Communications

How students' record keeping during problem solving can support cognition and communication

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Two middle school students, Jessica and Stephanie [1], were asked to find the area of an irregular polygon (Shape A, shown in Figure 1) individually and then share their work with each other. Although the students used different strategies, both made written records of their work while finding the area, determined successfully that the area was 25 square units, and used their records to share their ideas with one another.

Jessica first counted whole squares, touching each one with her pen cap. She then removed the pen cap and recounted the whole squares, placing a dot in each one, as shown in Figure 2. After recounting the whole squares, Jessica matched up partial squares to form additional whole squares. For pairings of adjacent partial squares that would form a whole square, she drew short lines or arcs connecting them. Finally, she indicated with dots two non-adjacent partial squares that she combined to form a whole square. Because Jessica originally counted squares without marking the paper and then marked the paper as she recounted, we concluded that the act of marking the paper—keeping a record of what she had counted, and for the partial squares, how she counted them—helped her correctly determine the total area of 25 square units.

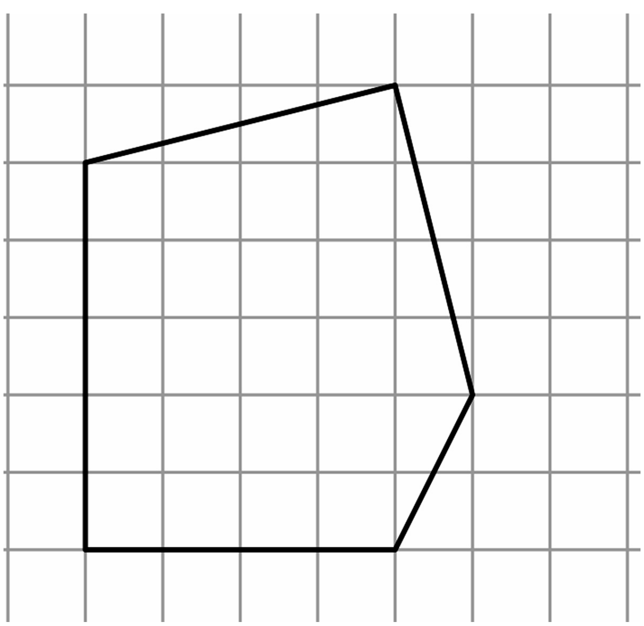

Figure 1. Shape A.

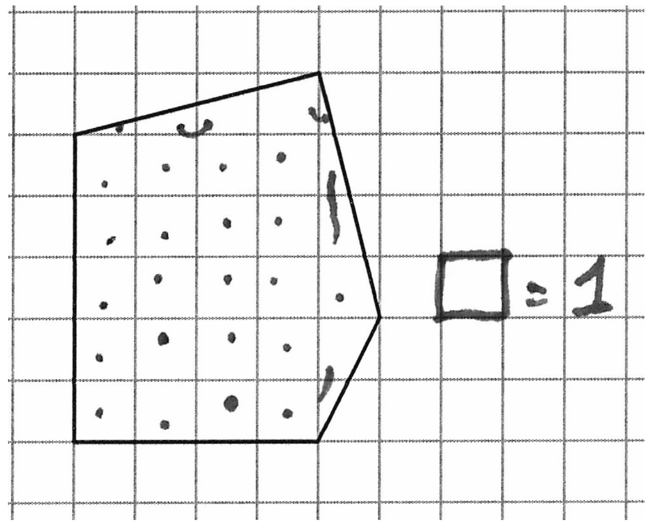

Figure 2. Jessica's work on Shape A.

Stephanie approached the problem differently; she separated and "moved" a 4 × 1 right triangle from the right-hand side to the top of the polygon to complete a 6 × 4 rectangle, recording that action as shown by the curved arrow at the top of Figure 3. To determine the area of this new rectangle, she used the area formula (length × width, or 6 × 4) and recorded this partial solution. She then spent time examining the polygon further before moving the small triangle on the bottom right to the top of the adjacent partial square and recording that action in a similar fashion. From there, she added an area of one square unit to the area of the rectangle to determine that the area of the entire polygon is 25 square units. Stephanie's work illustrates the use of multiple record keeping steps to solve a problem. The record she made of her initial move to form a rectangle and the subsequent calculation of rectangular area appeared to allow her to concentrate on the remaining portion of the polygon without having to keep track of her earlier work mentally.

