COMBINING APPROACHES FOR THE ANALYSIS OF COLLABORATIVE MATHEMATICS LEARNING

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Collaboration is more and more at the heart of mathematical learning. By collaboration we mean "a coordinated joint commitment to a shared goal, reciprocity, mutuality, and the continual (re)negotiation of meaning" (Mercer & Littleton, 2007, p. 25). A considerable amount of research has focused on the negotiation and joint construction of mathematical meanings by analysing students' peer interactions. The role of language is prominent in such analyses, as it is viewed not only as a means [1] but also as the object of the interaction and the negotiation. Starting from the 'purely' linguistic approach of Austin and Howson (1979), recent research – following the impact of Lev Vygotsky's work – has begun to acknowledge the importance of other social factors by including them as an element of a broader analytical system (Dekker, Elshout-Mohr and Wood, 2006; Ernest, 1994; Pimm, 1987). The role of sociology is more prominent in analyses of the norms of classroom interactions (Yackel & Cobb, 1996), its meta-rules (Sfard, 2000) and issues of positioning and identity (Wagner & Herbel-Eisenmann, 2009; Sfard & Prusak, 2005).

However, collaboration is not considered an infallible method of learning: Mercer and Littleton (2007) note that "in certain circumstances peer interaction can result in regression as well as development" (p. 12). Tatsis and Koleza (2006) have shown how particular actions of a student in a collaborative pair may inhibit her partner's participation. Weber *et al.* (2008) in their literature review state that, "Exactly how discussion should be promoted in mathematics classrooms or what types of discussion are likely to promote learning opportunities are important open questions" (p. 248). Eventually, we started considering the above issues while we were re-analysing a series of discussions coming from a pair of two 16-year-old students working collaboratively on an investigation task about probabilities using the TIC-TAC game (see fig. 1). During the first lesson Peter and Susan worked on the following problem by playing the game on the computer:

> One can play with a little ball by clicking with the mouse on its starting point at the bottom left. There is a 50% probability that a ball will go up, and a 50% probability that it will go to the right. What is the probability of ending up in the bow with 100 points?

In that lesson Susan suggested that there are nine possibilities; this was criticized by Peter. They started reconstructing their work, but at the end of the first lesson they returned to Susan's initial answer and expressed it in percentages
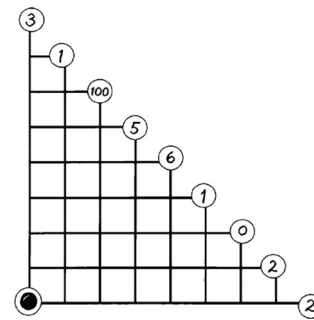

Figure 1.   The TIC-TAC game.

(9.333%). In the next lesson Peter returned to the problem and showed a different answer:

1 Peter: I assumed that every time that, every time that the ball moves toward the middle, that the probability that it, er, one upward and then this way and that way, so here the probability is 1, 2, 3, 4 and 5, and then I've added those chances, that's 25, and then this is three 25ths, so twelve 100ths

2 Susan: Yes.

3 Peter: Is 12 percent.

4 Susan: Yes.

5 Peter: I think, but I'm not sure, so it would be, yes, 20% and that 4%. Well, shall we […]

6 Peter: You may come up with another theory, but anyway I think it's better than 1/9.

7 Susan: Okay.

8 Peter: At least, for me. I don't know what you think.

9 Susan: Yes, it's okay, add it to our stuff. … And do you also have something for this one? Er, have a look.

10 Peter: Wow. Erm, let me see, er …

11 Susan: Shall we proceed? Add it. … How do you write it?

12 Peter: Well, I'll write.

Pijls et. al (2007) in their analysis of the above dialogue noticed that Susan's participation was not supportive either for Peter or herself (in the sense of her involvement in the task which was expected to lead to her mathematical level raising):

> Peter came up with an idea he was not sure of. He started to defend it, but since he received no feedback from Susan regarding the content, he could not really explain, justify, or reconstruct it…. Susan accepted his theory without criticism. It seems that she was not interested in *why* his theory would work; instead, she wanted to go on with the next task. (p. 321)

