CONTEXT MATTERS IN ASSESSING STUDENTS' MATHEMATICAL POWER

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> The greatest need in our present-day scientific age is for men and women who can use their minds as well as their knowledge of mathematics; for men and women who can use their understanding of the uses that have already been made of mathematics and apply it to new and unsolved problems in physics, biology, astronomy, the social sciences, and to new fields of technological knowledge still to be identified. (Hildebrandt, 1959, p. 370)

In the USA, the 1996 and 2000 National Assessment of Educational Progress (NAEP) mathematics assessment framework highlights the need to evaluate students' ability to *reason* with mathematics, *communicate* about mathematics, and

> *connect* the mathematical nature of a situation with related mathematical knowledge and information gained from other disciplines or through observation. (NCES, 1996, p. 37, *emphasis added*)

These are elements of what is known as *mathematical power*.

> This notion is based on the recognition of mathematics as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication, and notions of context. (NCTM, 1989, p. 5)

Despite the suggestion that evaluating students' mathematical power may provide a comprehensive view of the state of mathematics education, this essential way of knowing and using mathematics has not yet been measured. Perhaps the most compelling reason for mathematics educators' delay in addressing this topic in research is the complexity of the enterprise.

First, what types of items might provide information about students' mathematical power? Mathematics assessments such as NAEP focus on evaluating what students know and can do in mathematics. Items generally provide 'clean' contexts that are not useful in developing solution strategies and that do not demand the application of knowledge other than mathematics. Examples drawn from released items from NAEP mathematics assessments provide evidence of this assertion (see Figure 1). [1]

In reference to items of this type, van den Heuvel-Panhuizen (1996) asserts that

> the reality referred to by these contexts has been replaced by a mathematics textbook context [that serves simply as] window dressing for the mathematics. (p. 20)

This is not to say that students do not use practical reasoning on such items – they may (Cooper and Dunne, 1998). However, to evaluate mathematical power, items that are situated in contexts integral to the problem are far more likely to produce evidence of connections between mathematical, practical, and other content knowledges.

Second, do items that demand the use of mathematical, practical and other content knowledges to craft a correct response exist? Some items released from NAEP in areas such as civics, science, geography, and history have encouraged, and in some cases required, the use of mathematical skills such as computation, data analysis (D'Ambrosio, Kastberg, McDermott and Saada, 2004), and measurement. Responses to these items are one source of data from which an evaluation of students' mathematical power can be drawn.

A collection of NAEP assessment items released between 1994 and 2001 in the areas of geography, history, civics, and science provided students with opportunities to use their mathematical knowledge in context. In a survey of released items from these disciplines we identified 11 items for which task demands or student interpretation resulted in the application of mathematics as part of the solution process: 2 from civics, 4 from geography, and 5 from science. We then requested and secured access to student work on these items.

Subject: Math Grade: 4 Block: 2003-4M6 No.: 11

11 Six students bought exactly enough pens to share equally among themselves. Which of the following could be the number of pens they bought?

A) 46
B) 48
C) 50
D) 52

Subject: Math Grade: 8 Block: 2003-8M6 No. : 12

12. Carl has 3 empty egg cartons and 34 eggs. If each carton holds 12 eggs, how many more eggs are needed to fill all 3 cartons?

A) 2
B) 3
C) 4
D) 6

Subject: Math Grade: 12 Block: 1992-12M15 No.: 01

1. Rulers cost $0.85 each, including tax. How many rulers can Tom buy if he has $7.00?

Answer:____________________

Figure 1: Examples of contexts used in selected NAEP mathematics assessment items.

The discussion in this article is based on the analysis of student work on constructed-response items as well as multiple-choice items.

Student reasoning and mathematical power

Research on student reasoning on contextual items provides several plausible sources of the reasoning students use (Gravemeijer, 1997; Greer, 1997; Hatano, 1997; Verschaffel *et al.*, 1999). Among these are the materials and teaching to which students have access. While textbooks and teacher practices are certain to have an influence on what students learn, it is the student who largely determines what he or she learns.