When Jessica and Stephanie shared their work with one another, both used their written records to aid in explaining how they solved the problem. For example, Jessica said, "The other ones that were incomplete, I just put them together into the shapes that were […] that could make it full," as she circled with her finger on the paper to indicate examples of specific partial squares that she combined to form whole squares. Similarly, Stephanie pointed to the triangular partial square in the bottom row and the square it completed in the row above as she said, "That little spot fits in here" (see her lower arrow in Figure 3). These students' records of dissecting and transforming the polygon appeared to help them solve the problem successfully and communicate their solutions effectively.

We observed student record keeping in this and related area-finding tasks during a series of 23 problem solving

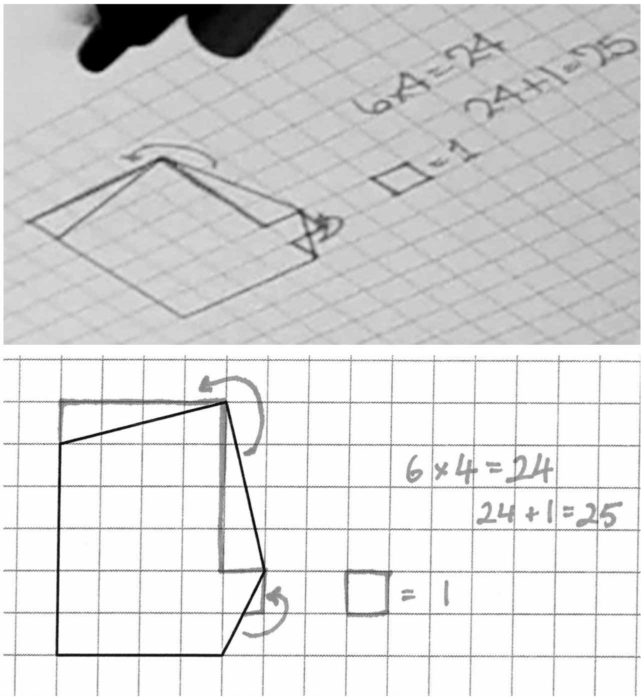

Figure 3. *Stephanie's work on Shape A.*

sessions conducted and video-recorded with pairs of students in public middle schools across the US as part of a larger research project. Although some students attempted similar strategies, not all were successful in keeping track of their work to solve the problem or communicate their strategy, prompting the question: how did students' record keeping help them complete these measurement tasks and communicate their thinking?

Managing cognitive load for problem solving and communication

Our theoretical framework for addressing this question is guided by Cognitive Load Theory (CLT), a learning and instructional theory based on the temporary, limited nature of working memory and the permanent, effectively unlimited capacity of long-term memory (Sweller, 1988). Working memory draws on long-term memory, but can store only about seven chunks of information and process only two or three chunks of information at a time. If these limits are exceeded, working memory gets overloaded.

We concentrate on two types of cognitive load that need to be managed for successful learning and performance, *intrinsic load* and *extraneous load* (Sweller, van Merriënboer & Paas, 1998):

> *Intrinsic load* is the cognitive load generated by the elements that must be considered simultaneously in working memory to understand the problem. This load varies according to the nature of the problem itself and the problem solver's level of relevant knowledge.
>
> *Extraneous load* is generated by processes that are not necessary for problem solving. For example, using new vocabulary or an unfamiliar representation to present a problem introduces extraneous load unless learning the new vocabulary or representation is a goal of the problem (in which case it is intrinsic).

From a CLT perspective, problem solving can generate a high intrinsic load for students due to the information that needs to be stored and processed simultaneously in working memory. Strategies to reduce or eliminate extraneous load can improve problem solving by making space available in working memory for intrinsic load. Communicating mathematical thinking can also generate high cognitive load as students simultaneously juggle the features of their solution and language for communicating their thinking in their working memory. Multistep explanations, in particular, are challenging to describe (Hufferd-Ackles, Fuson & Sherin, 2004).

Recording mathematical ideas

The impulse to record mathematical ideas and use these records in communication begins in early childhood. Examinations of children's forays into "mark making" reveal that preschool-age children create a variety of marks in mathematical contexts, including pictographs and iconic drawings, and they sometimes "read" these marks to adults as numbers or units of measurement (Carruthers & Worthington, 2006; Hughes, 1986). The mark-making of preschool and early school-age children may be important as a precursor to more formal mathematical activities and for the development of abstract thinking more generally (Carruthers & Worthington, 2006; van Oers & Poland, 2007).