According to Pijls *et al.* (2007) the acts of explaining, justifying, reconstructing, giving and receiving feedback are connected to mathematical level raising (MLR) (*cf.,* Yackel & Cobb's, 1996, sociomathematical norm of justification and the findings of Tatsis & Koleza, 2008), thus their lack creates obstacles in MLR. Although the authors have identified the communication breaches in that discussion, their object-level analysis could not account for Susan's behaviour. It was this observation that led us to look for a theory and an analytical framework that could complement the authors' analysis. We sought one that would consider the intra- and interpersonal aspects of the interaction. Our main consideration was that this theory should account for the students' behaviours and perhaps suggest possible ways for the teacher to monitor such behaviours and even regulate the interaction towards a more 'fruitful' exchange. [2] *Role theory* as introduced by Goffman (1971) and implemented by Tatsis and Koleza (2006) was our choice. It is not the first time that Goffman's work has come to the fore in mathematics education: Sfard (2001) has related her *meta-rules* with his notion of *frames*, while Hersh (1988, mentioned in Ernest, 2008) drew on Goffman's dramaturgical model to describe the work of mathematicians as having a front (following strict norms) and a back end (messy and chaotic). Our choice was based on the assumption that students and teachers are social actors whose actions are influenced – if not shaped – by a number of interpersonal and intrapersonal concerns, from identity formation (Sfard and Prusak, 2005) to face-work (Goffman, 1972). These actions eventually constitute the person's *role performanc*e (Goffman, 1971).

Role theory: face-work and respective roles

Role performance is the key concept in role theory and is defined as "all the activity of a given participant on a given occasion which serves to influence in any way any of the other participants" (Goffman, 1971, p. 26). But why would anyone want to influence the other participants in a given situation and how can this be relevant to collaborative mathematics learning? Both questions can be answered by introducing another key concept in role theory, the concept of *face*. Face is defined as "the positive social value a person effectively claims for himself by the line others assume he has taken during a particular contact" (Goffman, 1972, p. 5) and can be further categorised into positive and negative: *positive face* is related to a person's need for social approval, whereas *negative face* is related to a person's need for freedom of action (Brown & Levinson, 1987). The participants not only have these needs, but recognise that the others have them too; moreover, they

recognise that the satisfaction of their own face needs is, in part, achieved by the acknowledgement of those of others (Maslow, 1943). For this purpose, they may employ some politeness strategies in order to reduce the potential threat to the other's face (Rowland, 2000). These strategies can be identified by the use of particular hedges (*e.g.,* "I think", "probably", "maybe") and modals (*e.g.,* "might", "could").

Returning to our collaborative session, it is obvious that Susan and Peter were faced with some contextual demands concerning their expected behaviour: they were in a classroom and they were expected to solve a task by working collaboratively. Thus, they were bound by a set of social and sociomathematical norms (Yackel & Cobb, 1996). At the same time, they had to consider the face needs of themselves and of their colleague. This consideration led them to a continuous interpretation of the situation: each person's act was consciously or subconsciously interpreted according to its effect on the speaker's and/or the hearer's face. The following categories are an expansion of those used by Tatsis and Koleza (2006) for dyadic interactions: [3]

> (a) face-threatening act: explicitly threatens the other's face (*e.g.,* requests, orders, rejection of the other's suggestion, expressions of sarcasm and irony);
>
> (b) face-empowering act: explicitly or implicitly empowers the other's face (*e.g.,* acceptance of the other's suggestion, expressions of appraisal)
>
> (c) face-weakening act: implicitly weakens one's own face (*e.g.,* expressions of uncertainty, withdrawal of one's own suggestion, admittance of being mistaken);
>
> (d) face-maintaining act: implicitly aims at maintaining one's face, even when it is not being explicitly threatened (*e.g.,* initiation of talk, expression of one's ideas);
>
> (e) face-saving act: aims at 'repairing' one's face after having received a face-threatening act (*e.g.,* argumentation, justification of one's own acts, repetition or elaboration of a suggestion, expression of face-threatening acts against the other).

All the above were used to produce a joint analysis of Susan and Peter's dialogue as shown later on. But before that we describe the basis of the original analysis: the process model for interaction and mathematical level raising.

