THE EMERGENCE OF A 'BETTER' IDEA: PRESERVICE TEACHERS' GROWING UNDERSTANDING OF MATHEMATICS FOR TEACHING

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For the last few years we have been exploring the power of improvisational theory as a theoretical tool to talk about the growth of collective mathematical understanding and the ways in which shared understandings can emerge and exist in the discourse of a group working together (Martin & Towers, 2009; Martin, Towers, & Pirie, 2006). In doing this we employed and extended the theoretical work of Becker (2000), Sawyer (2000, 2001, 2003, 2004), and Berliner (1994, 1997) and suggested that by using the lens of improvisational theory it was possible to observe and account for acts of mathematical understanding that could not simply be located in the minds or actions of any one individual, but instead emerged from the *interplay* of the ideas of individuals, as these became collectively woven together in shared action, as in an improvisational performance.

In this article, we shift our focus somewhat and explore the growth of understanding of a group of preservice teachers, and their instructor, as they work together on what we describe as a 'mathematics for teaching' task – specifically, the collective development of a question that would assess students' knowledge of decimals. We analyse how the preservice teachers' understanding of the mathematical concept as a teachable idea emerges as they work together. We do this through drawing on elements of improvisational theory, in particular the notion of the 'better idea,' to detail the way that individual contributions mesh together and are collectively built on by the group. We build on this analysis to suggest that particular kinds of improvisational actions can be a powerful means through which student teachers might think about and learn mathematics for teaching.

Improvisational actions and the 'better' idea

Improvisation is broadly defined as a process "of spontaneous action, interaction and communication" (Gordon Calvert, 2001, p. 87). Ruhleder and Stoltzfus (2000), in talking of the improvisational process, draw attention to "people's ability to integrate multiple, spontaneously unfolding contributions into a coherent whole" suggesting that many of our everyday actions and interactions are improvisational in nature (p. 186). Sawyer (2003) talks of improvisational activity as being conceived of "as a jointly accomplished co-actional process," and for us the use of the term *coaction* rather than interaction emphasises, in a powerful manner, the notion of acting with the ideas and actions of others in a mutual, joint way. Improvisational coaction is a process through which mathematical ideas and actions, initially stemming from an individual learner, become taken up, built upon, developed, reworked, and elaborated by others, and thus emerge as shared understandings for and across the group, rather than remaining located within any one individual. But how do groups achieve this sophisticated form of interaction? In studying this process in the domain of improvisational jazz, Becker (2000) has highlighted the requirement that everyone pay attention to the other players and be willing to alter what they are doing

> in response to tiny cues that suggest a new direction that might be interesting to take…. As people listen closely to one another, some of those suggestions begin to converge and others, less congruent with the developing direction, fall by the wayside. (p. 172)

Such *listening to the group mind* is described by Sawyer (2001, p. 18) as "The Third Rule of Improv" and surfaces the vital role of listening in improvisation. Listening to the group mind places a responsibility on those who are positioned to respond to an offered action or innovative idea, as much as on the originator, and it is this "social process of evaluation" (Sawyer, 2003, p. 92) wherein the group collectively determines whether, and how, the idea will be accepted into the emerging performance, that we suggest is the key to the emergence of collective understanding of mathematics for teaching demonstrated in the data we offer later.

When an image is challenged and an innovation offered, a coacting group must collectively determine whether the innovation is to be accepted into the emerging performance. They achieve this by listening to the group mind – being willing to alter their emerging action by being responsive to tiny cues from the other players. One implication of listening to the group mind is that when one person does something that is obviously 'better' (in the view of the group) then "everyone else drops their own ideas and immediately joins in working on that better idea" (Becker, 2000, p. 175). Interestingly, Becker also suggests that in collaborative improvisations, as people follow and build on the

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leads of others, they "may also collectively change their notion of what is good as the work progresses" (p. 175) leading to a creative production or performance that could not have been predicted prior to the activity. Of course this requires some understanding of what "better" might look like and of how to recognise it, and in this article we develop this construct in the specific context of engaging with a mathematics for teaching problem.

The growth of understanding of mathematics for teaching

Following on from Shulman's (1986) influential description of the concept of *pedagogical content knowledge* and prompted by the work of Ball and Bass (2000, 2003), there has been a recent upsurge of interest within the mathematics education community in the notion of the specific kinds of mathematical knowledge needed to be an effective teacher, and how this might be identified and developed. Mathematics for teaching describes this "distinctive form of mathematical knowledge, produced in, and used for, the practice of teaching" (Adler & Davis, 2006, p. 272). Research in the field has focused on the differences between how teachers need to hold and use their mathematics and how mathematicians do so (Ball & Bass, 2000); on the nature of the mathematics and pedagogy tasks, courses, and programmes in which preservice teachers are supposed to learn mathematics for teaching (Adler & Davis, 2006); the continued theoretical conceptualisation of mathematics for teaching (Davis & Simmt, 2006; Hill, Ball, & Schilling, 2008; Silverman & Thompson, 2008), and how it might be meaningfully assessed and measured (Chamberlin, Farmer, & Novak, 2008; Hill, Ball, & Schilling, 2008).

