

# Probes and Fragments

JOHN HEALEY MASON

Shuffling feet; ripples of chatter as friend greets half-remembered friend; scrapes and crackles of programmes as the day's timetable is sought and choices debated; "Probes and Fragments—that's a funny title. what does it mean?", "Who is he?", "I hope he doesn't run over the hour." "?", "What did you think of last night's lecture?", ...; butterfly attention darts from thought to thought; the sounds fade

And me? Sitting awkwardly, present yet not present, slipping in and out of that about-to-be-spoken-from world of shifting possibilities for what to do in the coming event. Stomach heaves. How shall I start? How finish?—aye, that's my real weakness! Will it be too complicated, too obvious, too simple, too ? I see myself stand, awkwardly, twice as large as life but out of touch; struggling to contact the audience as a whole; later there will be a few coy sorties, to try to get response. Memories of ivy'd college walls in a freshman autumn evening, seeking contact/connection with the freshies; gravitating to warmth or flow; ion-charged ends of my sensitivities attracted to some and away from others; it's the same with any crowd, wooing the audience as a whole, responding to patches of appreciation.

## Introduction

My opening is prepared. Pensively I move to the OHP and flick on a switch. The hall growls with the quietly rumbling opening notes of Healey Willan's *Introduction, Passacaglia and Fugue*, like a monster awakening from a long sleep. No one knows it, so will this parallel structure work? Is it for them, for Grandad, or for me, that I inflict this extra medium? I turn on the OHP and up come the titles of the three sections. I scan the audience, then place my pen on the acetate pointing to "Introduction" just as the hall explodes with fortissimo. My eyes sweep to the ceiling, listening, and conveying listening. (I gulp at the use of a conduit metaphor like "convey" for what is actually participation.) The music fades

What do I say next? Part of me wants to start with the personal, to build up quietly like the music:

*It is a singular pleasure and honour to be invited to talk with you this morning. I can think of no more fitting way to mark my twenty-fifth year of teaching mathematics than by being here.*

*Each year I have sorrowed at my inept efforts, and each*

*year I have sought, and found, some new avenue of hope*

*Each year for the last ten of those twenty-five years I have tried to express to others something of my search, in the hope that they might benefit, glossing over the pits and cracks in my veneer, trying to put on a positive and hopeful face.*

*As with each preceding year, this year I think I have learned something, and several directions seem attractive. I hope to convey to you (there is that metaphor again!) something of the rich tapestry of interconnected awarenesses which manifest themselves as me in the classroom.*

I speak from a virtual hill looking out over the landscape of mathematics education, yet I know that my paltry words cannot come close to painting the scene; that my audience must be lost in a sea of generality. I am talking myself in, like an athlete doing warm up exercises, except that the audience and I need to warm up together. My mental screen fills with a picture of Jenny sitting at a table after dinner; a remark I can barely remember about how to communicate results of research to teachers; a flooding, swaying, opening; my eyes fill with moisture; a partial seeing, so fleeting. She goes on talking. I try to say the seeing but it chokes in my throat and is washed away in the conversation's tide. A similar case now presents itself: *I see a little; you see something; but can we see, together? How? It's no good having large numbers of people talking at each other, or writing reports at each other, because that forces an upward drift to superficiality and consensus. How can we dive down deeply together?*

*I have learned that I cannot tell you anything about teaching, just as you cannot tell a pupil anything about learning. Or rather, I can tell you, but can you hear the bell I'm tolling? Can it have significance for you?*

Mustn't get carried away, drifting into Kelley and the re-construction of meaning; into Plato and the re-collecting of knowledge as in the *Meno*; into Gattegno and the re-leasing of awarenesses. That is yet to come. Another time, another place perhaps to develop those, yet they attract me so that I want to make a fresh start and get into them more directly. REMEMBER the title—"Probes and

Fragments". How can I get back to them? Have I ever left them?—no, but the audience don't yet know that!

*Research into teaching is not about how to teach better. It's about broadening; about extending awareness and options. It's about awakening to alternative ways of seeing the pupils, of hearing and appreciating what they say and do. The result can at least be more satisfying for pupils and teachers.*

*Pupils don't learn better from one treatment or another. It's no good trying them out in different combinations like seeds sown in the Latin squares of an agricultural research plot, seeking the best nourishment combination. The numerous schemes of work and the plethora of undergraduate calculus books support my contention that pupils are not machines waiting for the perfect grade of fuel to be injected.*

I see in my mind Lancashire cotton mills and automated assembly lines. The industrial revolution spawned more than mechanisation of labour and universal education. It established the image of a machine which, given fuel and maintenance, performs whenever and wherever required. People are not like machines! How often does Orwell have to tell us? Yet we continue to think this way, writing schemes of work for students and teachers we will never meet, and talking about whether this or that activity is engaging. An activity is not engaging. Engagement is a product of people, activity and context. Schemes are of course necessary, but there is a large and significant missing component. Perhaps we can approach it together today, bearing in mind that our brains are far more sophisticated than mere speech-generating devices, and that not everything of importance can be said (yet I keep on trying, or at least I keep on speaking, partly to occupy that speech-receiving part so that something else can operate).