Process model for interaction and mathematical level raising

Dekker and Elshout-Mohr (1998) modeled processes of mathematical level raising and thereby created an instrument to analyze the interaction of students during collaborative learning. In the model three types of activities are discerned, namely key activities, regulating activities and mental or cognitive activities. Key activities are ones that are central for level raising: to tell or show one's work is a step in becoming conscious of one's own work and in taking some distance from it; to explain one's work triggers reflection on one's work; to justify one's work means checking if your work is consistent with certain external norms and if that is not the case reconstruction is needed, which can imply level

raising. Students can provoke these key activities by asking each other to show their work, by asking each other to explain their work and by giving critic. These activities are described as regulating activities in the process model. The process model is fully presented and described in Dekker and Elshout-Mohr (1998). A summary is presented in Table 1, which shows the regulating and key activities for mathematical level raising in students' interaction.

Regulating activities	Key activities
Students ask each other to show their work	Students show each other their work
Students ask each other to explain their work	Students explain their work to each other
Students criticize each other's work	Students justify their work
	Students reconstruct their work

Table 1.   Regulating and key activities

The combined analysis

Following Sfard (2001) we have decided to focus on "the activity itself and to its changing, context-sensitive dimensions" (p. 24). This does not mean that we underestimate the students' past histories or the influence of the factors such as the presence of the observer. We believe that all these manifest themselves in students' communicative actions.

Let us now go back to our initial dialogue and to the students' activities. [4] Peter starts by *showing* the thinking he has done after their first attempt to solve the problem during the previous lesson. [5] His answer reveals that he went back to their *reconstruction* and has continued it; moreover he tries *to show* his idea in an explicit way, following the norm that you are expected to be clear and comprehensive in a (mathematical) discussion. He uses the first singular person ("I assumed", "I've added"), which is normally used when students present their own work or express their feelings and beliefs (Rowland, 2000). He also demonstrates in that way that he has been thinking about the problem in between the two lessons.

Susan's positive replies (2, 4, 7) do not provide any essential feedback to Peter; she *doesn't ask for an explanation* or *doesn't criticize* him; she might be uncertain about her own work and/or even her ability to participate in the joint construction of the solution, thus she decides not to do anything that could damage her face in front of Peter. We cannot be sure if she understood his idea or if she was convinced that their initial answer had to be *reconstructed* thus she 'trusts' that Peter has done that well. Although these positive replies allow Peter to elaborate his idea (3, 5, 6) and *show* it more fully, at the same time they increase the risk of it being faulty (in the sense that a discussion could reveal possible weak points).

Peter realises that risk (5) and he takes some action, particularly he deploys some face-maintaining strategies: firstly he uses the hedge "I think" to decrease the degree of certainty and then he claims that he is not sure about the elaboration of his initial idea. In that way, he protects his face in case his suggestion proves faulty and implicitly invites Susan to pro-

vide some *critic*. At the end, he makes an explicit suggestion using a modal ("shall we"), trying to include Susan in the process, but we see no reaction from her.

Her reluctance to join the discussion leads Peter to make another attempt by inviting her to "come up with another theory" (6) – or in other words, he *asks* her *to show* another solution – which could be the start of a level raising discussion. This utterance could be interpreted as a bit provocative, thus face-threatening for Susan; that is the reason why Peter deploys a politeness strategy by using the modal "may" and the hedge "I think". He believes that his solution is not only different from Susan's (1/9) but also better; but he expresses it in such a way that:

a. he firstly 'leaves room' for Susan *to show* another idea but not by a direct request. In that way he appears as a non-dominant partner (face-maintaining act) and avoids posing a direct threat to Susan's face (politeness strategy).

b. he uses again the hedge "I think" in order *to show* his own idea, which is in contrast with Susan's. In that way he appears as a collaborative and non-absolutist partner (face-maintaining act) and softens the potential threat to Susan's face.

Peter's behaviour in that episode is that of a collaborative initiator. [6] Although such a person usually boosts the discussion and the elaboration of mathematical meanings this is not case in this episode where Susan acts as an insecure conciliator, and thus hinders the joint elaboration of mathematical meanings and the process of level raising. In the previous lesson it was Susan who discovered that for calculating the chances they had to count all routes, and that there is symmetry (which reduces counting). But Peter gave no attention to her solution then and *did not ask* her for an explanation, but showed another solution, avoiding the counting. It could be that Susan still feels that they actually should count, but that is a lot of work. And testing Peter's theory by experimenting is also no option, as at this second lesson the TIC-TAC game is not at hand.