By middle school, students have learned some conventions for recording mathematical ideas, including written expressions and diagrams. The role of diagrams in problem solving has been a topic of considerable study. Researchers working with students from early elementary school through college have found that successful problem solvers are often able to develop a representation of a problem rather than tackling the problem statement directly (*e.g.*, Fischbein, 1977; Nunokawa, 1994). Progress toward a solution is both indicated and facilitated by modifications to the recorded diagram that successfully capture the inherent structure of the problem (Nunokawa, 1994).

Diagrams and other records can be an important multimodal communication resource, combining with gestures and language to form a semiotic bundle (Arzarello, Paola, Robutti & Sabena, 2009) used by students working together to solve a mathematical problem (Chen & Herbst, 2013). Using records of students' work can also support sharing results with peers, who may be challenged to follow explanations without a representation that captures and depicts key aspects of the explanation (Hufferd-Ackles *et al.*, 2004). For example, students in a US after-school mathematics program for English learners reported that looking at another student's written number sentences helped them understand a solution that the student was trying to communicate verbally (Turner, Dominguez, Maldonado & Empson, 2013). The visual and shared nature of records can also foster communication between a student and a teacher; for instance, a teacher may be able to isolate features in a particular diagram that led to an incorrect solution (Murata, 2008; Ng & Lee, 2009).

In our work with students in grades 6 and 7 (aged 11-13 years), we have observed them making records and using these records in both solving problems and communicating their thinking. We define *record keeping* as the act of noting pieces of information during the process of solving a mathematical problem in a manner that allows the problem solver to retrieve the information later. Such pieces of information include: important information from the problem statement, such as quantities or measurements; additional information that might be useful for solving the problem, for example, noting that a square on a grid is one unit; ideas about solution strategies; and/or partial solutions, such as Stephanie's record of the 24-unit partial area. The information could be physically inscribed or electronically captured, and it might take various forms including words, symbols, equations, drawings, or diagrams. Its defining characteristic is that record keeping secures information external to the mind of the problem solver; it does not take the form of a mental note.

Applying CLT to student record keeping

When asked to find the area of irregular polygons, students in our study used various forms of record keeping as they decomposed the polygons into a rectangular portion and a set of perhaps less familiar, but easier to manage, portions. Some altered their diagrams to visually distinguish these sub-problems. Many made notations to indicate which portions of a shape had already been accounted for and which remained, as Jessica did, in addition to recording interim results. These records may have helped students manage cognitive load by spreading the intrinsic load across multiple sub-problems and/or offloading mental accounting onto paper.

Another student in our study, Marissa, altered the diagram she was given to find the area of Shape B (a rotation of Shape A) using an encasement strategy. She enclosed the irregular polygon within a rectangle (see Figure 4) and calculated the area of that rectangle. To do so, she counted the dimensions of the enclosing rectangle by making a dot in each square along the length of 5, and a dot in each square along the width of 6, and then multiplied 5 × 6 and recorded the result, 30, next to Shape B. She then mentally combined the ten partial squares that were within the enclosing rectangle but outside the original shape to form five whole squares and subtracted those from the rectangle's area of 30 square units to find that the area of the original polygon is 25 square units.

Marissa's record keeping included changes to the diagram, which altered the structure of the problem. These alterations enabled her to use a formula she had in long-term memory to calculate rectangular area, which she recorded on her paper. She was then able to focus the resources in her working memory on determining the area of a small number of partial squares, rather than the area of the entire original polygon.

In sharing their work with their partners or explaining it to researchers, many students in our study gestured to their records, as Stephanie did; traced over portions of their drawings; or created additional records to show their thinking as they described how they solved the area problems. Marissa, for example, explained to the researcher that, "We made it into a rectangle, and then we subtracted pieces that were in here," as she pointed to the triangles formed by encasing Shape B within the rectangle she had drawn.

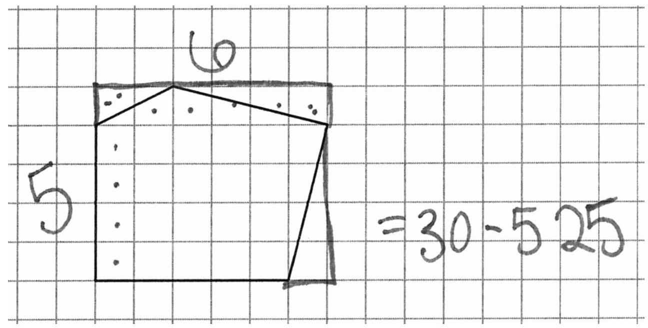

Figure 4. Marissa's work on Shape B.

