THE VERY MULTI-FACETED NATURE OF MATHEMATICS EDUCATION RESEARCH[1]

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It is tempting to claim that mathematics education as a research domain is turning fifty. Along with a host of other significant undertakings and events around 1970, the world's first journal devoted to the scholarly and scientific study of mathematics education, *Educational Studies in Mathematics* (ESM), was established in 1968, and soon after, in 1970, another leading journal, *Journal for Research in Mathematics Education* (JRME) was founded. The first *International Congress on Mathematical Instruction* (ICME) was held in 1969. There was, of course, also mathematics education research to be found before 1968, but that time marks a significant milestone. However, one might well ask whether the papers published in ESM and JRME around 1970 would pass as research papers according to the predominant perceptions of research today.

During the past fifty years, what has been published under the label of mathematics education research constitutes a very broad, diverse and extensive body of contributions to the field. And not only that, these contributions display an immense variation in *nature*. However, during the last three decades I have detected a gradually reduced spectrum of variation in the nature of 'mainstream' published research, especially in our journals. It sometimes seems as if we are moving in the direction of that somewhat monolithic perception of research that is characteristic of an old and established discipline. One that has undergone centuries of development, which has clarified its foundations and purified its methodologies, resulting in a rather uniform standard for research publications and for the kind of research that lies behind them. But is mathematics education today really such a discipline?

In this article, I offer my account of this development, attempt to explain it, and finally offer a critical analysis of its consequences. By saying this I have already indicated that I have strong reservations towards parts of this development.

Aspects of the development of mathematics education research

If we look at papers published in ESM and JRME around 1970, they were typically, albeit not exclusively, of three kinds: *position and opinion* papers concerning mathematics as an educational subject, in general or in some of its aspects (for example, Klamkin, 1971 and Scandura, 1971a); papers offering an *exposition and/or discussion of a piece of mathematics* in an educational context (for example, Roganovskij, 1971 and Shepler, 1970); and papers presenting *teaching experiments,* sometimes but not always involving a control group, and typically accompanied by some quantitative data and an analysis thereof (for example, Rector & Henderson, 1970 and Smith, 1970). The latter category was much more prevalent in JRME than in ESM. Only seldom, at that time, did ESM and JRME publish papers of a conceptual or, even more so, a theory-oriented nature, such as Scandura (1971b).

The first two categories mentioned, position/opinion papers and mathematical-exposition-for-teaching papers, have largely disappeared from today's research journals. They would hardly count as research contributions at all. Only the third category of papers, empirical investigations of teaching experiments, is still alive, even though modern papers of this category are very different in approach, structure and style from the ones published in the early days.

In those early days, empirical mathematics education research was largely inspired by psychology (cognitive, developmental and educational). It would not be unfair to claim that psychology provided the conceptual and methodological umbrella for such research, together with the standard statistical testing apparatus employed in psychological and educational investigations. This is also the reason why mathematics educators in those years assumed that the scientific basis for research was to be found in psychology, as reflected in the establishment of the *International Group for the Psychology of Mathematics Education* (PME) in 1976. The notion of mathematics education research as a separate and independent domain, borrowing from a multitude of other fields, but subsumed under none, took several years to develop. I return to this issue below.

When teaching experiments and innovative curricula are placed on the agenda, the question of whether these experiments and curricula do in fact enhance learning becomes crucial. This gives rise to an even more fundamental question: How can we tell whether learning has improved? In the beginning this was gauged by means of test outcomes. If students taught according to approach A performed significantly better than students taught according to approach B, and it could be argued, with reference to some pre-tests, that students' performance at the outset was not significantly different in the two groups, then it was seen as safe to conclude that approach A was responsible for the difference in performance, and that, hence, approach A was more efficient than approach B.

This way of operating, however, places all the responsibility of gauging learning on the test instruments employed, where it is taken for granted that test results are valid indicators of learning. But soon these test instruments themselves became subject to questioning and criticism.

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Many tests were seen as focusing too narrowly on routine procedures and factual knowledge, whilst paying far too little attention to conceptual understanding, problem solving, and mathematical reasoning, aspects of mathematical learning that are not easily detectable and measurable by usual timed tests, let alone multiple-choice ones.

