photos). While often obscured, each mathematics text has a narrator who relates the contents of the story. As Love and Pimm (1996) and Herbel-Eisenmann and Wagner (2007) explain, mathematical texts, and particularly textbooks, are usually narrated in the omniscient third-person point of view, which means that its directives and voice are given without an acknowledgement of the existence of an author.

Consider the sequence of tasks condensed and adapted from a textbook of which I am a co-author shown in Figure 1. Even without a "story problem" as defined by Gerofsky (1996), these signs on paper (or on a computer screen as the case may be) can be interpreted as the text layer of a mathematical narrative. The narrator's contributions can be identified (*e.g.*, claims, questions) and the configuration of signs described (*e.g.,* the way the tasks are numbered, the placement of the diagram).

1. A rational number written as a fraction can also be written in an equivalent decimal form. For example, $\frac{1}{9}$ can be written as $0.\overline{1}$. Convert the numbers in the sequence below to their equivalent decimal form. Assuming the pattern continues with constant growth, what will be the ninth term?

$$\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}, \ldots$$

2. Now check your decimal values from question 1 with a calculator. Do they match your predictions? Are there any that are different?
3. Is $0.\overline{9} = 1$? How do you know? Discuss this with the class and justify your response.



A representation of 0.9999

Figure 1. A sequence of tasks adpated from Making Connections Course 2 (Dietiker, Kysh, Sallee & Haey, 2010).

Mathematical story

When the set of mathematical concepts and images generated by a reader when reading a text changes throughout a sequence, then the mathematics text offers a reader a chronologically experiential layer similar to that of a literary story. That is, a reader encounters and recognizes mathematical ideas [2] (*e.g.,* mathematical objects, relationships, properties, and procedures) in a chronological sequence during his or her reading of a mathematics text. Distinguishing points along the sequence are changes and transitions of the mathematical states of these ideas. For example, suppose that when answering the first question in Figure 1, a reader extends the pattern $0.\overline{1}$, $0.\overline{2}$, $0.\overline{3}$,… to conclude that $\frac{9}{9} = 0.\overline{9}$. When the calculator later returns 1, this same reader may wonder if $0.\overline{9} = 1$, investigate, and eventually be convinced that $0.\overline{9}$ and 1 are the same mathematical object/character. This recognition extends an understanding of the mathematical representation of 1 of this reader and, therefore, represents a change brought about by the comparison of numbers.

Since literary events are changes throughout a story, then a *mathematical event*, likewise, can be conceptualized as a transition from one mathematical state to another. Thus the initial and changed states have a mathematical relationship. For example, the generation of new terms of the pattern by a reader is an event that changes the given list of numbers by increasing its number of terms.

Metaphorically, then, the *mathematical story* layer describes the chronological sequence of mathematical events encountered and experienced by a reader throughout a mathematics text. How might the mathematical story of the tasks above be expressed? As an experience of a reader, it is the accumulation of his or her temporal mathematical experiences and realizations while reading. One way it can be represented takes its inspiration from Sinclair's (2005) reading of a proof. For the sample mathematics text above (which will differ from that of other readers), my reading of the mathematical story exposes its potential drama:

> This tells me that fraction can be also written as a decimal and that $\frac{1}{9}$ can be written $0.\overline{1}$, which I know is 0.11111… So, with a sequence, I can change $\frac{2}{9}$ to a decimal (2 divided by 9) and I see it is $0.\overline{2}$, while $\frac{3}{9}$ becomes $0.\overline{3}$. I notice a pretty straightforward pattern with the decimals; the first has "1" repeating without end, the second "2," the third "3." So I assume $\frac{4}{9}$ is $0.\overline{4}$ and verify by dividing 9 into 4. Extending it to nine terms, I get:
>
> $$0.\overline{1}, 0.\overline{2}, 0.\overline{3}, 0.\overline{4}, 0.\overline{5}, 0.\overline{6}, 0.\overline{7}, 0.\overline{8}, 0.\overline{9}$$
>
> So the ninth term is $\frac{9}{9} = 0.\overline{9}$.
>
> Checking with my calculator, I find that my pattern seems to work. My calculator gives me $0.\overline{5}$ for $\frac{5}{9}$ and $0.\overline{6}$ for $\frac{6}{9}$.
>
> Nothing seems out of the ordinary and I hardly think I need to check the rest. But just in case, I see that
>
> $$\frac{7}{9} = 0.7, \frac{8}{9} = 0.\overline{8}, \text{and } \frac{9}{9} = 1.$$
>
> Wait a minute—that is not the same. Is the last term in the pattern $0.\overline{9}$ or 1? Is $0.\overline{9}$ an imposter? Or perhaps a pseudonym? The pattern suggests that the calculator should show $0.\overline{9}$ and yet it definitely shows 1. What is going on?
>
> To find $\frac{9}{9}$ with my calculator, I type 9 divided by 9, which I remember is 1. Is this correct? I think so, since grouping 9 into groups of 9 results in exactly 1 group.
>
> Now I suspect my pattern is wrong. Perhaps it only works for the first 8 terms? Because if it holds for all nine terms, that means 1 is equal to $0.\overline{9}$. Could that be? In the diagram, I imagine how each successive digit in the decimal 0.9999 fills nearly all of a 1×1 square. Extending the decimal to an infinite number of nines may fill the square. But how can I know?
>
> One thing I know is that the pattern starts with $\frac{1}{9}$ (which I'm told is 0.111…) and each term increases by $\frac{1}{9}$ (also 0.111…). Since the decimals repeat without end, adding 0.111… will increase each digit after the decimal place (all infinite of them) by 1. This helps me know that $\frac{2}{9}$ is 0.222… So, by continuing to add 0.111… to each term, I know that the ninth term (for $\frac{9}{9}$) can be written 0.999… Since by division, I also know that $\frac{9}{9} = 1$. Therefore, I conclude that $0.\overline{9} = 1$.

Mathematical fabula

As explained earlier, a literary fabula is a reader's re-construction of the literary events beyond the story based on a logical chronology. Metaphorically, then, the *mathematical fabula* represents a reader's logical re-construction of the mathematics events beyond the text and story layers. Although recognizing a chronological story layer of mathematical texts allowed a metaphorical correspondent to literary story to be defined, a second chronology with respect to mathematical texts is difficult to identify for reasons asserted by Solomon and O'Neill (1998). The mathematical ideas dealt with in mathematical texts (*e.g.,* the equivalence of number representations) are assumed to be timeless [3]. Mathematical objects, properties, and relationships are encountered and interpreted by a reader in the eternal present and can be viewed as independent of time. For example, though a reader may recognize that $\frac{9}{9} = 0.\overline{9}$ and that $\frac{9}{9} = 1$, which of these realizations necessarily comes first independent of the order they are presented in the story? Unlike a literary story, which might require a reader to use verbal or situational clues to recognize that the events described in the story may have occurred in a different sequence, a mathematical story makes no such demand on its reader.

However, even without a second chronological referent ordering events, a reader can *still* logically relate mathematical events beyond the story. That is, when reading a mathematical story, a reader may recognize that a later revelation can be used to logically support a prior assertion. For example, a reader may use logic [4] to recognize that, although $0.\overline{9}$ and 1 may appear to be different mathematical characters found in separate "acts" or tasks, they are instead different names for the same mathematical character. Therefore, I propose that the mathematical fabula, or the re-construction of mathematical events while reading a mathematical story, is not based on time but rather a logical line of reasoning. Since there are many deductive lines of reasoning that can lead to the same conclusion, then the *mathematical fabula* is a reader's reorganization of the logic around how certain mathematical ideas support or connect the meaning of other mathematical ideas. It is the logical confluence of the mathematical ideas that unfold and develop throughout the story layer.