Evidence of the impact of personal experience on student reasoning on contextual problems peppers the mathematics education literature (Irwin, 2001; Kazemi, 2002; Lo and Watanabe, 1997; Lubienski, 2000). These reports illustrate that students may not interpret a given scenario as mathematics educators intend (Davis and Maher, 1990). Instead, their attention can be drawn by what are, for them, realistic or contextual considerations. Students accounted for context by relating problem situations to their personal experience. For example, when students were given the initial condition of "four quarters for 10 candies" some students reasoned that the price per candy would increase if fewer candies were purchased.

> They argued that the shop might charge a higher price for buying a few; for example, 'One for 35 cents and three for a dollar'. (Lo and Watanabe, 1997, p. 219)

Lubienski (2000) characterized students' reasoning about problem contexts in this way as approaching "problems in a way that caused them to miss their intended mathematical points" (p. 454). This example and interpretation illustrate difficulties and tensions involved in an evaluation of students' mathematical power.

One difficulty is the multidimensional focus of a contextual problem. Because contexts elicit responses that are based on integrations of mathematical and practical knowledge, answers are difficult to score. If this reasoning produces a solution different from that of experts, should the solution be interpreted as incorrect? It is our view that interpretations of students' solutions should be based on the assumptions that each student makes about the problem context and his or her ability to integrate personal, mathematical, and other content knowledges to craft a reasonable solution. This is the very method Pollak (1970; 1987) suggests he and other applied mathematicians use when they attempt to build mathematical models to solve real world problems.

One tension that results from the use and examination of student work on contextual problems is a unidimensional focus of some educational researchers on the mathematics alone. If the problem is contextual, should students ignore practical considerations and simply identify the mathematical structures experts see as 'embedded' in the problem? Is learning to ignore context or account for it only superficially an indication of mathematical power? We, with some other mathematics educators such as Greer (1997), would say no. Much has been written about students' ability to ignore practical considerations on problems such as the 'bus problem' [2] from the third NAEP mathematics assessment. Students were asked to find the number of buses needed to transport a quantity of persons. The division of the numbers in the problem resulted in a mixed number and students were expected to account for the fraction. Was another whole bus needed or only a portion of a bus? Many students reasoned that a fractional portion of a bus was needed, effectively ignoring the practical consideration that there cannot be a portion of a bus. We consider this reasoning as lacking in mathematical power when mathematics was used to solve the problem, but connections to practical knowledge were not made.

A careful analysis of problem statements and student work may be a starting point for an evaluation of students' reasoning and mathematical power. Solutions that draw on the practical, the mathematical, and knowledge of other content areas seem to provide evidence of mathematical power and suggest that the increased use of contextual problems in student assessments may be warranted.

Student reasoning on NAEP

Our interpretation of the data suggests that students use their mathematical and practical knowledge alone and in concert to craft problem solutions. The results of these approaches are mixed. In some cases, when mathematical knowledge is emphasized and other content knowledge is ignored, students are successful on items. In other cases, the use of mathematical reasoning and practical knowledge produces faulty conclusions. Evidence of both cases will be discussed.

Emphasizing mathematical knowledge

Student success on the items we discuss in this section was mixed, however on all of the items we argue that a large portion of the students applied their understanding of mathematics in an effort to make sense of the problem. The use of mathematical reasoning alone on a subset of these items resulted in the selection of distractors. For the remainder of the items mathematical reasoning alone could result in the elimination of distractors. We will show that many students interpreted the problems they faced as mathematics problems. This interpretation illustrates one element of mathematical power, the application of mathematics in other content areas.

Mathematical reasoning and the selection of distractors

Our analysis of student responses to items in this section suggests that students often apply mathematics alone without connection to other content areas. In the examples below, students emphasized mathematical reasoning yielding incorrect answers.

On the 2000 NAEP science assessment, fourth grade [3] students were given information about a quantity of toad eggs and asked to select a graph that appropriately represented the number of tadpoles and toads the eggs would produce (Figure 2).

The problem was designed to assess basic knowledge and understanding of life cycles. Four percent of students omitted this item, making it the single most omitted item of the 43 multiple-choice items released from the 2000 science assessment (all grades). Seventy-eight percent of students

8. An adult toad lays 6,000 eggs at a time in a pond. Which graph shows the number of tadpoles and toads that will most likely result in the pond from these eggs?