Another insight gradually emerged amongst researchers. Quantitative studies are based on counting entities, or on measuring entities by way of some sort of scale. For this to make sense, it is essential to know exactly what it is that we are counting or measuring. Counting requires the presence of discrete objects, or entities resulting from discretising a continuum, such as student responses classified as belonging to different categories, types of errors made, categories of problem solving strategies chosen by students, types of mathematical arguments preferred by students, response categories in questionnaires, *etc.* If the demarcation lines between different groups, categories or types are ill-defined or blurred, then counting results based on entities being put into categories are neither valid nor reliable, because even slightly different decisions on where to place a given entity may result in widely different counting results. Similarly, when ordinal or numerical scales are used to capture student background variables, performance outcomes, levels of reasoning, percentages of correct answers in a test, *etc.,* it is important that the measurements made according to the scale adopted are accurate and robust. If perturbations arise in the use of the scale, the outcomes of its use may change dramatically.

Therefore, for quantitative methods to be put to valid and reliable use, it is crucial that the entities counted or measured are well-defined and well delineated. This is particularly true if comparisons are involved. It is of utmost importance to go beyond the surface of things. But this is fundamentally a qualitative issue. This was one of the reasons why mathematics education researchers from the mid-1980's began to develop and employ a wide range of qualitative methods, a trend which has gained momentum ever since.

Qualitative methods in mathematics education research generate their own issues. How can we document what we see in interviews, classroom observations, or video-recordings, especially when the amounts of data typically are so immense that sample excerpts must be made (but on what grounds)? How can the reader of a qualitative research study be convinced that such excerpts are balanced and representative, given that it is unrealistic for the reader to check this against the entire pool of data?

The most significant issue is how qualitative data can be interpreted with reference to the initial aims of the research study, and to the questions posed in it, in such a way that this interpretation can generate justifiable and robust findings that can survive scrutiny with regard to possible alternative interpretations. Depending on the nature of the study, such interpretations often involve a variety of conceptual and methodological considerations drawn from diverse disciplines, mathematics included.

The key role of interpretation in qualitative studies generates the need for establishing some grounds for making interpretations. This is where theoretical frameworks, or even what some call 'theories', enter the stage. By invoking existing theoretical frameworks or theories, or by putting forward new or modified ones, researchers hope to be able to provide a platform on which interpretations can rely. Different scholarly traditions and schools of thought take different positions towards these issues, ranging from British pragmatism, which largely discards the significance of general theoretical frameworks, over the plethora of 'singular' theoretical frameworks found with individuals or groupings of researchers in Germany, to the French school of *didactique* based on Brousseau's *Theory of Didactical Situations* (TDS) and Chevallard's *Anthropological Theory of the Didactic* (ATD). Some very influential quarters attach an essential and overarching role to theoretical frameworks. In the current description of 'Characteristics of a High Quality JRME Manuscript' (NCTM, 2018) one reads, under the heading of 'A Coherent Theoretical Framework':

- The study is guided by a theoretical framework that influences the study's design; its instrumentation, data collection, and data analysis; and the interpretation of its findings.
- The literature review connects to and supports the theoretical framework.
- Make it clear to the reader how the theoretical framework influenced decisions about the design and conduct of the study.

In other words, according to the editors of JRME, a high-quality research study is *necessarily* guided by and subsumed under a coherent theoretical framework. This implies that studies conducted in order to answer research questions that are not derived from or embedded in such a framework, but for which possible theoretical frameworks are to be chosen *post festum* in response to the research questions posed, stand little chance of being published in JRME. One fundamental problem here is what counts as a 'theoretical framework' or a 'theory'. Looking into the ways in which these terms are used in the literature one sees that they are very far from being clear and well-defined. As a matter of fact, there is a plethora of meanings of these terms, and hence a plethora of uses of them, which in itself greatly weakens the capability of theoretical frameworks or theories to serve as a platform for the formulation of research problems, the design of a study or the interpretation of qualitative or quantitative data and findings. In a later section of this article a more detailed analysis of the notions of theory and theoretical framework in mathematics education research is offered.

