TALKING ABOUT EMBODIMENT AND CARING IN RELATION TO COMPUTER USE IN MATHEMATICS EDUCATION

AMY HACKENBERG, NATHALIE SINCLAIR

Technology [..] the knack of so arranging the world that we don't have to experience it (Frisch, 1959, p 178)

This quotation captures a central focus of a year-long conversation between us, which was triggered by an interaction over technology, sustained via technology, and focused on issues of technology in mathematics education As is characteristic of conversing, we soon meandered into many other areas, but found ourselves returning time and time again to two themes regarding computer use in the practice of mathematics education: embodiment and mathematical caring We draw on our conversation to outline our critique of the Frisch quotation, and, more importantly, to explore the possibility of creating new narratives for questioning, supporting and even advocating the use of computer- based technology in mathematics teaching and learning.

Nathalie: So to begin, Frisch's quotation reflects that, for many people, the computer represents a cold, lonely, rigid object that gets in the way of achieving their goals. Although the advent of the internet and email is changing people's interactions with computers quite drastically, the machine itself still causes anxiety when it comes to mathematics – especially for mathematics teachers In fact, despite its now well-entrenched availability and well-accepted effectiveness, mathematics teachers rarely use computers in their classrooms (Becker, 2000). What might change this state of affairs? Better software programs? Better computer labs in schools? Better pre-service or in-service education? A new generation of teachers? Or do we need new narratives about computers and learning? Our existing narratives either involve the computer as the solution to existing problems or as the means to facilitate an escape from, or barrier to, experiencing one's world

Amy: In fact, we have been struck by how themes of embodiment and mathematical caring in relation to computer use open possibilities for new narratives. The notion that computer use can facilitate, rather than thwart, bodily experience in mathematics teaching and learning suggests a warmer and earthier image than the view you just described And the notion that computer use may have direct bearing on establishing care between human beings engaged in mathematical interaction is also novel, and even antithetical, to the view you described, since computer use is often envisioned as a solitary, perhaps even anti-social, affair

Nathalie: It might be useful to look at how these themes play out using a specific example Let's recount our first computer-related interaction

Episode 1: Embodying linearity

Nathalie and Amy met over a computer As part of a larger group, they had been asked to watch two colleagues push chairs along the floor to model the horizontal and vertical force combination that would result in the chair moving along the line $y = 3x$. This was an embodied experience of linearity, they were told The movement of the chair looked about as linear as the path of the drunken sailor's random walk, whether because of friction, gravity, or uncooperative participants Amy questioned whether the notion of making a line was being taken as the beginning point, rather than starting with the physical and mental activity that could produce a line (e.g, moving at a constant rate), and then considering why the result should be a line She surreptitiously pulled out her computer and used *Sketchpad* to make a representation of a rate of 3 miles per hour by constructing a moveable point in a plane This point (point A in Figure 1a) represented all the possible pairings of the lengths of a time segment (horizontal) and a distance segment (vertical, on *y*-axis), where the distance segment was always 3 times the length of the time segment. The trace of this point over all values of time results in a line (Figure 1b).

At the same time, Nathalie surreptitiously pulled out her computer and used *Sketchpad* to quickly model the "pushing" of the chair. To represent the two people pushing, she used two cars orthogonal to each other pushing on the same point, and also made it possible to pre-determine the 'force'
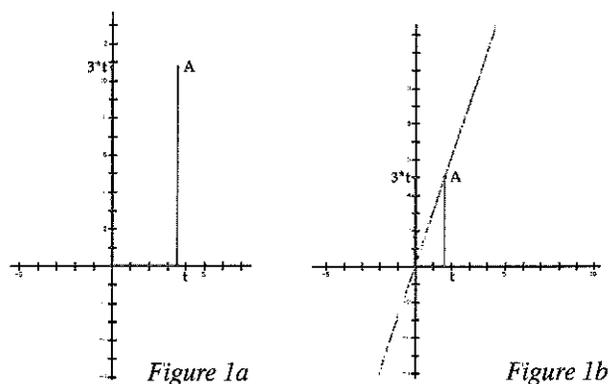