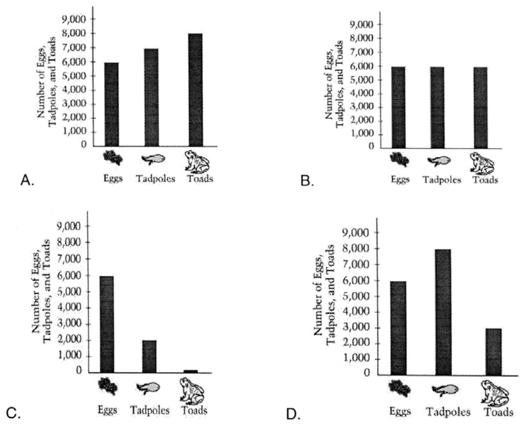
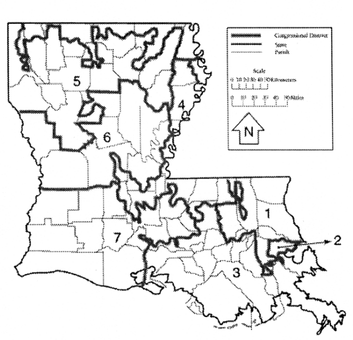


Figure 2: Number of tadpoles and toads (2000-4S21).

responded incorrectly to this item, with 52% choosing graph B. In mathematics class, in all probability, a static quantity of eggs, tadpoles, and toads would have been expected. Hence, one explanation for the popularity of the incorrect representation of the life cycle of a toad is students' interpretation of the item as purely mathematical. An alternative interpretation is that children considered the rate of decline depicted in response C to be unbelievable. Hence, the only other plausible alternative to this unrealistic rate of decline was the static model. In both alternatives however, the students' mathematical interpretation of the situation dominated their science content knowledge.

On a civics item, a group of twelfth-grade students met with a similar difficulty applying mathematical reasoning alone. The item was meant to assess students' ability to analyze and explain how the government embodies the purposes, values, and principles of American democracy (see Figure 3). To answer the item using the reasoning intended by the item developers, the student would have to identify district 2, note its size relative to the other districts,

Refer to the map below, which shows congressional districts in Louisiana.
The numbers on the map refer to congressional districts.



7. From the map, you can conclude that congressional district 2 must

A. include a large urban area.
B. have fewer people than the other districts.
C. have been drawn to protect an incumbent.
D. be a very old congressional district.

Figure 3: Twelfth-grade civics item (1998-12C8) requiring inference based on area.

and then account for this difference using his or her knowledge of civics. Forty-seven percent of the students selected the correct answer, A, for this item. Forty percent however selected B, a response indicative of the students' use of proportional reasoning. The smaller the district, the fewer people must be living there. When these students were asked to select a rationale for the size of the district, they reasoned mathematically.

In a common exponential decay problem, students used mathematical reasoning alone to craft their answers. This science item was developed to assess students' basic knowledge and understanding of matter and its transformation (see Figure 4).

3. Carbon-14 has a half-life of approximately 5,700 years. Analysis of the carbon in a piece of charred wood found in an excavation revealed that the carbon has 25 percent of the amount of carbon-14 that is found in the carbon of living trees. Which of the following is most nearly the age of the excavated wood?

A) 160 years B) 5,700 years C) 11,400 years D) 23,000 years

Figure 4: Traditional half-life problem (2000-12S11) given to twelfth-grade students in 2000.

Forty-one percent of the students correctly selected response C, however 34% of the students selected response A. Because this type of problem is so often presented in mathematics texts and classes, students may approach it as a mathematics problem characterizing a linear relationship between the amount of carbon-14 a sample of organic matter contains and its age. Using this model, one might take 25% of 5,700 years and compute an age that is "most nearly" 160 years. Of course, as Greer (1997) suggested, the students could also simply take the numbers provided in the problem and multiply them, without reasoning about relationships very carefully, and incorrectly select response A.

A rather large portion of the students responding to the items discussed in this section seem to have emphasized mathematical reasoning with unproductive results. Their application of mathematical knowledge and reasoning disconnected from knowledge of other content areas explains their choices of distractors. Although power lies in the ability to apply mathematics in context, there is the underlying assumption that knowledge of the context will be integrated with knowledge of the mathematics. Failure to do so shows limitations in students' mathematical power.