Qualitative studies serve at least two different purposes. One purpose is to provide an existence proof of a certain phenomenon, and to offer explanations of the occurrence of the phenomenon, without necessarily making any claim of universality. Another purpose is to pave the way for subsequent quantitative studies conducted to chart the prevalence, generalisability or scalability of the qualitative findings obtained, or to investigate interrelations, correlations, and ultimately causalities, amongst the phenomena and processes identified in the qualitative study.

So far, this brief survey of the development of research in mathematics education has focused on the nature of the *research approaches* adopted. From the 1990s on, psychology ceased to be the most significant discipline informing such research. Mathematics education research has developed an independent identity, or to be more precise, several independent identities, seeking inspiration from a wide array of disciplines: mathematics, statistics, computer science, psychology, cognitive science, neuroscience, philosophy, linguistics, semiotics, history, social science, political science, general education including curriculum studies, and psychometrics.

Alongside this development, which represents a massive expansion of the network of links between mathematics education research and a multitude of other fields and disciplines, we have also witnessed a huge expansion of the domains, issues, questions, educational levels and target groups that mathematics education research sets out to deal with. Research has moved from focusing on primary and lower secondary mathematics education to also focus on pre-school and upper secondary mathematics education and on tertiary mathematics education at large, including mathematics as a service discipline and advanced university mathematics. Research has moved from being primarily interested in mathematics teaching to a massive interest in mathematical learning. Research has moved from paying particular attention to concept formation and procedural skills to dealing with all aspects of mathematical work and activity, such as problem posing and problem solving, mathematical reasoning, mathematical exploration and formulation of hypotheses and conjectures, mathematical modelling, and the role and use of technology in the teaching and learning of mathematics, as well as mathematical beliefs and affect. It has moved from focusing on rules, algorithms and procedures to focusing on meaning, sense making and understanding in mathematics, and on mathematical proficiency, competencies and practices. Research has moved from predominantly focusing on students to also focusing on teachers, their education and professional development, their beliefs, and practices.

This enormous expansion of mathematics education research in several different dimensions (see Niss, 2007b) implies that, today, its nature is multi-faceted and highly diverse. Yet, this fact is not adequately reflected in the dominant paradigms in research studies as published in leading journals in our field.

The structure of mathematics education research articles

Of course, there is considerable variation in the structure and organisation of research studies published in journals and books. Nevertheless, I am ready to risk my skin by claiming that an *ideal-typical* (in the sense of Max Weber, 1904/49) journal paper can be characterised as follows below. To be sure, an ideal-type is meant to be a descriptive notion attempting to capture *what is* the case across a host of particular specimens, not a normative notion meant to prescribe *what ought to be* the case.

The study presented in such an ideal-typical paper is a small-scale, qualitative, empirical case study, oftentimes accompanied by a quantitative survey of, say, item responses, student or classroom types, groupings, correlations between categories, *etc.*, conducted within the boundaries of the cases involved in the study.

In the introductory section of the paper (which may or may not be labelled 'introduction') the theme of, the rationale for, and the background to the study are presented, and key literature pertinent to the theme is briefly reviewed so that the study reported in the paper is situated in the existing research landscape. Depending on the underlying philosophy and nature of the study, one or a few research questions are then stated, possibly accompanied by comments on the purpose and goals of the study. Sometimes stating the research question(s) is postponed until or after a theoretical framework for the study has been presented. This happens when the research questions posed refer to and draw on concepts, terms and perspectives belonging to the theoretical framework(s) adopted, so that these questions cannot really be formulated outside the framework(s). At any rate, the outline of the theoretical framework(s), perspectives or constructs constitutes a centrepiece of the paper. In that section the author introduces the key concepts and terms that are used in the article and invokes the theoretical philosophy and perspectives meant to position and underpin the study. Next comes the section usually called 'method(s)' or 'methodology', but sometimes also 'the study', in which the approach to the empirical data generation and/or collection is presented, as is also the case with the statistical or other quantitative methods of analysis employed to analyse the data collected, provided quantitative issues form part of the study. Typically, the methods section also explains how the data generation or collection instruments were specifically put to use in the study, how analyses of the data were conducted, and how problems and challenges arising along the way were resolved. The next section in the paper, typically called 'results' or 'findings', is devoted to stating the outcomes of the data collection and analysis and to interpreting these outcomes as answers to the initial research questions (provided these were stated explicitly). It is usually in this section that one finds excerpts of transcribed interviews, classroom dialogues, open-ended questionnaires and so on and so forth, serving as empirical illustration of and evidence for the findings. This section is followed by one or two sections containing a 'discussion' and presenting a 'conclusion', in which the author discusses the scope, validity and robustness of the findings obtained, identifies possible weaknesses and limitations, and summarises what new knowledge and insights the study has produced. Finally, many papers have a section devoted to speculations on possible educational or other implications of the study, or to pointing out future research perspectives, either in terms of new questions to investigate in a continuation of the study, or in terms of applications of the research methods employed in the study to new contexts or for new purposes.