Figure 1a   *Figure 1b*

Figure 1: A representation of a rate of 3 miles per hour. Point A (Figure 1a) is a moveable point and the trace (Figure 1b) of this point over time is a straight line.
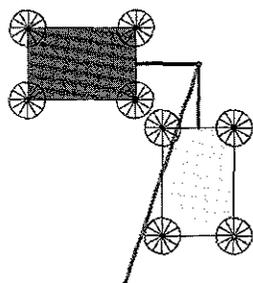
x force = 1.00
y force = 3.00
| Push point! |

Figure 2: Two cars exerting force on a point resulting in a linear trace.

with which each car would push. By pressing 'Push point!' the two cars started exerting force on the point causing the point to move and leave behind a perfectly linear trace (see Figure 2). Amy noticed Nathalie's activity, and quietly sidled up in front of her computer. Both were oblivious to the discussions in the room about who was pushing faster or harder than whom.

Nathalie: One reason we pulled out our computers was that it seemed quite clear that the chair's motion along the classroom floor was far from linear. In fact, if you think about it, mathematical linearity is quite difficult to experience given the 'imperfections' of the real world (such as friction, bumps, gravity, .). In *Sketchpad* though, these imperfections are minimised and I could push the car to produce a perfectly straight line. Whether tracing out the line in your sketch or in mine, motion and vision combine to give rise to the experience of linearity. Said differently, the visual and haptic (relating to the sense of touch) interaction with the computer provides a way for a human being to experience the constraint – and perhaps the privilege – of having to travel along a line. It opens the way for the body to 'feel' this phenomenon that can otherwise be approached perhaps only in travelling long stretches of straight highway – and then, of course, the visual trace of the path is not evident. In fact, linearity is just one example of a vast number of phenomena that the computer can allow the human body to experience. If we take seriously the theories of embodiment, these technological affordances aren't just about catering to students' varied learning styles. Rather, they are about closing the ontological gap between the body and mind and acknowledging the epistemological import of the body.

Amy: In addition to experiencing linearity by each of our (individual) interactions with our computers, we were also drawn into an interaction with each other. I wanted to see what you had done with your sketch, and you seemed interested in showing me.

Nathalie: Yes – I am reminded of Papert's (1980) observation that "although the work of the computer is usually private it increases the children's desire for interaction" (p. 180). Indeed, I think we have both witnessed this compulsion among students to show and show off their handiwork and ideas – the striking way in which commands and techniques can spread like wildfire throughout a classroom. Rather than arranging the environment to prevent us from

experiencing the world, Papert suggests that technology may well be able to arrange the environment so that we want to experience it together – so that we are urged or compelled or curious to communicate about our mathematical ideas, even to build relationships through mathematical interaction. Doesn't this seem relevant to your work around mathematical caring relations?

Amy: Yes. In my work, which I've adapted from Noddings's (1984/2003) care theory, I've tried to identify what makes care mathematical for me. What seems critical is attending to both the affective and cognitive responses of another person and tailoring mathematical interaction to these current states of the other, with an eye for what might allow the other to make progress in some way – for expansion of the other's mathematical world beyond what the other might envision, which could be primarily an affective 'advance,' or primarily a cognitive advance, or more likely a complicated mixture of the two. In approaching you that day, I was in some sense caring for myself mathematically – seeking out interaction with a person from whom I expected I could learn. This mathematical care for oneself can also translate into establishing mathematical care for others, in that as I seek to rethink and expand my own ideas, there is potential to open more opportunities for those I teach. In addition, a sort of mathematical care developed between you and I. It was different than in the example above in that neither you nor I were expressly charged with overseeing the other's mathematical progress. However, we each became interested in interpreting each other's mathematical ideas and computer work. Making interpretations involves shifts in one's thinking, even if they are quite subtle, allowing for the kind of advances that can be made when two colleagues come together to work on particular mathematical and/or educational ideas.