Mathematical reasoning and the elimination of distractors

In this section we present items on which mathematical reasoning alone may have helped students eliminate distractors.

In an item drawn from the 2000 NAEP science assessment, twelfth-grade students were asked to interpret a scientific equation representing nuclear decay (see Figure 5). This item was designed to assess basic knowledge and understanding of matter and its transformations. Fifty-eight percent of students correctly selected response C. Fifteen, fourteen, and twelve percent of the students selected the remaining responses A, B, and D respectively.

X → Y + Z + energy

2. The equation above represents a nuclear decay, in which nucleus X decays into particle Y and nucleus Z and releases energy. Which of the following can explain why energy is released in the decay?

A. The mass of X is less than the sum of the masses of Y and Z.

B. The mass of X is less than the difference between the masses of Y and Z.

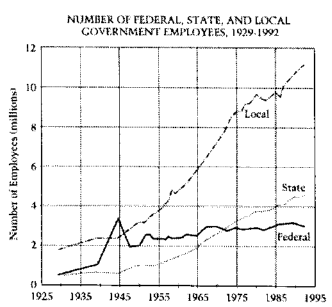
C. The mass of X is greater than the sum of the masses of Y and Z.

D. The mass of X is greater than the difference between the masses of Y and Z.

Figure 5: Science item (2000-12S11) representing nuclear decay.

There are at least two plausible reasons explaining students' success on this item. The student may understand and be able to interpret science equations associated with nuclear decay. We suggest, however, that the mathematical appearance of the item may have prompted students to analyze the item using mathematical reasoning. If one assumes that X, Y, Z, and energy are all positive integers and that X is greater than or equal to Y + Z + energy, then, from a mathematical standpoint, responses A and B can be eliminated immediately. D can then be eliminated because it refers to the 'difference.' Since subtraction is not represented in the problem, D is an inappropriate response. Thus in this item, interpreting the given relationship as a mathematical inequality, without considering any science, can produce a correct response.

On a 1998 NAEP civics assessment item, twelfth-grade students were expected to read and interpret a graph (see Figure 6). Like the congressional district item, the item in



10. Which statement helps to explain the data presented in the graph above?

A. Federal government has been growing much faster than state or local governments because increasing numbers of people rely on the federal government for different services.

B. Local governments employ more people than do state or federal governments because local governments meet the direct needs of so many people in so many different places.

C. State governments employ fewer people than do local governments because state governments run much more efficiently than are *[sic]* local governments.

D. Federal, state, and local governments have increased at the same rates over a 70-year period because the system of federalism divides responsibilities among different levels of government.

Figure 6: Civics item (1998-12C8) that can be simplified with mathematical reasoning.

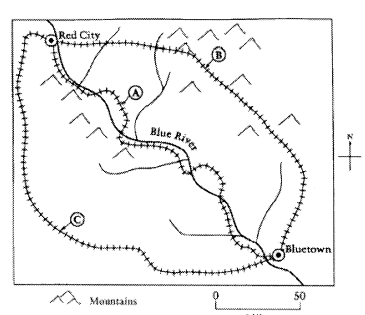
Figure 6 was designed to assess students' ability to analyze and explain the government's embodiment of the purposes, values, and principles of American democracy. In this item however, mathematical reasoning alone cannot be used to select the correct response. Each response includes an analysis of the data in the graph and a justification for the interpretation. A correct analysis of the graph quickly reduces the number of possible responses from 4 to 2. Choices A and D include an incorrect reading of the graph. Only 8% of the students selected A or D. Students may have eliminated distractor C by applying their knowledge of state government or by drawing on their cultural experiences in the USA, related to governance. Hence, not surprisingly, 83% of students selected the correct response to this item by integrating knowledges.

Students' use of mathematical reasoning alone produced mixed results on multiple-choice items set in contexts. In some cases, students were led to distractors using this reasoning, while in others the reasoning supported the elimination of distractors. Regardless of students' success on these items, we suggest that students are applying mathematical reasoning and knowledge to items set in context. This finding suggests students are gaining a weak form of mathematical power in their education. We conjecture that many students have not yet developed the strongest form of mathematical power, integration of mathematics and other content knowledges.