However, as Hill, Ball, and Schilling (2008) note,

> this domain remains underconceptualized and understudied. Although most scholars, teachers, and teacher educators would agree that teachers' knowledge of students' thinking in particular domains is likely to matter, what constitutes such 'knowledge' has yet to be understood [… and that the research base is] far from complete. (p. 395)

Similarly, Silverman and Thompson (2008) suggest "while mathematical knowledge for teaching has started to gain attention … there is limited understanding of what it is, how one might recognize it, and how it might develop in the minds of teachers" (p. 499). There is a recognised need for "theory that would help us examine the nature of teachers' mathematical-pedagogical reasoning about students" (Hill *et al.,* 2008, p. 396).

In this article we contribute to this emerging body of work, and offer a theoretical tool for considering one mechanism (the 'better' idea) through which such reasoning might emerge and be developed in the context of pre-service teacher education. In particular we are interested in the process through which student teachers engage in "unpacking or decompression of mathematical ideas" (Adler & Davis, 2006, p. 271) to enable the growth of understanding of mathematics for teaching, and of how to characterise and account for this. Cavey and Berenson (2005) note that

> opportunities for image sharing about what and how to teach are critical to prospective teachers' growth in teaching mathematics understanding … it seems critical to provide beginning prospective teachers with multiple opportunities to share their images of school mathematics and teaching strategies. There is evidence that such image saying directly leads to self-awareness of understanding and subsequently initiates growth in understanding for the learner. (p. 188)

Our work recognises the importance of such image sharing and self-awareness, and complements and extends this body of research by proposing to view the necessarily complex nature of growth of understanding of mathematics for teaching as an improvisational process. In particular, we offer one example of how collective image sharing, and the emergence of a 'better' idea, can occasion the growth of collective understanding of mathematics for teaching.

The emergence of the better idea in one teacher education classroom

In the description of the data extracts that follow, we show how a better idea develops among a class of preservice teachers and their instructor in the context of developing a question that would assess students' knowledge of decimals (the mathematics for teaching task). The data reveal how this is a collective phenomenon – in the extracts that follow no one student announces that he or she has a better idea and that everyone should drop the previous image and adopt the new one, and yet the group does recognise the cues, drop or modify the current image, and coact to create a new, better collective image.

The study on which we draw focuses on the ways in which preservice teachers in an inquiry-based teacher education programme learn to teach mathematics, and on their pedagogical practices during their first year of teaching. The participants were elementary-, early-childhood- and secondary-route preservice teachers enrolled (all routes together) in an optional course in the final semester of their two-year, after-degree teacher preparation programme that focused on the teaching and learning of K–12 mathematics, taught by Towers. The research participants (and instructor/Towers) were videotaped in class sessions throughout the semester, and preservice-teacher-participants were interviewed individually about their preparation for teaching and later followed into their first year of teaching where further data were collected. Here we focus only on an episode from their learning of mathematics for teaching in the university course, with the purpose of illustrating how a better idea emerges as they work together. In this episode the first author used as a prompt an adaptation of a task proposed by Ball and Bass (2003) (see fig. 1).

Students were invited think about the lists and then volunteer a rationale for why they had chosen A, B, C or D as the correct response. After some deliberation and discussion in which the students revealed collective dissatisfaction with the offered lists, and again borrowing from Ball and Bass (2003), the instructor proposed that the group try to create their own list of four numbers that would assess student understanding of ordering decimals. The students

Which of these lists would you choose to help you understand how children understand ordering decimals?

a)	0.5	7	0.01	11.4
b)	0.60	2.53	3.14	0.45
c)	0.6	4.25	0.565	2.5

d) It doesn't matter, they all require students to read and interpret decimals.

Figure 1. A prompt

worked for several minutes in groups or individually, trying to create a list of four numbers. One student wrote his list of four numbers on the board: 4.5, 440, 0.45, 0.045. We join the episode as the instructor begins to solicit the students' responses, and we begin to see the emergence of what the class considers a 'better' set of decimals for the purpose of assessing student understanding of ordering decimals:

1 Instructor: Alright. We have our first suggestion. I'm not going to go around to ask all of you, but here are some things to be thinking about. Four numbers…. What do you think?

2 Noah: I would disagree…. Who is backing me up?

3 Heather: No, I was going to say I especially like the zero point zero four five and zero point four five, comparing those two.

4 Instructor: Alright, good. Noah, what don't you like about those?

5 Noah: My argument against has nothing to do with place value or decimals. In fact, they're similar numbers. And I would see that as a bit of a game, a bit of a trick. It would confuse some students….That's requiring that a different level of thinking.

