Research problems in mathematics education revisited

New clothes – and no emperor

ANNA SFARD

Dear Richard,

Let me begin with a comment on your request for reflections on the research problems in mathematics education that appear in volume 4 of FLM. In your call, you echo David Wheeler and his plea for "a number of specific problems whose solution would be likely to advance substantially our knowledge about mathematics education". Upon reflection, I cannot but wonder about what I see as the epistemological underpinnings of this request. These hidden premises, I'm quite sure, do not represent your own thinking. They have simply sneaked into the conversation together with the language that was in general use at the time David Wheeler was writing (and don't forget that Wheeler was paraphrasing an even earlier initiative by another David, David Hilbert, whose famous list of the 23 "remaining" problems of mathematics is, for me, an epitome of positivist thinking).

The expressions "solution to the problem" and "advancing substantially our knowledge" imply, at least to me, that research problems in mathematics education are to researchers what insufficiencies of natural conditions are to farmers: impairments to be identified, diagnosed and compensated for. According to this metaphor, our knowledge is like an agricultural crop: it systematically grows and accrues, and the effectiveness of this process depends on the farmer's ability to cope with whatever problems or difficulties nature may have in store for us. But can problems of mathematics education, like those of the depleted soil, be treated as residing in reality itself, as opposed to the proverbial beholder's eye? Can anybody really believe that these problems can be solved once and for all, that is, in such a manner as to simply go away, never to bother the researcher again?

The vision of mathematics education research that gives rise to the research-as-farming metaphor would be described by Bakhtin as *monological*, one according to which our steadily advancing knowledge is a collection of stories about the world, told by the world itself and ventriloquated by the researcher. My conception of research is different: it is what some writers, under Bakhtin's influence, call *dialogic* or *multivocal*. You can also call this approach *postmodern*, if you wish.

To describe the multivocal or postmodernist project let me use another metaphor, one that draws on a domain I am much better acquainted with than farming—the domain of clothing. I hope you agree that the researcher's job is to forge stories that give us a sense of understanding what is happening around us and may thus help us to go about our human affairs. These stories, I wish to claim, are to the world what clothes are to our bodies: they are human made rather than being a part of the world itself; they are supposed to "fit" what they are meant to "cover"; and although there is no "perfect fit", no ultimate story about the world, it is also not true that any story goes [1]. Some narratives may not fit at all, like a dress that is three sizes too small; for other stories, the "coverage" may seem so accurate that we start mistaking the "clothes" for the world's own skin. And, oh yes, there is another important parallel: our choice of story is no less a matter of fashion (and, in the background, of our desired identities!) than is our selection of garments.

Once the metaphor of research-as-farming has been replaced with that of research-as-dressing-the-world, our understanding of the term "research problem" changes. First, it becomes clear that it is not the world that faces us with problems, but rather, it is *us* who accord this latter name to whatever makes us less than satisfied with the world's present attire. And, as is generally known, especially among women, this latter need, being a matter of personal sensitivities, values, and tastes, is always debatable. True, we are all likely to agree that our purpose as a community is to add to people's well-being by helping them to become mathematically educated. We thus probably share the conviction that our choice of problems for research should be guided by this distant goal. But this is, I suspect, where our agreement ends. Once we begin "negotiating" specific research questions supposed to take us to our destination, we are likely to find out that there is no consensus about what it means to be mathematically educated or about what counts as a legitimate way of furthering one's mathematical competence.

Second, even if a certain research problem gains a wide following, it would be naïve to think that its solution may "advance our knowledge" in any absolute manner or can earn universal acceptance. Such hope would be as unreasonable as it is to believe that a single designer collection may ever gain exclusive control over the entire fashion market. But if not this, you may wonder, what could be the reward for one's research efforts? Is there any other reason for going through the pain of this complex endeavor than our wish to make academic careers? For me there is, as there is, I believe, for most of us. As a dressmaker for the world of mathematics teaching and learning, I am satisfied if the new dress I have sewn is pleasing to my eye and makes me hope that whoever chooses to use it feels more comfortable than before; I am fully gratified if this hope comes true, that is, if some people, and especially some teachers, actually tell me that they find the new garment more effective in fulfilling its role than the one in which they were dressing the world so far; and I am beyond myself with joy if there is a piece of evidence that the new dress made somebody, if only a child or two, happier. And no, I am not deterred by the thought that my offerings, even if successful now, will one day be deemed "unfashionable". As a dressmaker, I can only strive for solutions that are good for now.

My research problems, just like their solutions, are also for here and now, at least in the historical perspective. The list of my present personal favorites includes a number of long-standing queries that have always obstructed our (my)

understanding of the emergence and evolution of mathematical thinking. I presented five such problems in my book *Thinking as Communicating*, where they were called "quandaries": the quandaries of number, of abstraction (and "transfer"), of "misconceptions", of "learning disability" and of understanding. But I don't want to repeat myself. Rather than expanding on any of these old acquaintances for which I eventually managed to find some good-for-now new clothes, I prefer to talk here about a certain meta-level problem that had to be tackled before the crafting of the new "coverage" became possible. Here it is, my favorite research problem:

How to talk about mathematics and its learning?