Discussion

Our observations of students' work suggest that the use of record keeping can aid students in managing cognitive load in problem solving by offloading elements of the problem onto the environment. This action frees up working memory which can be directed towards the problem. Communicating their thinking about problem solving entails its own cognitive load for students. Our work suggests that record keeping can also support students in managing cognitive load when communicating their thinking. Students showed and gestured to their records when explaining their ideas to their partners and researchers.

Deliberate attention to record keeping promises to yield helpful information for teachers. Records can provide teachers with access to students' thinking that otherwise might not be available. Studying students' record keeping could reveal situations in which a student obtained an incorrect or imprecise answer due to a minor error, as opposed to an error in understanding, or obtained a correct answer through flawed reasoning. Beyond the record itself, observing the process of record keeping may be an avenue for teaching targeted, meta-cognitive strategies. Teachers can observe the kinds of information that students record, when this information is captured, how students translate concepts that they talk and gesture about into written records, and how they make use of these records in the process of solving problems and communicating their thinking.

The early results we present here suggest that students' record keeping can support two important elements of mathematics instruction: problem solving and communication about that problem solving. Additional research is needed to better understand the role record keeping plays and, on a practical level, how to design and implement tasks to promote students' productive use of records for solving problems and communicating their thinking. We contend that students' mathematical record keeping is an important area for future study, given its potential benefits to teaching and learning for individual students, their peers, and their teachers.

Notes

[1] All student names are pseudonyms.

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Uncritical ethnomathematics: a dangerous pedagogical approach?

PAMELA ROGERS

In FLM 34(1), Goossen Karssenberg explores the use of Persian mosaics when teaching geometry in a Dutch mathematics classroom, and claims that recently immigrated students from Morocco and Turkey could possibly connect with this non-Western content since it activates their prior cultural knowledge. While it is an interesting piece on implementing history and art in mathematics, what I find intriguing is the idea of "danger" mentioned at the beginning of the article. Drawing on the ethnomathematics work of Vithal and Skovsmose (1997), Karssenberg states, "there is a danger in taking subjects and methods out of their context and transplanting them into the context of Western mathematics education" (p. 43). However, instead of employing a critical or reflexive pedagogical lens to see if there are, indeed, any possible pitfalls when using this approach, he argues that the incorporation of Persian mosaics will nonetheless benefit students who have an Islamic background.

My central question is: could the incorporation of non-Western content, while well-intentioned, actually endanger, or further exclude students, especially those from cultural minority backgrounds who might struggle with learning mathematics? I argue that while Karssenberg offers interesting pedagogical explorations through an enriched classroom experience, critical reflection is necessary to support the implementation of these ideas, so that the ethnomathematics approach does not simply become a "didactical tool" as Pais (2013) suggests (p. 2). Utilizing the work of Pais, and Vithal and Skovsmose on critical ethnomathematics, I would like to engage with two areas that are potential pedagogical *danger zones* in Karssenberg's article:

- a simplistic historical understanding of Persian and Islamic histories as apolitical and unilinear;
- the essentializing of diverse Islamic student experiences into a homogenous group.

Simplistic historical content

Pais (2013) acknowledges that the broad discipline of mathematics is culturally diverse and needs to be understood in reference to political, economic and social contexts in which it has been developed. Mathematics, in this sense, shares similarities with historical inquiry: in order to have a nuanced understanding of historical processes, one must disentangle social, economic, geographic, and political aspects of its often uneven and multidirectional development. When combining history in a mathematics context, the nuanced understanding of mathematics *and* historical content begs the inquirer to create interdisciplinary connections in their analyses. In this way, students can possibly arrive at a complex product that is larger than the sum of its individual parts.