[Everyone talking at once. Then Anne's voice breaks through:]

6 Anne: That's, that's good because then they won't rank twenty-three, forty-seven, sixty-nine and seventy-five no matter where your decimal point is. *[Note that earlier the group had discussed the possibility of a student ignoring the decimal points completely, ordering the numbers, and getting the correct answer, as is possible with options A and B on Figure 1]*

Liz now introduces an innovation – the notion of increasing the number of decimal places under consideration:

7 Liz: My concern is that it's only getting them to think to the thousandths place and it's not getting them to think into the ten thousandth, hundred thousandth – way down.

This prompts another student, Liam, to offer a new list of four numbers ($0.4\dot{3}$, 0.43, $0.4\dot{3}$, 0.434) that initially seems to surprise and shock the group, but rapidly becomes seized upon as better and settles into becoming the list that the class sees as optimal for the purpose of assessing student understanding.

8 Instructor: Okay, so this … certainly it falls within what junior high and high school students need to know. What's your response?

9 Liz: That one's a tough one.

10 Deepak: I like this one. I like this … I like this one because you actually have to think about what is actually bigger and smaller in a sense. 'Cause I'm not really too focused on the hundredths and all this necessarily. I'm thinking about big and small.

11 Nicole: But big and small doesn't always work for decimals.

12 Deepak: Well, yeah, that's the limited knowledge I've based that upon but right now that's what I'm thinking about…. Yeah.

13 Liz: It's good because you always have the forty-three, forty-three, forty-three and then the four hundred and thirty four so if they do get confused then they would put that one first instead of the point four three repeating.

14 Instructor: Okay. You think they might confuse those two?

15 Liz: So you're going to be able to catch the students that, that just put them in order.

The students begin working on ordering the decimals and there are no further suggestions of four numbers from the group. They seem satisfied that Liam's is an optimal list.

Learning mathematics for teaching

As Davis and Simmt (2006) argue, "the mathematics teachers need to know is qualitatively different than the mathematics their students are expected to master" (pp. 315–316). In addition to knowledge of pertinent mathematics concepts, mathematics for teaching includes a capacity to "unpack" or "decompress" (Adler & Davis, 2006) those concepts. Hence we see that what constitutes a 'better' list of decimals in the above context is intimately bound up not only with issues of the preservice teachers' own knowledge of decimals, but also with knowledge about how students learn, the kinds of misconceptions they hold, and how to surface such misconceptions. For example, at various moments the preservice teachers show that they understand the kinds of misconceptions that students may carry about decimals (*e.g.,* see Anne's contribution, line 6) and what students may find difficult (see, *e.g.,* Liz, line 9). Similarly, Deepak (line 10) and Nicole (line 11) show their own understandings of ways to think about ordering decimals and what might be the limitations of those ways. Liz (lines 13 & 15) shows that, despite earlier acknowledging the challenge that Liam's four numbers might offer to students (line 9), she understands what is powerful pedagogically about the suggestion.

Mathematics for teaching also includes an understanding of classroom processes that might be most profitable in fostering student understanding. For example, Heather (line 3) understands that the mathematics at play in the classroom is not an individual construction and that it need not (only) be the student who proffers the initial idea who can take responsibility for the idea and its cultivation. Noah has learned to be unafraid to openly challenge a peer's contribution (line 2) and the form of his challenge indicates that he has learned that challenges to a mathematical idea gain 'currency' in the community of the classroom when they are supported by others. We can infer that he might therefore be disposed to encourage such challenges in his own classroom. [2] Learners are often swayed by the teacher's perspective and will privilege it over the perspectives of their peers yet here students are showing that they have learned not to succumb to a mode of thinking that says, "The teacher (line 4) is ignoring Heather (line 3) and going back to Noah's idea (line 2) so Noah must have the better idea." Instead, and despite Noah's rationale (line 4) which the teacher deliberately sought out, the class persists with the idea of similar numbers. Through their actions the preservice teachers are demonstrating that they have learned to trust their own capacity, as a group, to generate understanding and this will be important in helping them structure similar learning experiences for their own students.

