# Communications

## Ideas reinforced – ideas seen in a new way – ideas to be used: reflections on ICME-10

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ICME-10 was pleasantly surprising in the depth of thought and reflection it evoked and provoked for me. The reflections were generated principally during four lectures [1], which, based on their titles, would not seem related in any obvious way. The sessions were:

- John Mason's *Doing ≠ construing and doing + discussing ≠ learning: the importance of the structure of attention*
- Ubi D'Ambrosio's passionate expression about the place of mathematics in the world during the *Plenary interview session*
- Eric Muller's presentation *Future teachers use of technology to explore concept development in mathematics*
- Rafael Núñez's offering *Language, gesture, and the embodied mind: cognitive science and the foundations of mathematics*.

The theme I sensed, arising from all four presentations, was a focus on the capabilities of learners, manifested through an abiding respect for the humanity of learners. All these speakers, it seemed to me, recognized the power and fallibility of human beings to create and investigate mathematics.

Ubi D'Ambrosio highlighted, in a dramatic and passionate fashion, that (and I paraphrase)

> [...] mathematics is the backbone of the societal edifice and the edifice is in tough shape.

He was drawing attention to the fact that mathematics is an extremely powerful structure that underpins so much of modern society, yet we who live and work in the mathematics education area do not seem to recognize, and act upon, the social and cultural ramifications that the mathematical edifice engenders. *Mathematics is not neutral*, Ubi seemed be saying. We need to exercise the power we have in knowing and understanding mathematics to contribute towards a saner and more just society, one that honours the diversity that exists among us. He was acknowledging that we have the power to undertake this task, but that we have not, in the recent past, been good at addressing the task directly.

John Mason, though talking about the functioning of the human psyche, seemed to echo Ubi's theme arguing that:

> [...] human beings have the power to direct their attention, yet do not always exercise that control; they have the power to harness their energies, but do not always exercise that control. (p. 79)

Humans, though having the power to accomplish many great things, cannot be forced to do what someone else may wish them to do. This is not to say that Mason believes humans to be incapable of grasping ideas in powerful ways. He argues that for humans to exercise that power they need to recognize that their

> [...] attention needs to shift between holistic encompassing, discerning distinctions (stressing and ignoring, foregrounding and back-grounding), recognising relationships amongst discerned features, perceiving properties that objects or elements may possess, and deducing from definitions and axioms. These shifts can be rapid, but can be blocked; they are often subtle, but always vital to learning (p. 79).

Núñez postulates that the foundations of mathematical thought (and what powers the human capability of focusing attention that Mason talks about) are "cognitive in nature". Núñez's goal is

> to try to give a naturalistic account, biologically plausible and consistent with scientific findings, of the nature of mathematics and its peculiar and powerful conceptual organization. (p. 84)

Many years ago, Gattegno wrote a series of books entitled *The mind teaches the brain* [2]. A central tenet in those books was that the focusing of attention is a powerfully critical aspect of the human mind, one that can educate one's brain. Gattegno's thoughts seemed to have foreshadowed what researchers such as Mason and Núñez are now examining in a critical and disciplined fashion.

Underlying the comments made by D'Ambrosio, Mason and Núñez, it seems to me, was a fundamental belief that human beings had the capability and power to accomplish difficult and complex tasks and to learn extremely well despite many obstacles. Muller's presentation provided confirmation for that belief. Despite many obstacles in getting his program approved and operational, Muller persisted and developed a program for future teachers of mathematics that would

> develop their mathematical knowledge in an environment that nurtures concept development and the values the use of technology. (p. 82)

Muller states that the

> pedagogical goal of the MICA (Mathematics Integrating Computers and Applications) program is to help students internalize a unified framework of mathematical concepts by interpreting them computationally, visually and algebraically. (p. 82)

It was exciting to see the products Muller's students had produced, and the level of mathematical knowledge and sophistication they demonstrated. Traditionally speaking, and Muller said he heard these sorts of comments many times, skeptics would argue that the freshman students with whom Muller worked did not have the background or training (*e.g.*, they had not taken calculus yet) to be able to do this level of mathematics. The products generated by his students provided more than sufficient evidence to indicate that the students had a power to understand and do mathematics

well beyond anything they had previously been given credit for. Mullet believed in the power these humans had to do and understand mathematics, and despite many challenges, they demonstrated that Muller was correct

What did all of this say to me as a teacher educator? Why did these four presentations resonate so strongly with my beliefs and practices about the teaching and learning of mathematics? The answer to these questions lies in the reverberations I sense from those ICME-10 presentations for my work with novice teachers of mathematics and their mentors on eight, remote Pacific island communities [3] spread across 4.9 million square miles of the western Pacific, north of the equator. This work [4] is accomplished in consort with teams of locally based mentors drawn from the community colleges and departments of education on the islands.