Nathalie: When Papert was writing about social interaction around the computer, in the quotation I mentioned above, he was interested in students asking questions and sharing ideas and accomplishments. But viewing computer use from the perspective of mathematical caring relations may provide a larger umbrella for 'caring' about whether or not students interact when using technology; your perspective encourages us to think beyond the new mathematical ideas that may emerge and forces us to consider our own connection to these ideas and to the others with whom we share them. There's a sense of intimacy and empathy involved that evolves along with the rigour of the mathematical activity.

Amy: This is interesting in the sense that students' empathetic and generally affective experiences in computer-based environments are often considered epiphenomenal to the strictly cognitive gains offered by the computer. How many studies end by informing the readers that the students also 'had fun'? I can think of some exceptions in which researchers have explored affective aspects of computer use, such as Kaput's (1989) discussion of the changing affective content of error in the context of technology. But, in Episode 1, there seems to be another aspect of the affective or aesthetic at play.

Nathalie: It's true. I didn't create my sketch because I wanted to understand linear functions or just have fun! Like many other people, I wanted to express my experiential

world using tools I knew well – and enjoy using. Again, looking back to early publications on the use of computer-based technology, Resnick (1991) describes well the way in which people will choose to solve "challenging problems" not by using the most appropriate tool, but by using their most cherished one. Some people chose *StarLogo*, others *Cabri, Boxer*, and still others paper and pencil. Their tools of preference affected the way they approached the problems so that some saw a geometry problem, others a probability problem, and still others an algebraic problem. We see the world through and with our tools. Like in craftsmanship, there's a sense in which using the tool itself is rewarding, in addition to using it to present and represent the world. Mathematicians talk about this tendency toward craftsmanship – the mastery and enjoyment of deploying certain tools (algorithms, transformations, principles) to create and manipulate objects – in relation to their more abstract tools (see Dyson, 1982; Sinclair, 2006). They may even decline working on a problem if they cannot use their tools of choice. The idea of developing a set of tools for working with mathematical objects isn't usually at the forefront of the instructional goals of teachers (or curriculum developers), which are often bound up with concept acquisition. I think that current narratives for technology adoption follow the same path – they focus on better ways of learning mathematical concepts. Perhaps a new narrative about technology defines a connection between the mathematician's sense of craftsmanship and the pleasure and power of digital tools for mathematical expression.

Amy: Of course this idea of tools changing the way we see and experience the world is an old one, and underlies some of the current research theories related to technology such as *instrumental genesis* – a theory that focuses on the integration of artefacts into the structure of human activities. I'd like to introduce another episode, this one involving students in a classroom, in which the influence of the tool had lasting consequences for a mathematics learner, and not just immediate pleasure or understanding. As you'll see, there are no computers in the classroom, yet the computer is almost palpably present.

Episode 2: Partitioning as an embodied tool
In working with a group of pre-service elementary teachers in the second mathematics course of a three-course sequence, Amy has just posed the following *Streetcar Problem*:

> A streetcar travels 6 miles in 32 minutes; how many miles does it travel in 1 minute?

All of the students, at Amy's request and prodding, draw pictures of both quantities: a rectangular bar or line partitioned into six equal parts for the 6 miles and another bar or line partitioned into 32 equal parts for the 32 miles. And most of the students note that they can simply work with 3 miles in 16 minutes, because these two quantities are in the same ratio as 6 miles in 32 minutes. Some students say, "just divide 3 by 16." When Amy asks why, they respond, "you just do that" (usually with a rueful smile).

At this point in working on this problem and others like it, Amy can always distinguish the students who have worked with her in the first mathematics course on building up fractional knowledge with *JavaBars* (Biddlecomb & Olive,

2000) from those who have not. Some of these '*JavaBars* students' start rather excitedly chopping the air with their hands. "Ah, Melissa is doing this," Amy notes, imitating her chopping motion so that all can see. "What does that mean to you, Melissa?" Smiling, she says you can partition each mile into 16 equal parts and then you can take 1/16 from each of the miles; that's the amount of miles that goes with one minute. Other *JavaBars* students nod, and some remark that it is just like sharing 3 candy bars fairly among 16 people.