The contribution of improvisational theory

The theoretical notions of improvisational coactions and the pursuit of the better idea offer ways of characterising how talk might actually be "picked up" and acted on in ways that are likely to lead to powerful and perhaps unanticipated ways of thinking mathematically and pedagogically and therefore to the continued growth of collective understanding of mathematics for teaching. Improvisational theory suggests that participants must be willing to alter what they are doing in response to cues that suggest a new direction that might be interesting to take and that as people listen to one another, some of those suggestions begin to converge and others fall by the wayside (Becker, 2000). As the better idea emerges, the capability to take up the contributions of others, and, in improvisational terms, to be able to respond with "Yes, and" (Sawyer, 2001) – that is, to "accept the material introduced in the prior line, and add something new to the emerging drama" (p. 16) – is essential. In the above episode we see students picking up on cues suggested by others. For example, Heather (line 3) sidesteps Noah's challenge to the proffered list of numbers (line 2) and instead offers a "yes, and" support for two items in the list – 0.45 and 0.045, numbers that ultimately become the core type of 'similar' decimals included in the class' optimal list. Noah persists with his challenge (line 5) saying that including such similar numbers amounts to trickery, but his challenge falls by the wayside as the group mind continues to swell around the idea of 'similar' numbers, evidenced by the rest of the transcript. Liz (line 7) offers an innovation which, while it might at first seem to be a distraction or a completely new direction, is also taken by the group as a "yes, and" contribution – one which accepts the previous theme but adds something new to it. Indeed, the group itself responds with "yes, and" to Liz's contribution by accepting the idea of including more decimal places and building on it (see Liam's contribution of decimals and the ensuing responses, lines 9–15). In Becker's (2000) terms, "the players thus develop a collective direction that characteristically … feels larger than any of them, as though it had a life of its own" (p. 172).

This collective direction emerged because the group were able to listen to the group mind; when one person did something that was obviously better (in the view of the group) then "everyone else drop[ped] their own ideas and immediately join[ed] in working on that better idea" (Becker, 2000, p. 175). We note that while the teacher educator cannot mandate that groups listen to the group mind, they can foster the kind of environment in the classroom that encourages students to recognise the importance of attending carefully to emerging ideas and of trying to build on those seen to be "better" mathematically and pedagogically. Such classroom discussion would then be consistent with the kind of talk that Pirie and Schwarzenberger (1988) noted as "purposeful," wherein the goals of the interaction, whether set by the teacher or the group, are, implicitly or explicitly, accepted by the group, and where there are indications that "movement within the talk has been picked up by other participants" (p. 461).

We note that the instructor's interventions in the emerging growth of understanding about mathematics for teaching in the episode above are few. Both the relative infrequency and the nature of her contributions are, in fact, significant. The instructor had been deliberate in the choice of task and in the way in which she encouraged students to work together. This classroom example, then, shows us how a teacher educator is enabled to be part of the collective improvisational growth of understanding of mathematics for teaching without being what the improvisational literature refers to as a "driver" (a dominant role wherein one member of an improvisational group dominates the emerging direction), the presence of which typically flattens and ultimately halts an improvisational performance. Such teaching is delicate work, but improvisation theory offers a set of constructs that show how all members of a collective can (indeed, must) work together to create understanding. In the context of jazz improvisation, Sawyer (2001) suggests that novices listening to jazz improvisation often focus on the soloist, assuming he or she is the one 'leading' the group. In a classroom, the teacher might traditionally be considered the 'soloist', but a reading of the mathematics for teaching classroom as an improvisational space calls forth from us a more sophisticated interpretation of the role of the teacher, one in which the teacher is re-visioned as a full participant in the emerging cognitive structure of the learning unit (in this case, the group). Full participant, in this sense, does not mean dominant, nor even most important, participant, but one who is fully participating in generating understanding for and in the group.

Concluding comments

While we see this analysis as a significant contribution, such micro-level, classroom-based teacher education practices are, necessarily, embedded in broader systems of education. We suspect that it is not incidental that the students featured here, who were able to work collaboratively in a mode we have characterised as coacting, were educated within a teacher education programme that seeks to educate teachers to value and interrogate the contributions of their peers, to read their own experiences as a text for learning, and to learn actively and take responsibility for their own growth. Towers' ongoing research in this setting is considering the nature of the teacher education practices that foster the kind of classroom coaction we have described here and that help prepare teachers to enact and sustain sound, mathematically rich, inquiry-based teaching practices in K–12 classroom settings (see, *e.g.,* Towers, 2007, 2008, in press).

Hence, while our data serve as an example of the unpacking of mathematical knowing that is necessary in order to transform such knowing into a teacher's useable knowledge, and as an elaboration of a structure that allows us to "see" inside the practices of mathematics and pedagogy as these practices coalesce into the notion of mathematics for teaching (Adler & Davis, 2006), our analysis, drawing on improvisational theory, also serves to show how collective classroom processes can reformulate unpacked mathematics knowledge into knowledge of mathematics for teaching. Our study suggests, then, that further research employing the improvisational frame would enable us to gain a more nuanced understanding of the processes at play when groups interact to create knowledge of mathematics for teaching.

Notes

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[2] In fact, data collected in Noah's classroom during his first year of teaching demonstrates that his practice is consistent with an inquiry approach that emphasizes discussion, argumentation, and collaboration. Similar findings are emerging from other participants' classrooms, too (Towers, 2008, in press).

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