*It's how you are, you, with your class, in contact with the force of mathematics, manifesting it in your every act. That's what we need to work on—on Being; on Mathematical Being; on Being Mathematical with pupils.*

It won't do—too much preaching, even though I have reached a crescendo matching the music. What shall I do? The rest of me wants to start in my usual way, straight in with some mathematics, so that we all have an immediate common experience on which to reflect. How about:

*I'm not going to say anything you don't know already, but I shall invite you to re-collect things of which you are aware, even if not at the level of articulation. I hope to say things in a new way, making the familiar, strange. Those of you familiar with Plato's Meno, the epitome of Socratic dialogue, and those of you familiar with Gattegno's What we owe children, may recognise that my wish is to work on awarenesses; to activate powers that we all possess but which may not be suffi-*

*ciently developed yet to be at our beck and call.*

*I would like to draw attention to the fragmentary aspect of our experience, and certainly, of the parts of experience that we can recall. For example, take a few moments to recall last night's opening lecture. Stimulating, beautifully presented, yet when put on the spot you can recollect only the briefest of incidents. You may be able to piece these together, so that more details come back to you, but it is hard, is it not? That is probably what it is like for your pupils!*

Re-call, re-collect and re-member spawn potent images: existential, substantial and needing to be filled out. If I stop and dwell on them, everyone will lose the thread, including me probably!

*The past falls into fragments which with luck you can weave together again, producing your own version of what happened, and of what the "mathematics" is about. I shall try to build up a picture of these fragments, and of how we can help pupils construe meaning for themselves. This approach suggests three things.*

*Probes—for stimulating construal and for revealing uncertainties and misunderstandings. Diagnosis is one aspect, but often the purpose of a probe is to stimulate thinking, not to open a wound that you intend to try to heal.*

*Entries—for activating appropriate awareness and getting pupils thinking before technical and conventional details are imposed on them.*

*Helping pupils learn the difference between Working On mathematics, and Working Through exercises. The latter is intended to accomplish the former, but rarely does. I shall, given time, suggest alternatives.*

Pretty straightforward, direct signposting, but awfully boring! Ah well, perhaps the content is sufficiently jolting that I should at least appear to be giving a lucid and well-thought-out exposition, so that they can "take good notes" and feel that they go out with something (that conveyance metaphor once again!). I know that the only effect will be if somehow a little seed gets past their defences and begins to sprout much later!

However I get started, the next bit is prepared. I have been moving towards the OHP as my introductory remarks come to a close. I flick a switch, and music again fills the air, the eight bar theme of the *Passacaglia*. I casually roll my pointer-pen towards the *Passacaglia* cloud. A slow continuous dance rhythm, usually on a ground bass and in  $\frac{3}{4}$  time. Grandad developed eighteen variations—I will be lucky to have time for two, and there's the fugue to come as well! What could I use for examples/variations?

#### **Passacaglia**

Which of my many examples do I use? I want them to think about a mathematical topic in a new way, one which will challenge them a little, yet be neither too advanced nor too

elementary for anyone.

#### OPTION 1

I write silently on the OHP:

$$(-1) \times (-1) =$$

I pause, and look at the audience expectantly.

$$(-1) \times (-1) = 1$$

*How do you know?*

*Of what does your knowledge consist? Discuss in pairs together some of the images, the connections which were triggered in you by this statement.*

The room buzzes—it always does, and my immediate concern is to get them back again, to reassert my authority. Yet why must I participate overtly? Must I be present in order that something valuable should happen? As teachers we are constantly under pressure from ourselves and from “accountability”, to keep track, to keep records, to know what is going on. My presence actually inhibits, especially when everyone has something to say. I believe that it is important for everyone to try to express themselves because it often reveals unsuspected crevices and crevasses, and it helps each person to make the ideas their own.

After a few seconds:

*Tell me some of the things you came up with... (I make a list on the board)*

Funny how many more people have something to say once their mouths have got going in a pair than is the case if you ask directly for contributions. This is a tricky moment—control, yet listening to what each person says, not getting sidetracked, and finding a graceful way to draw on their experience, make my point (an interesting metaphor?) and move on.