Tracy, Frederic, and Kyung, who are new this term, appear to find this discussion difficult to interpret. A *JavaBars* student, Claudia, and another new student, Lauren, agree to help the class elaborate Melissa's explanations. The class uses their drawings first to establish why Melissa is taking 1/16 of all 3 miles – because 1/16 of 16 minutes is 1 minute, and that's the amount of time for which they would like to know the corresponding distance. Thus they need to find 1/16 of 3 miles. However, Tracy continues to look blank and slightly defiant; Frederic is frowning; and Kyung has not made any further marks on her paper.

Amy then asks for elaboration of the candy bar comments. Lauren says that you have to think about what it means to take 1/16 of 3 miles: You can find 1/16 of 3 miles by taking 1/16 from each of the three miles. This is 1/16 of a mile three times, which is 3/16 of 1 mile. In the same way, 1/16 of each of three identical candy bars will yield exactly 3/16 of 1 candy bar. Although several students nod, others, Tracy, Frederic, and Kyung among them, seem to find this sophisticated distributive reasoning rather mystifying, and Amy is left wondering what is required to engender ideas of distribution for them.

Nathalie: What is immediately striking about this episode is the presence and importance of Melissa's "chopping" hand gesture.

Amy: Right. The gesture Melissa makes seems to be representative of the partitioning activity she experienced in the microworld, and she can call it forth in situations she constitutes as involving partitioning, even when not using the microworld. The gesture is personal and private – it evokes her previous experience – yet it's also a public display for others. Her spontaneous creation and use of the chopping gesture seems aimed at communicating to others (at first perhaps just me, the teacher, then the class) something about how her mind has organized partitioning as a potent tool in this situation.

Nathalie: I am also struck by Melissa's use of partitioning in the *Streetcar Problem*. In explaining her solution, she uses partitioning in order to reason about the relationship between the two quantities, as well as the result of partitioning the distance into 16 equal parts.

Amy: Yes. The latter use of partitioning, which involves distributive reasoning, seems particularly influenced by experiences with *JavaBars* versus experiences solely with paper and pencil. In a paper-and-pencil world, the constraint of partitioning each unit of a 3-part bar in order to complete a partitioning goal, such as partitioning the entire bar into 16 equal parts, is not present. To solve the *Streetcar Problem* in this world, a student might take a pencil and mark a 3-part bar into approximately 16 equal parts, assessing where to make marks along the bar, or perhaps repeatedly halving. Although this activity may provide a reasonable

drawing for this situation, it has certain limitations from the point of view of developing robust concepts of division First, in marking the bar by hand as described, students do not generate the size of those pieces based on their partitioning activity. They may draw a fairly accurate solution, but there is no particular reason to know that the bar they drew measures 3/16 of a mile (other than that they may already believe that "you just divide.") In contrast, the default mode of *JavaBars* allows partitioning to occur only in each unit of multi-unit bars, not in the whole bar. Second, in marking the bar by hand, students will not necessarily be establishing a general way of operating so as to be able to take any unit fractional amount of a quantity made up of any number of units In this particular example, repeated halving is a fine idea. But if the streetcar went 3 miles in 17 minutes, halving would not be so handy. So, in *JavaBars*, constraints in how partitions can be made (*e.g.*, in each unit of a multi-unit bar) seem to be instrumental in building up images and concepts of division