What I find troubling in Karssenberg's example is the lack of historical complexity in the explanation of mosaic history. While one could argue that there is not enough time in the school year to provide students with such a rich description, there is a need to add some specificity to engage students in the learning process, which could otherwise fall flat. For example, mosaics were created across cultures and geographies, including Persia, Greece, and Rome, millennia before medieval Islamic geometric mosaics became widely known. Karssenberg notes that, "Islamic artists were restricted in their choice of subjects," but without the political, religious, and social knowledge from the pre-Arab conquest of the Persian Empire, students will not know why this is an important fact, or be able to articulate how mathematics, art, and history collide in certain times and places (p. 43). Before the conquest of Persia, mosaic art depicted intricate scenes of war, political leaders, women, and nature, with artists learning their craft from mentors, passed down orally and guarded, until Islamic mathematicians and Persian artists began collaborating to create large structures of religious devotion (Grabar, 1987; Sarhangi & Jablan, 2006). To what extent this process was peaceful and cooperative, or filled with violence and resistance could further the political and historical discussion of conquest, mathematics, and art, although, as Karssenberg states, there are few primary documents surrounding this process. In terms of being a *danger* to students, teaching a uniform and unilinear version of history without the intricacies of conflict and change, creates a shallow understanding of a geographic area that covered North Africa, India, the Middle East, parts of China and Europe during the middle ages, and, therefore, an essentialized view of non-Western culture and history (Sarhangi & Jablan, 2006; Vithal & Skovsmose, 1997).

Essentializing student experience

An important aspect of some versions of ethnomathematics is the ability of teachers to provide culturally relevant content for students to personally identify with, based on their experiences and/or interests. Vithal and Skovmose (1997) indicate that this is not a simple task, since teachers "not only have to access, understand and accept their students' social and cultural background knowledge, they need to be able to interpret these outside realities in terms of mathematics and transform them into curriculum experiences" (p. 145). Furthermore, teachers need to have a cultural understanding of the various social, political, and economic contexts their students come from, in order not to place all students into neatly defined inclusionary and exclusionary categories (Vithal & Skovmose, 1997). Karssenberg's example demonstrates the importance of a teacher's prior knowledge and cultural proficiency in implementing assumed culturally relevant content into a classroom with diverse student populations.

The expansion of the Islamic world during medieval times was geographically vast and, in contemporary history, includes culturally, politically, ethnically, and linguistically diverse societies. Karssenberg notes that his students are originally from Morocco and Turkey, yet he uses Persian examples of mosaics, in an effort to "motivate those with an Islamic background" (p. 44). It is assumed that the Islamic students from Morocco and Turkey will have the necessary cultural knowledge and experience to connect with the example of Persian artifacts, yet the student might not identify with Persian culture at all, as identification with a particular culture or cultures cannot be assumed. Students with "Islamic backgrounds" also include various linguistic, political, social, cultural understandings and knowledge of the world they live in, in addition to the ways they connect with schooling, and with mathematics in this particular example. What is pedagogically *dangerous* in this case is that the teacher did not appear to carefully think through the various complex cultural identities of each student, and assumed that all students with an Islamic background are part of a homogeneous group. This approach essentializes students into a fixed category that denies them their full and complex individual histories and identities, therefore creating an environment that ends up doing the opposite of what was intended: what was meant to be inclusionary becomes exclusionary.

An interesting point about Karssenberg's findings is that the students related to the content evenly, with little difference between Muslim and non-Muslim student responses, suggesting a lack of cultural connection to the content. While Karssenberg credits the cultural content to the relative success of the lessons, one student's response points to the process as being more important for learning: "I like this more than doing all these boring exercises alone!" (p. 48). Instead of sitting and working independently, students had the ability to collaborate on a project that might or might not have held cultural importance, which in the end seems to have been the most important finding of this experiment.

I reiterate the importance of creating interdisciplinary connections between mathematics, history, and art. However these connections need to be fully theorized, so as to not essentialize all students as sharing the same cultural experience and personal history. In the words of Pais (2013), "in the well-intentioned action of achieving a better world through mathematics education, ethnomathematicians often fail to acknowledge, in the corrupted reality they lament, the ultimate consequence of their own act" (p. 5). This is not to say that there no place for ethnomathematics in schools; on the contrary, the need for a continuously evolving, culturally responsive, pedagogically reflexive mathematics is necessary for change.