Moreover, she seems to be concerned with the continuation of their work (9). She wants to move on. On the surface, her eagerness to move on may be attributed to her being sensitive to the division of time (DT) perspective (Dekker *et al.,* 2006). Upon a closer look we may attribute it to a face-saving strategy. She might have felt that a discussion on Peter's solution was beyond her ability, and could damage her face if she appeared not to be competent enough. Thus, she withdraws from any discussion and stimulates Peter to be the producer of solutions (9) – a division of roles which is very counterproductive for the level raising process of both of them. In fact, they do not discover the flaw in his solution.

Discussion

The purpose of the above analysis was to demonstrate how role theory can be combined with the process model of MLR (Dekker & Elshout-Mohr, 1998) in order to get a richer picture of collaborative mathematics learning. What we have seen is that there is a connection between particular key and regulating activities and particular roles.

Susan accepted Peter's idea without criticism, thus giving no feedback to him. This may be attributed to her interpretation of the situation and the resulting role, which is that of an insecure conciliator. Peter took the role of collaborative initiator and came up with an idea, although he was not sure about it. He started to defend it, but since he received no feedback from Susan regarding the content, he could not really explain, justify, or reconstruct it. There was nobody to indicate the weak points of his theory.

Coming back to a question posed by Pijls *et al.* (2007): *What does Susan need in order to participate more actively?* Our combined analysis has shown that her behaviour could be altered if her attitude towards collaborative work changed. In other words, if she felt that her voice (Wagner, 2007) should be heard and that her face would not be damaged by a possible mistake. Peter did his best to enable this by employing some politeness strategies, but they proved insufficient. A possible interpretation could be that it was the effect of the observer's presence in the setting. That Peter was not affected in the same way takes us back to the face-saving approach.

The order of regulating activities is also important. For example, if a discussion starts with a criticism of someone's solution – as Peter did in the first lesson – instead of asking for explanation, it may be interpreted as face-threatening. So the hearer may adopt a face-saving strategy, which could lead the person to be an insecure conciliator.

How can the teacher be aware of all this? Our analytical framework is based on simple assumptions about how students interpret collaboration. A teacher who is able to identify the students' acts under the prism of face-work and MLR, may be also able to provide feedback to the students about their participation and, if necessary, intervene in crucial moments. After all, mathematics is a human activity, so it can be regulated according to human's inter- and intrapersonal needs, without neglecting the mathematical aspects of it.

Notes

[1] Along with other means, such as visual representations (see *e.g.,* Pirie's (1998) categorisation) or gestures (*e.g.,* Radford, 2008). Sfard (2001) overcomes this distinction by using the term *discourse* to refer to "any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system" (p. 28).

[2] What we are suggesting here is not a strict adherence to a set of meta-rules concerning participation; we rather believe that the students should be aware of these rules (*e.g.,* Wagner, 2007) and also of the potential effect of their speech acts in the flow of the interaction.

[3] It is important to stress that speech acts do not function in one direction, as far as face-work is concerned; a face-saving act for the speaker may be a face-threatening act for the hearer as it is shown in the fifth category of acts.

[4] Implicit in our analysis are the students' intentions (*e.g.,* concerning the saving of their face) guiding or forming their actions; however, we stress the fact that "the use of such terms as intentions is safe as long as it is understood that the status of any claim about other people's *intentions* is the researcher can make is *interpretive*" (Sfard, 2001, p. 32).

[5] The italics designate actions related to mathematical level raising.

[6] Following Tatsis and Koleza's (2006) categorization, we identify four types of roles: a) the dominant initiator, who makes many suggestions but rarely asks for the other's opinion; (main face-saving strategy: domination), b) the collaborative initiator, who makes many suggestions but at the same time asks for the other's opinion; (main face-saving strategy: initiation), c) the collaborative evaluator, who makes relatively fewer suggestions compared to the initiators but promotes the discussion and evaluation of all suggestions; (main face-saving strategy: evaluation), and d) the insecure conciliator, who makes few suggestions and accepts the others' suggestions without evaluating them (main face-saving strategy: avoidance).

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