Let me explain. Feeling that my inability to deal with the quandaries on my list has to do with the way I talk, and so think, about them, I realized that a revision of my dressmaker's tools may be a condition for any further progress. I began asking: is my present way of talking good enough to allow me to tell stories as insightful and convincing as I want them to be? Am I not stymied by assertions I am tacitly adopting against my own better judgment? Am I not failing myself and misleading others by saying things I do not intend to say? Am I not going in circles only because, unwittingly, I am using the same words in a number of different ways or because I do not really know what I am talking about?

You may find it strange that I view all these questions as constituting a *research problem*. First, is answering these questions really a matter of research? Well, I think it is, even if our own discourse is the object of investigation. Some people call this kind of enquiry "theory building". Second, some may doubt the importance of these questions, saying that I am just dealing with words, words, words. As you may guess, my answer is simple: one should not underestimate words. They are the fabric from which we make our stories and, as such, they enable and constrain what we are able to tell. While combining words into assertions about specific things and situations, we may be informing others, unwittingly, about our entire worldview. My discussion, above, of David Wheeler's call aptly illustrates this claim. It also substantiates my present assertion that thinking about the way we talk as researchers is a necessary part of our endeavor. Had I tried to answer David Wheeler's appeal without changing the question, I would have entered a discourse incommensurable with my own. And I would have ended up entangled in contradictions. Epistemological premises hiding in our sentences doom us to failure, just as Greek soldiers hidden in the wooden horse brought about the eventual surrender of Troy. In my book, I identified several linguistic Trojan horses, which, I thought, might have been responsible for our persistent difficulty with the five quandaries on my list. I made the utmost effort to change my language. Ever since my colleagues and I began working with these new tools, our "clothing" business has been flourishing and we were able to forge many good-for-now stories about mathematics and its development [2].

I hope to have answered your question, even if only in an altered form. Still, I cannot conclude without some footnotes, two of them in the form of disclaimers and one of them a correction to what I said before. This latter footnote will raise a new question for our collective consideration.

Footnote 1. My first disclaimer is meant to forestall misinterpretations as to my intentions. I hope you do not suspect me of preaching one discourse for us all. Let me assure you that I am doing no such thing. While asking "How to talk about mathematics teaching and learning?" I am not looking for a newspeak for the mathematics education community. My recommendation is neither to "advance knowledge" by adding stories not yet told, nor to limit ourselves to a single kind of story, created with a single set of tools. All I am proposing is that we invest as much thought and creativity in shaping our storytelling tools as we do in the storytelling itself [3]. What I am trying to say, in short, is this: let us all be thoughtful and explicit about how our discourses work and, at the same time, let a thousand discourses bloom.

But while advocating freedom of discourse, I create, of course, another problem to be always kept in mind: in the absence of a common form of talk, how do we communicate and keep our community together? And this problem forces me to add the second footnote.

Footnote 2. Freedom of discourse does not doom our conversation to failure. I am sanguine about the possibility of communicating across discourses. I believe that all it takes to be understood according to my intentions is to be explicit about how I use words. If I was a non-Euclidean geometrician, I would probably open every story with an explanation of what the basic word *line* means in my language (which I would do by presenting my position on Euclid's fifth postulate). As a researcher in mathematics education, I feel the urge to explain what I mean while talking about *learning*.

Footnote 3. What has been said so far obliges me to problematize my earlier claim that our purpose as a community is to add to people's well-being by helping them to become mathematically educated. Indeed, after issuing the call for the freedom of discourse I cannot escape the obvious question: if we are so liberal with regard to our own discourses as researchers, why are we supporting the idea of imposing mathematical discourse on everyone? The query seems easy to answer: mathematics is one of the hegemonic discourses of our Western society, and thus not being mathematically competent means a serious disadvantage. There is also a simple explanation for the special allure of mathematical discourse: with its ability to impose linear order on anything quantifiable, this discourse is a perfect setting for decision-making. No wonder, then, that whatever is stated in mathematical terms stands a good chance of overriding any alternative story—enough to recall what counts as decisive "scientific evidence" in the eyes of the politician. But should the hegemony of mathematics go unquestioned? On a closer look, not each of its uses may be for the good of those whose well–being and empowerment we have in mind while launching our research. Thus, for example, when mathematical discourse, so effective in creating useful stories about the physical reality around us, is also applied in crafting stories about children, the results may be less than helpful. More often than not, the numerical tags these stories accord their heroes, rather than empowering the student may be raising barriers that some of the children will never be able to cross. The same happens when the ability to participate in mathematical discourse is seen as a norm and the lack thereof as pathology and as a symptom of a general insufficiency of

the child's "potential". "Why should we teach mathematics and to whom?" is thus, in my opinion, one of the researcher's "must-ask" queries. If we want to make sure we do not perpetuate orders that we would rather like to change, we have to be always watchful of how mathematical discourse is used.