*These are one form of Fragments, the fragments which come together and congeal in your mind when you attempt to explain something to others. But is your story, your fabric of woven fragments available to others? Do not they have to weave their own story?*

*What are the threads?—the images, the sounds, the feelings and connections which are available to you? Are you open to more, or do you reject other people's images as being inferior or unnecessary?*

*How does it feel to be a student? Are you in the habit of evoking such fragments? Do you share yours with pupils and colleagues? How can images and awarenesses be shared? Reluctance to share is often justified by saying that each person's impressions and images are idiosyncratic and that you may do damage by inflicting yours on others. I put it to you that by your very presence in the classroom, your very Being there, you are manifesting images and attitudes. By all means interrogate your images before sharing them. See if they are potent or merely fanciful.*

*ful Ask yourself “what are the fundamental awarenesses that contribute to my sense of this topic?” But then share those images. That's what Probes, Entries and Fragments are all about*

#### OPTION 2

*$1/-1 = -1/1$ , so taking square roots of all four numbers and then cross multiplying I find that  $i^2 = 1$ . What went wrong?*

Could expect lots of discussion about principal square roots; so often algebra and complex numbers are introduced as “merely extended arithmetic”.

#### OPTION 3

The Intermediate Value Theorem. What does it say, why is it of interest? People like Darboux and Cauchy wanted all their functions to satisfy it so that they could assert that their equations had solutions. What functions do actually satisfy it? This led to continuity; continuous functions do satisfy the theorem, but so do lots of others. I could construct one for them, then ask them to explain to each other in pairs why it satisfies the theorem but fails to be continuous. Very Lakatosian, to try to characterise functions which satisfy the intermediate value theorem—the easiest way is to say “the ones that satisfy it”!

#### OPTION 4

Trigonometric functions seen as the shadow-projections of a point moving round a circle. Start with a point moving round a square. It certainly worked well last time, and there is plenty left to explore. It offers a direct perception of what sine and cosine are about. It might give a clear example of an awareness. Having shown them the sine animation and asked them to discuss it, I announce that I am about to show them the corresponding animation for tangent. *But just before I do, why not pause and construct in your mind what you think you're about to see!*

#### OPTION 5

Adding fractions. You can add them in the normal way, or you can add tops and add bottoms. We mark the latter wrong, yet it often appears on scripts when adding up marks. It is also the essential idea behind best approximations for the bisection method, and it leads to continued fractions and the origins of Greek number perceptions. It might demonstrate that mathematics is not all “right/wrong”, that it depends on context.

#### OPTION 6

I could do a very short presentation of the Pecking Order Theorem: either there is one chicken that pecks all others, or there are three chickens such that

every other chicken is pecked by one of them. The audience could then see how much of the argument they could reconstruct in pairs. It might show up fragments and fragment-processing skills

The music is reaching the triumphant variations sixteen and seventeen, with the theme in the pedal in broken rhythm under a manual reed fanfare, and then with the theme majestically restated in its original form in the pedal under flashing scale passages and crashing chords. I haven't managed eighteen, but I should have thought my six will do. Trouble is, there won't be time. There never is. "I've got 5 minutes, tell me all you know!" Very annoying! Which options will be most effective? In fact it is not the option which is effective, but the interaction between the audience, the option, and me, and there's no telling in advance. Have to see how it goes.

*Fragments are those directly and vividly recalled images and awarenesses. They constitute the basis of our understanding. When stimulated, the fragments are re-collected, and literally re-remembered, as details are filled in and further fragments accessed, leading to a "body of knowledge".*

Funny how once an idea comes up inside me I end up talking about it. It's almost as if the idea seeks a way out, an opportunity for expression. It generally works to my advantage, but what about the audience?

The *Passacaglia* fades away and the *Fugue* is about to begin. I approach the OHP and point to the fugue cloud.

### Fugue

Literally "fleeing", and used in psychology (unfortunately) for attempts to escape reality. What a pity because I am attempting to contact the reality of learning

My theme:

Working On and Working Through;  
Centripetal and Centrifugal study

*Having looked closely at what students do with O.U. texts and television programmes, and recalling my own study habit, I find it helpful to distinguish between "Working Through" exercises and texts bit by bit, and "Working On" the ideas, trying to make over-all sense of them and to fit them together. Working Through is, in the sense of Northrop Frye, centrifugal study, dealing with details as they arise, working on question after question without linking them together.*

On the OHP I show a typical text with its exercises, and observe that the authors probably intended pupils to generalise, to see an underlying pattern, but as we all know, most pupils tackle one after another and never notice any pattern.

*Working On is in Frye's terms centripetal, seeking the centre and source, the gestalt overview. Both are necessary, but I think you will agree that there is a prepond-*

*erance of the 'fugal over the 'petal.*

That is my theme, to be developed around the *Passacaglia* variations, counterpointed, inverted and strettoed in several voices.