Thus, beyond the given ordering of a mathematical story, there is also a potential logical re-ordering by the reader [5]. For example, if a mathematical story presents events in the sequence A, B, a reader may deduce that B is a condition for A to be true. Noticing how an algebraic generalization (*e.g.,* $x^2 - y^2 = (x + y)(x - y)$) can justify an arithmetic pattern, perhaps recognized years earlier (*e.g.,* $99 = 10^2 - 1^2 = (10 + 1)(10 - 1)$) is one of numerous possible examples. However, I caution that although Aristotelian logic nicely maps the mathematical fabula closer to its literary metaphorical correspondent (in terms of a linear sequence), this is not the only form of the logical re-sequencing of mathematical content possible. Re-sequencing occurs when re-structuring connected ideas, such as how a reader may recognize that integers are an extension of whole numbers, which are an extension of counting numbers. Logically re-constructing the content beyond the mathematical story can also include re-defining, noticing a pattern, connecting, and conjecturing. In short, any point at which a reader confronts a conflicting mathematical idea that requires the logical renegotiation of prior mathematical understanding, the mathematical fabula is involved.

The mathematical fabula can be represented in a variety of ways, each with a loss of information. However, using the notion of a concept map, the "timeless" mathematical ideas generated from my reading of the tasks is offered in Figure 2.

Narrative effects of changing mathematical layers

Beyond reading existing mathematical texts as narrative, Bal's distinction of narrative layers offers mathematics educators a way to recognize the potential effects of modifications of mathematics curriculum texts. For example, if changing the sequence of *Romeo and Juliet* affects its reading, it is reasonable to consider the effects of changing the order of events in the mathematical story layer. Consider altering the sequence of tasks shown in Figure 1 so that it instead starts with Task 3, followed by Tasks 1 and 2. In this mathematical story, a reader would confront the relationship of $0.\overline{9}$ and 1 first, potentially concluding that they are equal by studying the diagram. This result, however, would alter the encounter with the pattern in Task 1 since $\frac{9}{9}$ can now be read as both $0.\overline{9}$ and 1. The use of the calculator in Task 2 then confirms the result of Task 1.

This reordering has interesting consequences for the substance and organization of my fabula. Since the original mathematical story positions $0.\overline{9}$ and 1 as distinguishable quantities, my representation of its fabula was organized with two distinct realizations: that $\frac{9}{9} = 1$ and $\frac{9}{9} = 0.\overline{9}$. With the new sequence, $0.\overline{9}$ and 1 are instead introduced as different representations of the same value, eliminating the need to separate the treatment of these symbols. The logical connections are also affected by changing the sequence; my original fabula (in Figure 2) reveals my reasoning with the pattern, a logical path that is not available by the text in the altered sequence.

Beyond the fabula, the mathematical story also changes in several significant ways. Perhaps most importantly, the "meat" of this lesson, the recognition of equivalence in Task 3 (see Figure 1), is now completely unmotivated by the sequence. When a reader enters this reordered sequence, he or she has little reason to care whether $0.\overline{9} = 1$ or not [6]. Of course, the same can be said for Task 1 in the original sequence; the reason to rewrite the fractions and expand the pattern is not transparent. However, in the original sequence of tasks, the purpose of the pattern task becomes clear with Task 2: that, although it might not be recognized at the beginning, something mathematically interesting is going on worthy of investigation. Yet, in the altered sequence, the purpose of the pattern task is unclear. In addition, the role of Task 2 is now redundant; entering $9 \div 9$ into the calculator at this point to change $\frac{9}{9}$ into its equivalent decimal (*i.e.,* 1) does not reveal new information the reader does not already potentially have from Tasks 3 and 1. Rather than setting up a surprising contradiction necessitating resolution, it is now ineffectual. Thus, this change in sequence shifts the motivational effects of each of the tasks.