The foundation of this activity is the SUBTLE way of respecting the knowledge and customs of each island community. For many years I have steeped myself in an orientation developed by Caleb Gattegno [5] called SUBTLE (Subordination of Teaching to Learning), which is a way of fostering learners' engagement with mathematics. This approach is the core of the two NSF projects with novice teachers because of the respect and honour it has for them as learners. The central aspects of the SUBTLE approach include:

- a deep respect for and acceptance of the capabilities of learners. A SUBTLE teacher always expects learners to be able to grasp the concepts being presented. At the same time, a SUBTLE teacher does not expect the learner to do that every time, in every circumstance. Mason's presentation points directly at both of these aspects of learner behaviour.
- an acknowledgement that in the teacher/student dyad, the learner is central; that is, the learning of the student is of paramount importance, and that the teacher's performance, the teacher's lesson, must be subordinated to the learning of the student. Muller, in placing freshmen students at the centre of the learning experience, places his role of university instructor as secondary to that of the learning of his students.
- the recognition that it is the learner who must do the learning, and that the teacher's function is to create situations and experiences that (hopefully) focus the learner's attention on the key concepts of the mathematics being presented. Mason, Muller and Núñez all stress the constructive nature of learners' involvement with mathematical ideas.
- the discipline to provide the learner with the minimal essentials for understanding to occur, to not 'tell' the learner everything, or almost everything, in the belief that 'telling' fosters learning. In order for knowledge to be a permanent, accessible, and useable aspect of learners' repertoires, they have to play with it, mold it, modify it, and finally make it their own. While postulating that mathematics is cognitive in nature, Núñez implies that learners mold and shape their mathematics in line with their own biological and experiential make-up.
- the further recognition that conversations among and between learners is a valuable tool in a teacher's instructional repertoire, because often their peers can ask a question, or provide a focus of attention that enables the learner to 'see' something not previously seen through teacher-designed activities. Small group work among peers is a valuable method for fostering learning

D'Ambrosio's plea was for greater substantive conversations within the mathematics education community, and between that community and the general population. On each trip across the Pacific region served by the NSF projects, I see the impact such conversations can have on remote island communities. On these islands: college mathematics instructors and mathematics specialists now collaborate in their work with teachers; teacher understanding of mathematics and mathematical pedagogy is increasing; student performance is gradually improving; the larger community sees value in what is being done and lends its support; and the goal of having Micronesian children locally smart and world smart seems achievable. This is the kind of impact that D'Ambrosio was encouraging when he pleaded for the mathematics education community to enlarge its scope of interaction with the wider educational world.

In the words of Caleb Gattegno, "only awareness is educable" by which is meant that learners can only acquire knowledge of that which they are aware. The ICME presentations reinforced this idea for me, and gave voice to and support for work that is underway in the vast reaches of Pacific island communities. Each presentation provided for me, in its own way, confirmation for the mathematics education being encouraged across Micronesia

## Notes

[1] All quotations are taken from the ICME-10 booklet *Plenary and Regular Lectures Abstracts* except in the case of D'Ambrosio. His were impromptu remarks and are reported from memory, which in my case is admittedly weak. However, I trust I have not misconstrued the intent of his comments. The following are further references to writings by the four presenters:

D'Ambrosio, U. (1991) 'Ethnomathematics and its place in the history and pedagogy of mathematics', in Harris, M (ed ), *Schools, mathematics and work'*, London, UK, Falmer, pp. 15-25

Lakoff, G. and Núñez, R. (2000) *Where mathematics comes from: how the embodied mind brings mathematics into being*, New York, NY, Basic Books

Mason, J. (2002) *Researching your own practice: the discipline of noticing*, London, UK, RoutledgeFalmer.

Muller, E. (2001) 'Reflections on the sustained use of technology in undergraduate mathematics education', in Holton, D. (ed.), *The teaching and learning of mathematics at university level*, Dordrecht, The Nethelands, Kluwer Academic Publishers, p. 381.

[2] Gattegno, C. (1974) *The mind teaches the brain*, New York, NY, Educational Solutions Inc.

[3] The Micronesian communities involved are the Commonwealth of the Northern Mariana Islands (CNMI), the Federated States of Micronesia (FSM - Chuuk, Kosrae, Pohnpei, and Yap), Guam, the Republic of the Marshall Islands (RMI), and the Republic of Palau (Palau).

[4] The teams were formed and provided with extensive in-service education under the auspices of two projects funded by National Science Foundation grants (ESI 9819630 and ESI 0138916). The projects are the DELTA Project and the MENTOR project, directed by the author

[5] Gattegno, C (1971) *What we owe children: the subordination of teaching to learning*, London, UK, Routledge and Kegan Paul.

*[A second communication, 'From the editor', can be found on page 23 (ed )]*