Notes

[1] "Anything goes" is the slogan which, in the eyes of objectors, encapsulates the postmodernist stance. The critics derive it from the postmodernist rejection of the idea of "absolute truth". But "anything goes" does not follow from "no story is true in an absolute manner", just as the claim that every dress is equally good for me does not ensue from the fact that there is no dress that fits me in an "absolute" fashion.

[2] Some of these stories are offered in the double special issue (volumes 51 and 52) of the *Journal for Educational Research* titled "Developing mathematical discourse—Some insights from communicational research", published in March 2012.

[3] Storytelling is not an exclusive activity of researchers—it is something everybody does. What makes our research storytelling distinct is our being explicit and meticulous about our tools, and thus also about our assumptions. This, of course, makes us more accountable for our narratives than any lay storyteller

Explosive problems in mathematics education

OLE SKOVSMOSE

Dear Richard,

Thank you for drawing my attention to David Wheeler's collection of problems. It has been fascinating to reconsider them.

Wheeler did ask for problems "whose solution would be likely to advance substantially our knowledge of mathematics education". As examples I consider the following three, formulated by Geoffrey Howson, John Mason and Alan Bishop:

Howson:	It would help me to have a better theoretical framework within which to consider/ study/investigate mathematics education [1]
Mason:	Is mathematics really essential? [2]
Bishop:	How is mathematical meaning shared? [3]

These formulations are short and clear. Nevertheless, I have the feeling that it is impossible to reach any solution to be generally agreed upon. Howson might have had the same feeling, as he modified the task by adding: "Here I am for the moment not asking for a theory, just a framework which might help to develop one." But even with such modification, his problem appears gigantic.

Naturally, I came to think of David Hilbert's 23 problems presented in 1900, which, since then, have provided an ongoing challenge to mathematics research. I looked at a web page [4] where the problems are nicely classified: some were solved in a particular year, some are partially solved, one is too vague in its formulation to be solved, while three remain unsolved. Progress in mathematics has, at least partially, been related to the resolution of these problems.

Hilbert's famous second problem was: "Prove that the axioms of arithmetic are consistent." Gödel's second incompleteness theorem, however, showed that no proof of this consistency can be carried out within arithmetic itself. This was a most alarming result considering Hilbert's metamathematical aspirations. When I was young, I spent half a year studying Gödel's proof. I was fascinated by the perspectives it provided on mathematics.

While Hilbert's problems concern mathematics, Wheeler's problems concern mathematics education. Linguistically speaking there is not much difference between "mathematics" and "mathematics education"; epistemically speaking, there is. The difference becomes clear when we consider the difference in nature between Hilbert and Wheeler's problems. The formulation of Hilbert's problems seems be based on some shared conceptions of what a solution could mean, while what to consider a solution to a Wheeler problem appears to be ambiguous.

This brings me to consider the notion of explosive concepts. By this I understand a concept that can be defined only through concepts just as open and vague as itself. As an example, one can think of "democracy". In order to clarify what democracy means, one may consider the meaning of, for instance, equity, justice and inclusion. However, these notions are no easier to define than the notion of democracy. Thus a definition of an explosive concept does not "narrow down" its possible meanings. Instead, it opens into new landscapes of possible meanings.

Hilbert's problems are formulated through concepts that one could call *solid*, like: integer, real number, axiom, function, arithmetic, consistency, polynomial, group, algorithm, prime number, *etc*. Naturally, the definition, of such concepts are not fixed; they develop during history. What might be considered a proper definition in one period might appear inadequate later. Think of the notion of function, for example, which has been part of a fascinating conceptual development. But, although "function" demonstrates a historicity, it is not explosive. Its definitions and redefinitions are based on notions generally accepted in the mathematical community at a given time. In this sense, we can consider "function" a solid concept. (Let me just add that the notion of a proof being "finite", as used by Hilbert in formulating what methods could be allowed in metamathematics, can hardly be called solid. Let this, then, indicate that explosions might also occur in mathematics.)

Contrary to Hilbert's problems, many of Wheeler's, including those presented by Howson, Mason, and Bishop, are formulated through explosive concepts. In order to define a "theory" one needs to address broader questions about knowledge, justification, and power. A discussion of mathematics being "really essential" leads us to consider the possible roles of mathematics in society, in technology, and in people's life-worlds. A discussion of "meaning" brings us into a turbulent trip through the history of philosophy, even before we come to address "shared meanings".

Let us call a problem that is formulated through explosive concepts an explosive problem. Thus many of the problems collected and presented by Wheeler are explosive. Hilbert's problems, however, can be characterized as being solid. They are formulated through solid concepts, and there exists a general agreement, within the mathematical com-