This is not to say that changing the sequence is the only way to impact the mathematical story. While not a major focus of this article, it should be noted that changing the text layer can also impact the mathematical story and fabula. Consider how the experience of reading the mathematical story would change if, rather than asking the question in Task 3, the narrator asserts the relationship, stating "$0.\overline{9}$ is equal to 1." Without posing a question, a reader may not question the relationship and, thus, might not even notice the logical conundrum it suggests. Other changes to the narration that might have implications for the story and fabula layers include removing the diagram in Task 3 or changing it to a dynamic representation on a computer.

Identifying the actors of mathematical texts

Although Bal's (2009) layered framework offers a useful distinction between a reader's temporal experience with emerging mathematical content and his or her logical re-

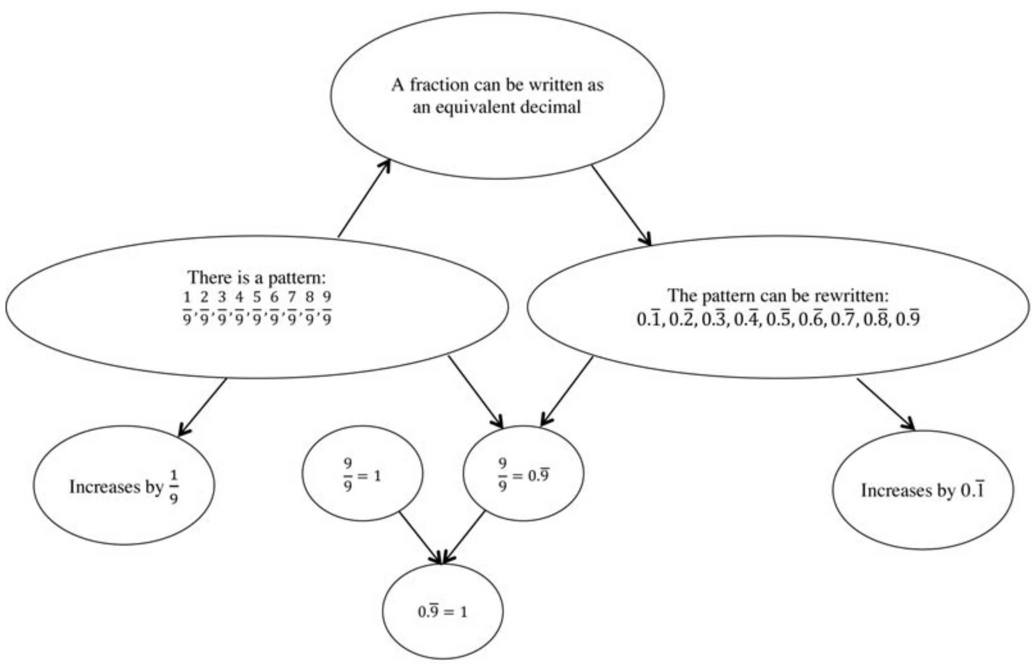


Figure 2. A sample mathematical fabula, representing my logic of the mathematical ideas generated by my interpretation of the tasks in Figure 1.

construction of the content, critical differences between literary narratives and their metaphorical mathematical texts can also offer insights into the way in which mathematical texts operate. An important example is the way in which a literary story is advanced. Bal's (2009) framework explains that a literary narrative must have distinguishable events and cannot solely be a setting or a character. That is, *something must happen* which enables the story to make progress toward a conclusion [7]. The identification of events is the recognition of these moment-to-moment changes within a sequence. For example, at one point of *Romeo and Juliet,* Juliet is alive and in the next, she is dead. How does this change occur? Although a reader of *Romeo and Juliet* is active in the reading of this narrative (*e.g.,* writing the scene in the mind), this reader is not the agent causing Juliet's demise. Instead, characters of a story have the agency to act to change the temporary conditions of the sequence (*i.e.,* Juliet's life or death) from one part to the next. An external narrator (one who is not a character of the story) can also act to change the circumstances in the moment, such as the arrival of the tornado and its impact on Dorothy in *The Wonderful Wizard of Oz* (Baum, 1900). While characters act in this story, it is the work of the narrator to describe what occurred with, "Then a strange thing happened. The house whirled around two or three times and rose slowly through the air" (p. 9). Thus, the characters and narrator are the potential actors of literary narratives.