*After you've introduced a topic, rehearsed the technique, and had the pupils do several exercises, what do you do? Do you act the same way they do, rushing on to the next topic? Do you help them integrate what they've done? Do you help them see what patterns are suggested by the exercises?*

I reflect back on each of my six *passacaglia* options, referring to what arose in the discussions and relating it to patterns of study. It's as if I'm gazing at a large and sumptuous Persian carpet. The busy border recedes, turning into a gate to paradise. I sense the depth, the richness, the interconnection of what I'm saying with my experience, knowing full well that I have not learned to actualise it effectively in classes — but I suspect that with training pupils would become more receptive and we would all learn. What can the audience make of it? Each brings his or her own experience and concerns (that conveyance metaphor again). How can I trigger potent common images — or maybe I can't. My six options are intended to do just that, but each person attends to the event differently, and it seems I can never know for sure what they have stressed and what ignored.

I know my time is short. I feel an urgency. I resort to exposition knowing full well that what I say is fully self-referent. To make useful sense of this event they will have to go through the very processes I am going to enumerate!

On the OHP I place my prepared slide:

#### Fragment Processing Skills

1. Can your pupils recall the main fragments of the topic?
2. Can they fill in details between recollected fragments?
3. Can they account for specific fragments in relation to their previous experience?
4. Can they give a reasonably coherent account of how the fragments fit together?
- 5a. Given a problem or an idea, can they identify some fragment from the topic which provides a specific instance? (Specialising)
- 5b. Given a problem or an idea, can they identify some fragment of their own which provides a specific instance? (Specialising)
- 6a. Given one or more fragments from the topic, can they indicate what is being illustrated? (Generalising)
- 6b. Given an idea or example from elsewhere, can they indicate a connection with the topic? (Generalising)

None of these are spoken. Everyone present can read — all I do is read them quietly to myself, then place a pointer at one and illustrate it from a *Passacaglia* option.

*These skills are stated in very general, context-free terms. We each need to find our own way to ask them as pertinent context-dependent questions. If we also draw pupils' attention gently to the effect of these questions, to the Work that they then do. On their experience, the pupils may learn invaluable study habits. They may also begin to find studying more attractive because they actually have the opportunity and the assistance to make things their own, to feel confident about what they do know, and confident that they can cope with not knowing what they do not know.*

*The most I can hope to do in this short time is to draw your attention to the desperate need that our students have for assistance in Working On rather than just Working Through*

*I invite you now to pause and reflect on the chunks of mathematics that we have looked at briefly together. Some could be used as diagnostic probes; not setting out to find errors, so much as revealing "where you are" with respect to certain curiosities and stumbling blocks. I don't need to impose my views on you, and there are no "right" ways of dealing with them. If you focus on what you see, and I focus on how you say what you see, I can learn quite a bit about your mathematical awarenesses. This in turn may suggest particular activities, problems or investigations which might appropriately challenge you—in other words, something for you to Work On.*

*Some probes can be directly diagnostic (two fractions to add, two decimals to multiply, a situation in which the empty set is the only counter-example). These might accurately be called instrumental, following the language of Skemp. Some probes might be designed to reveal whether pupils can reach fragment processing skill 4—relational understanding. Some probes can be designed to reveal specific conceptual misunderstandings, though it is my experience that once a mathematical discussion gets started, all sorts of weaknesses and misunderstandings emerge. That's why I include Entry*

*Points as Probes.*

*Entry Points are mathematical questions like "What kind of equations can I be certain have solutions even if I can't find them?". They offer a sense of direction, and even though they may be too hard for pupils to resolve immediately, if ever, they indicate why someone might engage in study of the topic.*

*Diagnosis usually follows exposition and practice. Some probes can be chosen to give entry to a topic, by focussing attention on the specific awareness which supports or underlies the topic*

I would love to develop this subsidiary theme, but there is not the space. It's a pity it takes me so long to get to this point. It would be nice to find a group of people who could begin to work on how to become aware of latent powers and awarenesses that produce a given mathematical notion, and how to foster these in pupils.

Algebraic pattern in the case of  $-1 \times -1$ ; geometry of complex numbers in the case of the false proof; the search for when the Intermediate Value Theorem holds and for capturing continuity; trig functions as projections of uniform circular motion; ...

I have prepared my ending. I move to the video-player and as the fugue recalls the passacaglia theme, I start the machine.

*If I had time, I would get you to work on this with me. But I haven't. So I won't.*

A silent animation appears on the screens and the music comes to a crashing close

Suddenly I hear my name. The hall is silent. "... Probes and Fragments." Clapping. I stand, awkwardly, unsure of where I am, twice as large as life but out of touch. ... The lecture is about to begin!