From her research on students with mathematics learning difficulties (Karagiannakis *et al.,* in press), and in particular when engaging in interventions with low achievers, Anna is learning to combine neuroscientific findings with the outcomes of the intercultural semiotic analysis discussed in our research group, to smooth the scarce transparency of the Italian wording.

This very short episode from a study in progress shows the synergy between intercultural dialogue, neuroscience and technology for defining effective teaching-learning situations. We hope that this synergy will be further and more deeply developed in the future, and applied in mathematics teacher education and development.

Notes

[1] motionmathgames.com
[2] www.youtube.com/watch?v=hmm0D90vcYI

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Zed: the structural link between mathematics and mathematics education

BHARATH SRIRAMAN

Zoltan Dienes (1916-2014), known as Zed, passed away on 11 January 2014. Some of us see it as a culmination of an era. Mathematics education is prone to forgetting its origins within the realm of mathematics, and Zed's passing away serves as a reminder to those who have witnessed the weakening of these origins. My intersection and interest with Zed's work began in the mid-1990s when there was excessive focus on the social turn in mathematics education. Being trained in mathematics, it was difficult to stomach at that point in time the associated set of sociological problems that were being addressed by mathematics education. I initiated a correspondence with Dienes which led to my discovery of *Building Up Mathematics* (Dienes, 1960) and *Thinking in Structures* (Dienes & Jeeves, 1965). Both these books have been influential to a generation of mathematics educators who entered the field in the 1970s and they remain classics to this day.

Trained as a mathematician in England, Zoltan became interested in the psychology of learning in the 1950s and earned a second degree in psychology. The field of mathematics education, seen through its origins in mathematics, is often outlined in terms of the classical tradition of Felix Klein followed by Freudenthal's re-conception with an emphasis on the humanistic element of doing mathematics. While the approach of Klein, steeped in an essentialist philosophy, gave way to the pragmatic approach of Freudenthal, Zoltan's approach influenced by structuralism and cognitive psychology remains unique from the point of view of developing a theory of learning which has left a lasting impact on the field. Most importantly this theory was grounded in fieldwork with school children that experienced the multi-embodiment approach to a mathematical idea through manipulatives, games, stories and even dance, before they were encouraged to abstract the essence of the activity leading to mathematical generalizations. The six-stages of learning consisted of free play, games, commonalities, representation, symbolization and finally formalization. I have always considered his approach to mathematical learning (and teaching) as falling within psychological structuralism à la Wilhelm Wundt because of its nuanced and layered approach to encouraging abstractions, with formalizations only occurring at the very end. This, to me, was similar to the focus on introspection as the method used by structuralists to understand conscious experience.

Zed's use of his theory of learning was powerful (to put it mildly). He had grown up surrounded by mathematicians. His father was a mathematician by training and gave Zed a book he authored on Taylor series for his 16th birthday. Zed's PhD thesis generalized one of Baire's category theorems by using Brower's intuitionist approach. In other words Zed believed in constructive mathematics in which *reductio ad absurdum* was viewed as a logical trick and frowned upon. When I met him in 2006, he was pushing 91, with a mind keen and fertile to talk mathematics (Sriraman & Lesh, 2007). I complained about being unable to find multiple embodiments to facilitate the learning of ideas in analysis. Two months later he sent me a paper on "A child's path to the Bolzano-Weierstrass theorem" (Sriraman, 2008), which essentially contained a structured story which allowed one to discover this deep theorem!

Zed embodied the common ground between mathematics and mathematics education, in a life that was dedicated to exploring the beauty of mathematics by making it accessible to schoolchildren. Given the climate of the "math wars" in the US and similar debates elsewhere in the world, it seems ironic that his seminal work on *Building Up Mathematics* remains forgotten. This book would appeal to both mathematicians and mathematics educators because of its focus on the foundational structures of mathematics.

Zed lived an adventurous life that included fieldwork spanning over 50 years with school children in the UK, Italy, Australia, Brazil, Canada, Papua New Guinea and the United States. His body of work will remain an inspiration for generations of mathematics educators who place mathematics at the center of mathematics education.