Since this is an analytic reconstruction of the ideal-typical paper published in today's front-line research journals, I do not claim that every published paper is written in exactly the way just presented [2]. The full pool of papers does, of course, display deviations from this structure, especially when it comes to the relatively few purely theoretical papers. What I do claim is that the ideal-typical paper captures the essence of a highly predominant segment of published journal articles. I further claim that reviewers tend to strongly adhere to the template constituted by the ideal-typical paper and explicitly criticise papers that deviate from it.

Theory, theoretical frameworks and constructs

One of the key components in the ideal-typical journal article is the theoretical framework(s) and constructs invoked to position and underpin the study. Elsewhere (Niss, 2007a) I have discussed the concept of *theory* in mathematics education and concluded that this is a highly ill-defined concept, which covers a wide variety of meanings, ranging from nothing but a limited set of singular notions and terms, as is very usual, over a more elaborate network of interrelated notions and distinctions, through to what I define as a theory (Niss, 2007a):

- A theory consists of an *organised network of concepts* (including ideas, notions, distinctions, terms, *etc.*) *and claims* about some extensive domain, or a class of domains, of objects, situations and phenomena.
- In the theory, the *concepts are linked in a connected hierarchy* (oftentimes of a logical or proto-logical nature), in which a certain set of concepts, taken to be basic, are used as building blocks in the formation of the other concepts in the hierarchy.
- In the theory, the *claims are either* basic hypotheses, assumptions, or axioms, taken as *fundamental* (*i.e.,* not subject to discussion within the boundaries of the theory itself), or statements obtained from the fundamental claims by means of *formal or material* (by 'material' we mean experiential or experimental) *derivation* (including reasoning). (p. 99)

This understanding of the concept of theory is rarely encountered in mathematics education research. Instead, what mostly passes as a theory is a much weaker construct in which one or more of the above three characteristics is relaxed, usually the hierarchical structure of the concepts, or the distinction between fundamental and derived claims. Many constructs termed 'theory' consist only of a few basic concepts and perhaps, but not necessarily, a few claims involving these concepts. Such a 'theory' amounts to putting forward some notions and terms, oftentimes degenerating to mere scientific term or name dropping.

The fact that 'theory' has turned out to be a rather vague and ill-defined notion may explain why most articles prefer to speak of *theoretical frameworks* or *perspectives* rather than of theories. A theoretical framework typically consists of an outline of a domain of entities, phenomena, or issues supposed to be captured by the framework, as well as a number of *constructs*, that is, a set of more or less connected concepts and terms by means of which it is possible to speak about the domain under consideration. Sometimes, a theoretical framework also includes certain basic but usually general claims about how to understand the entities and phenomena of the domain and their interrelations. Still, what counts as a theoretical framework, even though the notion is less ambitious than that of a theory, is highly diverse.

Theories or theoretical frameworks seem to serve at least six different, yet not contradictory, purposes (Niss, 2007a, p. 100): to provide *explanation* of some observed phenomena; to provide *prediction* of the possible occurrence of certain phenomena; to provide *guidance for action or behaviour;* to provide *a structured set of lenses* through which aspects or parts of the world can be observed, studied, analysed or interpreted; to provide a *safeguard against unscientific approaches,* or, differently put, against "ad hoc empiricism that is theoretically vacuous" (Schoenfeld, 1992, p. 181); and finally to provide *protection against attacks* from sceptical colleagues in other disciplines.