Nathalie: In fact, in using *JavaBars*, students can experience partitioning in a way that is not necessarily easily experienced in "regular" life – just as we discussed in Episode 1 with respect to experiencing linearity. The constraint in partitioning activity in the microworld is more stringent than what students experience with pencil and paper, and yet it seems to help at least some students generate ways of thinking that are productive outside the classroom too, such as determining an average rate – say a cost per ounce or a number of kilometres per litre – based on reasoning, using partitioning to multiplicatively coordinate the two quantities involved So, in a way, technology in the form of *JavaBars* has been used to "arrange the world so we don't have to experience it," as Frisch put it, by creating a microworld that is stricter in at least one sense to the free-form partitioning of a paper-and-pencil world. But by experiencing this stricter world, there is potential to actually enrich our experience of the non-microworld world – to see and act in ways that help us think about this world

Amy: Yes, and then there's the idea that Melissa's activity with the computer actually taught her body to move - to create that hand gesture. It's worth questioning whether Melissa would have constructed partitioning as an embodied tool –*i.e.*, symbolized it with her hand gesture – without her *JavaBars* experiences This point is related to how technology may be able to help humans organize their mental and physical experiences in ways that will promote making certain abstractions, such as being able to take any unit fractional amount of any multi-unit quantity. Whether this is better or not is debatable (*cf.* Turkle and Papert, 1992), although it can be argued that students like Melissa have developed considerable power in their thinking That does not mean that other ('non-*JavaBars*') students can't make these abstractions without the technology Nonetheless, I see providing students with opportunities to organize their thinking in ways that promote making abstractions as an example of caring mathematically for students. In general terms, establishing and maintaining mathematical care necessarily involves a teacher in making second-order models of students' mathematical ways of thinking (*cf.* Steffe and Thompson, 2000), as well as constructing students as affective beings

Nathalie: And there's certainly some telling affective fluctuations at play in Episode 2. Melissa and some of the other *JavaBars* students seem to be awake or enlivened (*i.e.*, they appear to be experiencing some level of stimulation). This affective state is married with their cognitive activity in the sense that using their ideas about partitioning in considering the *Streetcar Problem* induces a sense of fit, of making sense, and perhaps even of power – and here I'm reminded of the power of a good tool, and the mathematician Lewis's explanation of the pleasure of using "well-worn tools in often routine ways, like a well-oiled piece of engineering" (in Sinclair, 2006, p 96). In a similar way, the perfect partitioning experienced in *JavaBars* offers Melissa a tool, satisfying to use, to make a ratio of two quantities. On the other hand, Tracy, Frederic, and Kyung appear to be experiencing some level of depletion, a decrease in energy or aliveness This affective state is also conjoined with their uncertainty about what to do, and perhaps their sense of not having a way to proceed, compounded by not understanding what their classmates find so pleasing or sensible.

Amy: As I have argued elsewhere, stimulation is not "good," and depletion is not "bad" – in fact, both are necessary and useful But prolonged depletion may impede mathematical activity, learning, and the development of mathematical self-concepts, in both the short and long term. So a question the teacher-as-mathematical carer naturally asks, then, is: What kind of mathematical activity might help Tracy, Frederic, and Kyung to alleviate some of the depletion they are experiencing? In fact, this question can become a source of significant depletion, as well as stimulation, for the teacher, in that it can trigger sticky, consciously-conflictive perturbations It can lead to mathematical activity quite different from what was originally planned, and the teacher has to create this new activity, these new problems So important sub-questions are: Can the students who are experiencing depletion and cognitive difficulty construct right now the kind of thinking that I can attribute to Melissa and others? If so, what kind of learning trajectories (which includes problem situations and activities) are involved? Or, if that kind of thinking has to be deferred for the moment, what sort of mathematical thinking can these students work toward right now that might be an advance for them? Answering these questions often requires the teacher herself to construct new mathematical ways of thinking Her current ways of thinking may be insufficient to "see" the students.

Nathalie: And is technology, or can it be, a tool for that? That is, I am left wondering about how a teacher's sensitivity to cycles of stimulation and depletion, as well as to students' current mathematical ways of thinking, are related to technology use. Noss and Hoyles (1996) point to the ways in which expressive technologies can provide teachers and researchers with a "window" on student mathematical meaning-making. They root their notion specifically within expressive technologies in the sense that it is through student action and in interaction with the technology that the teacher-researcher can understand aspects of how the student is thinking.