What causes the transformation and development of content within a mathematical text? That is, who are the actors of mathematical stories? Unlike their literary counterparts, mathematical characters (the objects of the mathematical story) do not have agency to act. Mathematical objects (such as $\frac{2}{9}$ in the example) and other ideas (*e.g.,* conjectures) will remain untransformed unless acted upon. Certainly, narrators of mathematical stories can "tell" of change. For example, the number $\frac{1}{9}$ in Task 1 was transformed to its decimal equivalent through the statement, "For example, $\frac{1}{9}$ can be written as $0.\overline{1}$." Other more common examples of the narrator working as a mathematical actor are found in worked examples found in textbooks whenever the narrator acts upon a mathematical object (such as by solving an equation). Unless a narrator acts to change the mathematical content, it is up to a human actor to advance the story by acting on mathematical objects and relationships such as completing a pattern and rewriting a fraction as a decimal.

Can all mathematical texts be read as narrative?

Returning to Sinclair's (2005) concern about whether all mathematical textbooks make stories, I am not suggesting that all mathematical texts, and particularly textbooks, make for good reading or that all can be understood as a mathematical narrative when read in this manner. For a reader to be able to read a text as a mathematical narrative, he or she must be able to recognize a sequence of mathematical events that connects a beginning with an ending or else there is an absence of plot. An example of a mathematical text that would not constitute a mathematical narrative would be a mathematical glossary of terms (the "cast of characters", if you will). Although mathematics textbooks certainly contain definitions that are similar to those in a glossary, they are hardly limited to them.

Even with multiple events, a mathematics text is not necessarily a mathematical narrative unless the reader can discern connections between the events. In the case of mathematics curriculum texts, when the events appear to be a seemingly random collection of mathematical activities that could be sequenced in almost any order, then a sense of progress across the sequence is lost and it is rendered incomprehensible by its reader (see Fernandez, Yoshida & Stigler, 1992, for examples drawn from data). In my reading of the example mathematics text shown in Figure 1, a sequence of events can be summarized as the progression from my conversion of fractions to decimal equivalents, to the use of those decimal equivalents to recognize a pattern, to the generation of new terms of that pattern, to the raising of a contradiction by the checking with a calculator, and so on. Each mathematical event serves to set up the next: the change in the representation of the numbers serves to produce a pattern, which serves to introduce a contradiction, which serves to encourage the reader to raise and answer an important question: is $0.\overline{9} = 1$? This progression of connected mathematical events helps to form a path that enables a reader to recognize it as narrative. An example of events without connections would be a set of mathematics tasks placed on cards in no particular order. This is not to say that a random mathematical sequence can have no mathematical or curricular value, only that they do not offer a reader a mathematical narrative as discussed here.

However, not every mathematical event must be related to those before and after. Again, a literary example is instructive. Even when literary events are disconnected, these separate parts eventually connect. Consider, for example, a story that alternates between scenes featuring two different groups of characters, only to then reveal that these groups are neighbors and that earlier conversations were about each other. Similarly, there can be breaks in the mathematical progress (*e.g.,* a development on linear functions might follow a study of probability).

Finally, it should be emphasized that having a mathematical plot does not imply that a reader must find a mathematical question and its answer(s) relevant, interesting, or important (and thus, making it a "good" story). Just as a children's story may pursue a question that an adult may not find intriguing (such as how to tie your shoes), similarly, it is assumed that a mathematical story will not appeal to all readers. Therefore, its appeal is not a quality that distinguishes between what is and is not a mathematical story.