Frank Lester (2005) suggests abandoning theoretical frameworks in mathematics education research and replacing them by what he calls *research frameworks*:

> A research framework is a basic structure of the ideas (*i.e.* abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated. (p. 458)

Lester joins forces with Gravemeijer (1994), and later Cobb (2007), when they make a plea for researchers to act as *bricoleurs*, "by adapting ideas from a range of theoretical sources to suit our goals" (Lester, 2005, p. 466). As long as we are so far away from having overarching, stable, and exhaustive theoretical frameworks to underpin our research in mathematics education, I, for one, want to position myself in the same camp as these researchers.

Research questions, methods and methodology

Another key component in the ideal-typical journal article is the research questions to which answers are sought. In cases—rarely found in an ideal-typical article—where these are stated at the beginning, after presentation of the background, but before mention of any theoretical framework, the study is largely problem-driven (Schoenfeld, 1992, p. 180; Arcavi, 2000). If the research questions are formulated within or in close association with the predominant theoretical framework(s) adopted for the study, the study is largely theory-driven (Schoenfeld, 1992, p. 180). The place and role of the research questions have a significant impact on the methods adopted to answer them.

In an ideal-typical study for which the research questions are formulated outside any theoretical framework, the choice of an extant method or the design of a new one can either be made right from the beginning, aimed directly at dealing with the research questions posed, without being informed by any particular framework, or it can be made after theoretical frameworks to deal with the research question(s) have been chosen or established, in which case the methods at least have to be compatible with, if not simply derived from or embedded in, the frameworks adopted. Of course, this is true also of the much more prevalent cases in which the research questions have been formulated within the chosen theoretical framework(s).

Looking at ideal-typical papers, the most significant challenge with respect to methodology is whether the methods adopted are actually suited to answer the research questions, entirely or in parts. More often than not, there is a marked discrepancy between the nature and scope of the answers sought and those actually obtained, which is often due to the inadequacy of the methods adopted.

Critical issues concerning the predominant paradigms in research articles

By insisting on explicitness and clarity, the ideal-typical mathematics education research article has several merits. Also, it is not difficult to understand how it came about in the development of mathematics education research, namely as an attempt to vaccinate the field against a number of actual or potential internal illnesses, such as: unspecific and imprecise terminology and language; lack of clarity concerning the purposes and aims of given research studies and of the questions they set out to answer; sheer 'opinionating' substantiated by nothing but prejudices or impressionistic anecdotal evidence; and shaky theoretical or empirical bases on which inferences are made and conclusions drawn, implying that findings are less than satisfactorily supported. On top of all this comes the need for protecting our field against criticism from outside, which some believe is achieved by defining, adopting, and implementing very rigid standards such as the ones represented by the ideal-typical research paper. So, there are lots of good reasons for *discussing* criteria for quality in mathematics education research papers. However, there are serious problems in *establishing rigid standards* for such papers.

The most important problem is that our field is very far indeed from having arrived at an agreed-upon unifying foundation of mathematics education research, when it comes to the nature and place of significant issues and good research questions, to theories and theoretical frameworks to underpin and shape our research, to fruitful and effective research designs and accompanying methods that are conducive to providing relevant, valid, and reliable answers to the research questions, and when it comes to the place and role in research of general reflections in and on our field. It would be wonderful if our field had such a unifying foundation, but unfortunately, it does not. As Schoenfeld wrote in 1992:

> In times of normal science, researchers have it easy. Established paradigms shape the vast majority of inquiries undertaken, and established methods appear to deal more or less adequately with the phenomena that are of paradigmatic interest. […] The learning sciences, also known as cognition-and-instruction or cognitive-science-and-education are decidedly not in a period of normal science. (pp. 179–180)