Connecting knowledges

To investigate our conjecture that students have difficulty integrating knowledges further, we examined student work on constructed response items. These items, taken from 1994 NAEP geography assessment, and the 2000 NAEP science assessment could only be answered correctly if the student integrated his or her knowledge of mathematics with knowledge of the content area being tested.

One of the most popular maxims in mathematics is that the *shortest distance between two points is a straight line*. This idea resonates with students since they have had personal experience minimizing their paths between positions. Student performance on a 1994 geography assessment item appears to provide evidence of the popularity of a "shortest distance" approach (see Figure 7).



5. Look at the map above, which shows three possible routes for a railroad line that will be built to connect Red City with Bluetown.
Which route would be the least expensive to construct?
Give two reasons why the route you chose would be the least expensive.

Figure 7: 1994 geography item (1994-8G6 and 1994-12G6) used with grades 8 and 12.

Fifty percent of the eighth-grade students and 30% of the twelfth-grade students provided solutions that were scored as inappropriate. A student response, representative of those scored as inappropriate, illustrates that some students believed that route A would be the cheapest to construct (see Figure 8).

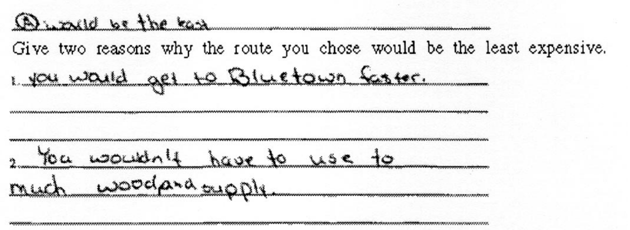

A would be the best
Give two reasons why the route you chose would be the least expensive.
1. you would get to Bluetown faster.
2. You wouldn't have to use to much wood and supply.

Figure 8: Sample student response – NAEP geography item.

One plausible explanation for the selection of route A is an assumption that the shorter the railroad-line the lower the construction costs. For these students, railroad construction costs are constant regardless of terrain. In the sample above, the student was not influenced by the implicit assumption of the item developers that construction costs vary depending on terrain.

Another plausible explanation for students' selection of route A or B is the use of their knowledge constructed from personal experience with railroads. Students who have ridden on trains know that railroads often go over rivers and through mountains. Hence it is plausible that these shorter routes are some how more desirable perhaps in lower construction cost. These two explanations illustrate and suggest that students whose responses were scored as inappropriate may have been interpreting the item using mathematical knowledge and reasoning alone or integrated with knowledge constructed from personal experiences.

The application of mathematical and personal knowledge was also demonstrated in student work from a twelfth-grade science item released in 2000 (2000-12S11). The item asks students to make inferences about the composition of a metal ring (see Figure 9).

One characteristic that can be used to identify pure metals is density. If you determine the density of a pure metal, you can determine what the metal is, as shown in the table below.

Metal	Gold	Lead	Silver	Copper	Tin
Density (gram/cm³)	19.3	11.3	10.5	8.9	7.3

13. Suppose that you determine that the ring has a density of 15.3 grams/cm³. Assuming that the ring is a mixture of some combination of the metals listed in the table, what can you determine about its composition from its calculated density? Explain your answer.

Figure 9: Composition of ring – twelfth-grade task from NAEP 2000 science.

According to the content classification guide, this item was meant to measure students' practical reasoning in the physical sciences. Zero percent of the students' responses were scored as complete, 41% were scored partial, and 35% were scored unsatisfactory. Student responses, scored as partial, did not include reference to the impossibility of finding the exact combination of metals. These responses often identified metals, including gold, based on averaging densities. Students' responses that asserted that the ring was pure gold, or was composed of a combination of metals not including gold, were scored as unsatisfactory.

The sample student responses provide some insight into students' reasoning on this item (see Figure 10).

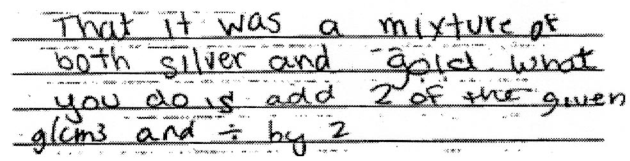

That it was a mixture of both silver and gold. what you do is add 2 of the given g/cm3 and ÷ by 2

Figure 10: Student response to composition of ring task.