New potential for mathematics curriculum

Despite vast differences in the content of literary stories compared with that in mathematics textbooks, in this article, I have introduced a set of metaphorical layers of mathematics textbooks that are consistent with Bal's layers of narrative. Although Solomon and O'Neill (1998) argue that "mathematics is constituted by a logical structure that is not reducible to a temporal sequence of events" (p. 217), I have proposed a conceptualization of fabula that depends not on the chronology of events but instead on the reader's re-constructed logical relations between events. Since it is with this logical dimension that a reader makes sense of mathematics text beyond its given sequence, it is consistent with the role of the fabula in a literary narrative. Rather than viewing the "timeless" reading of mathematical texts as the antithesis of narrative, it instead can be recognized as part of the "truths" of the reader after the temporal reconstruction has been experienced.

One of the purposes of this work is to increase attention to an important aspect of curriculum that is often neglected: the nature of the sequential unfolding of mathematical ideas for a reader. Recently, much of the mainstream mathematics education discourse regarding curriculum exists within the fabula layer. This article represents an effort to redirect at least part of this attention to the story layer with the aim of enhancing the potential dramatic effects of mathematical texts. As described with the re-sequencing of tasks, the way in which the mathematical content temporally unfolds can affect both the experience for the reader and the nature of his or her mathematical conclusions. Objectives such as "Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers" (Common Core State Standards Initiative, 2010, p. 46) are currently in the spotlight and reveal little about how mathematical ideas are expected to emerge through a reader's experience. In fact, it may be a particular challenge for mathematics experts (including teachers) to recognize the temporal qualities of some elementary mathematical texts, since the pre-existence of a fabula may obscure its temporal quality. However, this challenge of looking beyond the fabula only increases the need to offer the metaphoric conceptualization of mathematical story.

The generative aspect of this framework for curriculum designers (of which I include all teachers) is endless. The framework disrupts conventional ways of understanding the mathematical content of textbooks and invites the creation of inspiring new mathematical stories. If a novel can be appreciated for its rich characters or its sudden surprises, then why not a mathematics textbook? Although it may be unorthodox to consider mathematical objects and activity in these "novel" ways, conceptualizing the unfolding of mathematical content in a textbook as a mathematical story allows new questions to be pursued, such as what propels this mathematical story forward? How does this mathematical story build curiosity and desire to learn what will happen? What different (and new) types of mathematical stories can we find or design?

I wish to conclude by remarking on the numerous directions this framework could extend. Andrà (2013) offers an intriguing example, comparing the mathematical story "told" in the classroom with the fabula of students as represented by their note-taking. Other potential inquiries could include the analysis of how mathematical characters develop across a sequence or the identification of genres within mathematics curriculum. A theoretical conceptualization of mathematical plot could offer analytic tools with which to describe the aesthetic dimensions of the reader's experience of the unfolding story of a mathematics curriculum text. Ultimately, this framework invites the development of a rich variety of critical tools for mathematical texts that have similarly enabled a deep understanding of literature.

Notes

[1] In Russian formalism, the story is separated from the fabula, although the terminology varies and is often used differently than in Bal's framework. Often, Russian formalists refer to the fabula as "story" and to the story as "sjuzhet", which is closely aligned with plot. The aspect of the Russian formalism tradition that Bal recognizes is the separation of the chronology of the text (Bal's "story") with the chronology of the logic connecting the sequential parts (Bal's "fabula").

[2] The phrase *mathematical ideas* will be used throughout to refer to the milieu of mathematical concepts and processes, including objects, encountered by a reader when reading a mathematics textbook.

[3] Not counting a "genetic" or historical sequence, which is separate and often very different from a sequence found within a textbook.

[4] Again, this is not a claim that this particular conclusion will be made by a student, but instead that readers of mathematics textbooks (which includes students) can make connections and draw relationships between mathematical phenomena beyond the text and story layers, whatever those may be.

[5] This is not to suggest there is only one way. Netz (2005) explains the aesthetic choices the authors of mathematical texts (such as Euclid) make when choosing a mathematical sequence of reasoning to connect an A to a B.

[6] I make no claim about what all readers will do. It is possible the question, posed on its own, will interest some readers. The main point is that what the story offers to motivate a reader to want to answer this question is different between these two sequences.

[7] The notion of working toward a conclusion or resolution is directly linked to mathematical plot

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