Amy: I think this is very relevant, but in making second-order models of students' thinking and constructing students' affective states, the idea is to set one's own mathematical ways of thinking – and affective states – to the side,

in order to make space for formulating ways of thinking and feeling of the students. That doesn't mean that as a teacher you forget your own ways of thinking or your own affective responses to situations – indeed, that's not possible. But it does mean that you practice both cognitive and emotional decentering in order to, as Noddings says, enter students' worlds as if they were your own. Steffe talks about it as trying to be the student – to develop a way of thinking that is built up as a coherent system, so that if you follow that way of thinking, you would produce mathematical activity that you see the student produce. Of course, we engage in emotional decentering when we try, as the expression goes, to stand in someone else's shoes. I think cognitive decentering is actually harder, but quite necessary for establishing and maintaining mathematical care.

Nathalie: Ah, so perhaps our research should start paying more attention to providing teachers with tools they can use to create second-order models, rather than more tools for students to use. Or at least, research in technology might focus more explicitly on how teachers can appropriate technological tools so that they become an aid to practicing cognitive and emotional decentering. this use of the technology is likely related to, but still different from, how students use the technology.

Amy: Yes, *JavaBars* (and other *Tools of Interactive Mathematical Activity*, or TIMA, tools) have been designed so that students can make manifest their operative activity (*e.g.*, partitioning, iterating, .., *cf* Biddlecomb, 1994). That can be useful for students, but it can also be useful for teachers: using these microworlds can facilitate building second-order models of students' thinking, including the formulation of what problems and activities fall at the edges of what Steffe (1991) calls students' zones of potential construction. But you're right. That use requires the teacher to develop ways of thinking that go beyond their own first-order knowledge, to see and use the tools in the service of cognitive decentering.

Nathalie: It seems like this discussion is related more generally to issues of students' attention and focus in the use of technology. Many researchers have reported on the way in which students' interactions with expressive technologies enable and encourage them to go off in new mathematical directions, often ones different from what is explicitly directed by the teacher or the written curriculum. These expressive technologies offer a set of generative tools. If these tools prompt students to explore their experiential mathematical worlds, teachers who support this exploration honour the 'student part' of establishing a mathematical caring relation. Of course, you and I both know that sometimes students become interested in what may be construed – in the context of the desired activity – as non-mathematical features of a microworld: colouring all the objects just so, animating everything just to see. This kind of student activity often leads to two types of decisions: either the teacher closes down those avenues of play by constraining students' access to tools, or the teacher tries to figure out how to harness the attraction of colour and movement into more mathematically-relevant actions. Might we say that the latter decision involves entering into students' worlds and establishing mathematical care?

Amy: That's certainly another way of thinking that might provide teachers with new ways of responding to those dif-

ficult situations in which short term goals conflict with long term efforts to develop mathematical caring relations with their students. With this in mind, it sounds like we've hypothesised that technology may play a dual role in the development of mathematical caring relations. That is, it may help a teacher establish mathematical care via formulating second-order models of students' thinking and affective states, *and* help a student enter into a mathematically caring relation with a teacher, since a critical aspect of doing so is at least some willingness to engage in some mathematical activity and some level of trust in the teacher. I wonder, in addition, how working to establish mathematical caring relations with students may influence a teacher's use of technology? And in terms of the social interaction that Papert talked about, do students develop stronger caring relations amongst themselves when working in a technological environment, and if so, how does that function in relation to the teacher's mathematical care?

Nathalie: And in terms of our conversation around embodiment, I'm curious whether technology-based experiences affect the nature and number of gestures students create and use to communicate. And, thinking ahead, what opportunities might there be for expanding the range of embodied experiences with technology? We currently privilege the visual (and the dynamic) in our learning technologies, but perhaps our other sensory modalities, sounds in particular, have a role to play in these technologies and will also help us experience our mathematical worlds.

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