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From the archives

Editor's note: *The following remarks are extracted (and slightly edited) from an article by Hans-Georg Steiner (1985), published in FLM5(2).*

All our attempts thus far to exhibit various aspects of a comprehensive approach to mathematics education could yield the impression that there is an underlying structural *coherence* and *homogenicity* as an essential base for the intended integration and synthesis. However, there are *fundamental epistemological phenomena* which show us that the *opposite is the case.* As in physics, relativity theory and structuralism were followed by *quantum theory* with its *relation of indeterminateness,* and in N. Bohr's famous *principle of complementarity* it became clear in a much broader sense that:

> every relevant piece of theoretical knowledge being part of some idea or model of the real world, will in some way or another have to take into account that the person having the knowledge is part of the system represented by the knowledge. All knowledge presupposes a subject, an object and relations between them (which are established by means of the subject's activity). Therefore, all knowledge has an incoherent structure with metaphorical and strictly operative connections. (Otte, 1983, pp. 45-46)

In more recent times, this phenomenon has also been identified and confirmed with respect to mathematics and mathematics education (see IDM, 1981; Steiner, 1984). For example, most of the so called *"false dichotomies"* Peter Hilton was dealing with in his plenary talk at the Karlsruhe congress (Hilton, 1977), such as "skill vs. understanding", "structure building vs problem solving", "axiomatics vs. constructivism", "pure vs. applied mathematics", represent *pairs of seemingly opposing positions* which can be pursued through the history of mathematics and mathematics education. In educational practice and theory one has often tried to straighten out these paradoxes in a *reductionist way:* one has either given *absolute dominance* and principal importance to one of the two sides or one has taken a so called *multi-aspect position* which just says "do both" without really understanding and operationalizing the underlying antagonistic relationships which are actually connected in a fundamental way with the epistemological problem of the *relation between knowledge and activity* as the *kernel of all the complementarities.*

The concept of complementarity also turns out to be an adequate tool for better understanding the *relations between different types* and *levels of knowledge and activity* as they appear in contrapositions like "scientific theory vs. everyday knowledge", "meta-knowledge vs. primary knowledge", "empirical vs. formal", "the personal vs. the social", "perception vs. cognition", *etc.,* and also as they come up in the *regulation and control problem of systems-theory.* It plays an important role in the *foundations of cognitive psychology.* In his article titled "The Need for Complementarity in Models of Cognitive Behavior", Pattee writes:

> The classical idea that we can explain control in cognitive systems without complementary modes of description verges on a self-contradiction, or at least a conceptual paradox. Complementarity may be viewed as a recognition of the paradox. It has its roots in the subject-object dualism and in the basic paradox of determinism and free will. ... An idea that I would promote is that psychologists make the difficult effort to assimilate the basic concept of complementarity as an epistemological principle. It is by no means a clear and distinct concept, but it is rich and suggestive. The complementarity principle does not promote resolutions of the central binary oppositions of psychology: mind and body, structure and process, subject and object, determinism and free will, laws and controls, *etc.* On the contrary, …, the principle of complementarity requires simultaneous use of descriptive modes that are formally incompatible Instead of trying to resolve apparent contradictions, the strategy is to accept them as an irreducible aspect of reality. (Pattee, 1982, pp. 26-27)

This seems to be an adequate *description at a phenomenological level.* However, there are *deeper mechanisms* behind. Their analysis and reconstruction need conceptual tools as they are available from *activity theory*, which tries to build an understanding of cognition primarily on a concept of "*objective human activity*" rather than on "knowledge". Human object-related cooperation with its *practical wisdom, habitual features* and its *socio-historical reality* in Bourdieu's sense (see Bourdieu, 1980) plays a special fundamental role as a "*structuring structure*" in the regulation and control problem. M. Otte has described the interrelation between complementarity and human activity in the following way:

> Only within activity theory can the epistemological need for complementarity be productively developed and applied. On the other hand, a complementarist viewpoint…should prevent activity theory from detrimental reductionism and at the same time provide possibilities for the necessary and unavoidable relative reductionism. As far as the problem of cognition is concerned, we have to accept that we cannot know without knowing that we know. We cannot learn a particular theoretical concept without acquiring knowledge about theoretical concepts (their categorical characteristics, their function for cognition *etc.*). We cannot gain knowledge without acquiring meta-knowledge. But meta-knowledge is at one point the product of the evolution and at another its indispensable condition. Therefore, knowledge and meta-knowledge can neither be completely expressed nor represented as a closed and coherent system and in a uniform description. (Otte, 1984, pp. 53-54)

The *consequences for the development of* [Theory of Mathematics Education] should be clear: it can only be done successfully if it proceeds at a very general level and at the same time through very concrete examples; if it performs the systems approach simultaneously for the large system and for particular problem domains understood as sub-systems; if it is aware of and contributes to the elaboration of the inherent complementarities and related types of activities; if it is heading towards a simultaneous development of "the practical sense" and of "meta-knowledge", observing their fundamental interrelation.

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