A variety of plausible rationales for student reasoning on this problem can be given. The student response in Figure 10 suggests that students' knowledge of the composition of a ring constructed from personal experience may have been integrated with his or her mathematical reasoning. Suggesting that the ring was composed of two precious metals is consistent with an interpretation of a ring as a valued piece of jewelry. Other students who earned a score of *partial* on this item may have used mathematical knowledge and reasoning alone to respond. For example, simply taking the average of the first two entries in the table yields 15.3g/cm³.

Students whose responses were scored as inappropriate may have used mathematical reasoning alone to craft their answers. Consider the student response in Figure 11.

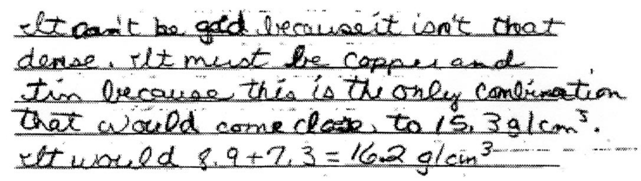

It can't be gold because it isn't that dense. It must be copper and tin because this is the only combination that would come close to 15.3g/cm³. It would 8.9+7.3 = 16.2 g/cm³

Figure 11: Inappropriate response to composition of ring problem.

This student relied on his or her mathematical reasoning to produce and justify the answer. The student suggests he or she summed densities and selected the sum closest to 15.3g/cm³. That the ring might contain gold may have been judged as implausible by the student because it comes into conflict with the students' mathematical reasoning.

The fairly small percentages of students who were successful on items discussed in this section indicate that many students may fail to integrate mathematical and other content knowledge as intended by the item developers. [4] Instead, many students use mathematical reasoning alone or integrate this knowledge with knowledge constructed from personal experience. We consider this evidence of mathematical power. These students can apply mathematical knowledge and reasoning to problems set in context and they do integrate this knowledge with their knowledge constructed from personal experience. Still, some students appear to find the integration of content knowledges difficult. Thus, this analysis suggests that students are failing to develop what may be an important element in mathematical power, *i.e.*, the integration of knowledge from various content areas.

Conclusions

The analysis presented in this paper illustrates students have gained two of the weaker elements of mathematical power. They are generally able to apply mathematical knowledge and reasoning to problems set in contexts. In addition, as indicated in the literature (Boaler, 1993; Cooper and Dunne, 1998) and from our analysis, many students bring their personal and mathematical knowledges to bear on contextual items. Both elements suggest students are gaining a measure of mathematical power from their personal and educational experiences. The difficulty lies in their ability to integrate these knowledges with knowledge of content areas other than mathematics. We contend that this skill is one of the most essential elements of mathematical power. Analysis of student work from problems in other disciplines suggests that some students tend not to integrate knowledge of mathematics with knowledge of other disciplines.

Given the limitations of the data used for this paper it is conceivable that, by interacting with students as they attempt to respond to the items, more information about how students integrate knowledges could be revealed. The methodology used by Cooper and Dunne (1998) could provide much needed insight in furthering the conclusions drawn in this article. As it stands, at best we have speculated about the ways in which students integrate knowledges. Further investigation would allow us to push beyond our speculations and begin to verify our conjectures regarding students' experiences in school that may support their integration of knowledges.

In summary, our analysis of student reasoning on released NAEP items invites us, and other mathematics educators, to question our success in developing students' mathematical power. In order to perform on NAEP assessments in mathematics and other content areas, students must be provided with opportunities to integrate mathematical knowledge and reasoning with other knowledges. These opportunities are an essential factor in the continued development of mathematical power.

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Notes

[1] All data (figures and student work) used for this article are considered public domain and can be accessed from the *NAEP Data Tool* available at http://nces.ed.gov/nationsreportcard/itmrls/. Retrieved, October 2002.

[2] An army bus holds 36 soldiers. If 1128 soldiers are being bused to their training site, how many buses are needed?

[3] In the USA, fourth-grade students are roughly between the ages of nine and eleven. Eighth-grade students are roughly between the ages of thirteen and fifteen. Twelfth graders are roughly between the age of seventeen and nineteen.

[4] *Railroad item* (36% of 8th graders and 54% of 12th graders); *Composition of ring item* (0% of 12th graders).

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