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My students deserve better

PETER TAYLOR

In my third-year course, *Mathematical Explorations,* designed for future high school mathematics teachers, I had my students submit journals this year. Ranging from 4 to 40 pages, they discussed the problems we had worked on and reflected on their own learning. I see now that some of the real struggles they (and therefore I) seemed to be having, especially during the first half of the course, worked out pretty well for most of them. I also see that there are some things that might be changed. For example at the early stages it was hard to get much participation; I have to rethink my expectations.

My experience with this class has given me new insights into my two large first-year courses: calculus and linear algebra. I've been thinking about those courses over the past few years, trying different kinds of problems, different ways of interacting with the class, and though things seem to be working pretty well, I've always felt that there was something fundamentally amiss. My main purpose here is to think about ways in which those courses could be more like my third-year course.

In my third-year course, the problems we work with involve mathematics that most of the students have seen before but they are challenging in the sense that one has to play quite a bit in order to begin to see what sort of strategies might work. They are chosen for their power to deepen the students' understanding of the ideas and to lead them to a new appreciation of mathematical structure. According to the students, the main difference between this course and others they have taken lies in its pace (slower) and thrust (deeper and wider). (Aren't deeper and wider opposites? Not really—lateral connections reveal new structural properties.) The objective is as much to give the students a chance to confront and develop their learning skills as to deepen their mathematical understanding:

> As I reflect on my learning in math throughout my university career and in this course, I find that [...] I haven't "done" or "learned" math since high school; I have memorized and regurgitated the knowledge of my professors in hopes of getting good grades and finishing courses. The knowledge that I retained from all of this felt minimal, and it probably was, but this class helped me to do and learn math for real again. I realized that I did learn in my first three years in university but I didn't know how to apply my knowledge. Math became a daunting, scary mountain that I couldn't climb because I had forgotten how to apply what I know and really do math. But MATH 382 reminded me that math can be fun, and reminded me how to really DO math. (Kirsten)

What I discover from the students' journals is that this experience of digging deeply, of taking things apart to see how they work and then putting them back together, of constructing simple concrete examples as a way of playing with ideas, was new to most of the students. Remarkably enough, after 14 years of formal learning, they have spent almost no time in play.

That's not quite right. A number of our students, perhaps a quarter, have certainly spent a lot of time in their lives in mathematical play. When kids are young, they bend the rules and twist things into the wrong shape just to see what happens—that's their job as kids. But later on this natural behaviour seems to get schooled out of many of them, and they increasingly adopt safe strategies which seem to offer short-term gain. Only a few resist these temptations and keep right on playing. Who knows what makes the difference? Perhaps some early success, a key learning experience, an unusual teacher, or just a natural appetite for risk-taking. In any event such students do well in mathematics partly because they develop powerful learning strategies, but also simply because they've put in the time because they find playing with mathematics more fun than texting. I believe that our current undergraduate program serves these students very well.

It's the remaining, say, 75% of our students that I am interested in here. I have no doubt that these students have the capacity for serious play, but somehow, in their early years, they abandoned it, and it's hard for them to get started again. I know that there is considerable work being done on the question of how to get more students to keep on with that mathematical play. The question I am asking here, however, is: given the students I have now, what should I be doing in my large first-year classes?

> I had to think about it right then and there in the lecture when usually I'm just trying to keep up with the professor's handwriting, hardly listening to what they are saying. (Ashna)

The answer seems clear enough to me. I need to *teach less and discover more.* Rather than deliver the *product of mathematical thought,* engage them in the *process of mathematical thinking* (quoted from a paper by Asia Matthews). I don't mean to disparage "the product of mathematical thought" (more simply described as "mathematics"). It's real knowledge, particularly in a world in which much of what passes as knowledge is suspect. It's solid and eternal and has beauty and structure to die for. Nothing else in the knowledge world comes close to touching it. But my primary job as teacher is *not* to convey knowledge (narrowly interpreted), but to interpret, to transform, to enable, to bring to life.