This is no less true today than it was then. Even though the ideal-typical research article is not the only type to be found in journals, it does give the impression that mathematics education research is much more monolithic in its requirements than is actually the case. This is highly unfortunate for at least two reasons. The fact that a rapidly increasing number of countries, universities, research agencies, and institutions only take articles published in journals into account implies that the researcher—and above all the novice researcher having recently entered the field—ought to act according to the ideal-typical paper standards, especially when these are formulated by one of the most prestigious journals, JRME, as characteristics of high quality papers. This clearly moulds research publications to fit the ideal-typical paradigm. The second, more important, reason is that research conducted according to different perceptions of what is significant and relevant will gradually disappear, because it cannot be published in research journals. This constitutes a serious danger to our field. If it prematurely congeals into a narrow prototypical understanding of what counts as research, it will lose the ability to develop and renew itself. This is particularly true of theory-driven research, which "tends to focus attention on issues that are essentially within the scope of current theory" (Schoenfeld, 1992, p. 180). If one of the main purposes of conducting mathematics education research is to pave the way for creating better theories that will eventually expand and consolidate our understanding of the teaching and learning of mathematics in all its manifestations and under all its boundary conditions, it is less than healthy to force mathematics education research to be locked up in extant theoretical frameworks.

Conceptually or theoretically oriented reflective research, without an empirical component, seems to be the kind of research that suffers the most from the predominant publication paradigm. Our field is simply not so far and well developed that we can let this happen. Fortunately, the waters are not yet completely frozen. Firstly, it is still possible to publish non-ideal-typical papers in some of our journals, including prestigious ones. Secondly, a much wider spectrum of research papers can be published in edited books. It is often in book chapters that we find the most ground-breaking and innovative research contributions to our field. Can our journals afford to let this be the case? Even though the waters are not yet completely frozen, we need to send out ice-breakers to keep the waters open for sailing.

What can we do to keep the waters open?

The most important thing for us as researchers, editors, reviewers, and supervisors, in working to keep the waters open, is to insist on avoiding narrow, rigid templates for research studies and papers, as represented by the ideal-typical article. Of course, I am not suggesting that such articles should be banned, but they should not stand alone. We must accept, foster, and adopt a multiplicity of approaches to all aspects of our studies and resulting papers, most importantly when it comes to the nature and role of research questions, theories and theoretical frameworks, methods and methodology, and interpretation of findings.

It is of particular importance that it is possible to publish non-empirical papers that focus on raising an issue, on putting forward new concepts and theoretical constructs, on proposing theoretical distinctions, on analysing, comparing or linking theoretical frameworks, or on presenting and analysing methods [3]. It is also important that it is possible to publish papers on curriculum design and development, even without empirical evidence to corroborate them.

Some might think that I have portrayed the state of affairs concerning publications in our field in too dire and pessimistic a way, because they can point to many published papers that do not fall under the ideal-typical category of papers introduced above, even papers that display some of the features I have pleaded for. Maybe so. I am definitely not claiming to have fairly captured all sorts of published papers. However, I maintain that I have captured a pertinent trend in research studies and publications that is detrimental and potentially dangerous to the development and future of our field.

In conclusion, the most important thing is that we work to keep the waters open while remaining/becoming open-minded, reflective, prepared and able to discuss the nature of our field, also in journal articles. This is not to say that anything goes. On the contrary, more than ever we must analyse and discuss quality, but we should not allow too narrow and rigid paradigms to jeopardise our discussions or our field of research.

Editor's Notes

[1] This article is based on a PME plenary lecture of the same name, (Niss, 2018).

[2] FLM articles should *not* be written in this way.

[3] FLM welcomes such articles.

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Editor's note

As the previous article by Mogens Niss indicates, academic publishing in mathematics education has changed a great deal over the years. In addition to the issues he raises, there are others: electronic and open access journals are playing an ever-increasing role; various measures of 'impact' have come into being and increasingly influence decision making; and colleagues in many places are coming under increasing pressure to 'publish or perish'.

What does this mean for the field of mathematics education? Does the existence of more journals publishing more articles in a wider range of formats indicate progress? Does it change the nature of our field? As my predecessor, Richard Barwell, put it in issue **31**(2), "There are more researchers, more publications and more topics than ever before. Do we know more as a result? Or are we simply making more noise?"

I invite FLM readers to comment, in a short communication of 1000-2000 words, on the state of academic publishing in mathematics education. What challenges do we face, what strengths do we have, in what direction are we going? Contributions may be written in English or French, and should be sent to editor@flm-journal.org.