Towards an Understanding of Mathematics Teachers and the Way They Teach

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"Chris Smith is a really good maths teacher!" What picture does this statement about Chris Smith conjure up for you? Does it make a difference to your picture if you are told that the comment above was made by a mathematics adviser? a pupil? another teacher? What do we *actually* mean when we say someone is a good mathematics teacher — because we do say it, and indeed teacher educators and inspectors spend quite a lot of time trying to identify "quality" and "distinction" within the teaching profession The question is undoubtedly a difficult one; it obviously has no single answer; but is it an impossible one? The Mathematics Teaching Project, a three-year research investigation based at the Polytechnic of North London, is attempting to identify the characteristics of good practice in secondary school mathematics classrooms

So what is meant by good practice? For the Project, good practice was originally conceived as that which stimulates mathematical involvement in pupils (that is, a focus on "task" and not on "self" [Hoyles, 1982; Bishop, 1981] and which also provides positive affective response from pupils. This definition has however been widened to include skills at engaging in mathematical dialogue with pupils and expertise in "seeing the meaning" of a particular pupil response.

General background

The incentive to try to characterize mathematics teachers arose from the finding [Hoyles, 1980] that, in the view of pupils, the teacher in the mathematics classroom has a considerable influence on their feelings about the subject and in particular on their confidence and autonomy. In addition, it seemed that such characterisation would be invaluable in teacher education as vehicles for the facilitation of discourse about practice and the incentive for development and change

There is not at present available any comprehensive theory of teaching mathematics which can be used as a framework for research. The nature, direction and choice of instruments of any research is therefore very much a matter for the individual researcher to decide. These decisions inevitably influence the kind of empirical data produced and in turn therefore yield a particular "image of the world", or as Todd [1981] has noted, "There is no scientific understanding of social reality which is independent of theoretical concepts" [page 212]. For these reasons it is felt necessary to make explicit and to justify the guiding principles which have committed the research to specific types of procedure.

The "job" of mathematics teacher is complex and wide-ranging and the first principle of this research is that it should itself reflect this complexity and not attempt, in the interests of simplicity of design, to limit (and thus distort) the investigation to a selected number of predetermined variables. An eclectic approach is therefore adopted which draws on appropriate ideas and borrows relevant research tools from psychology, sociology and interaction studies

The way an individual views a situation and interprets the "hidden agenda" underlying the actions of other actors in the situation is, it is believed, a powerful influence on future attitude and action. The second principle of the research is that the perspective of all the participants who interact in the context of the mathematics classroom should be studied

The importance of affective aspects in mathematics learning is widely recognised, as indeed is the need for more work in this area. The Cockcroft Report stated, "We have already noted the interest displayed by the Assessment of Performance Unit in respect of attitude to mathematics It is clear that there is a need for continuing investigation in this field." [Cockcroft, 1982. p 61]

However it has also been noted that there is often a "gap" between studies of the affective side of mathematics learning and the reality of the classroom [Bishop, 1980; Hoyles and Bishop, 1982] The third principle of this research, therefore, is to study teacher and pupil perceptions using a variety of methodological techniques and compare these with actual response in *real* classroom settings. The crucial issue therefore is firstly to find a way to "tap" those teacher perceptions which do actually appear to affect practice and secondly, to investigate how this happens, i.e how these perceptions together with those of the pupils are mediated in and illuminated by actual classroom episodes.

The fourth principle of the Project is that "good" mathematics teachers require certain qualities which vary, at least in rank, from those of teachers in general. The rationale for this principle comes from the personal experience and belief of the research team as well as from a variety of theoretical sources. Kyriacou [1983], for example, in a recent article on teacher effectiveness in British secondary schools pointed to the need in the future to pay attention to effectiveness within a specific subject area as opposed to a consideration of a global concept. Pollard [1980] uses the idea of "framing" to explain why some lessons may be

highly structured while others leave more room for individual response. The notion of "framing" comes from sociologists such as Bernstein, who has stated that, "Where framing is strong there is a sharp boundary, where framing is weak a blurred boundary, between what may and may not be transmitted." [Bernstein, 1981 in Pollard, 1980, p 44] Pollard argues that some subjects, including mathematics, "lend themselves to a higher degree of explicit behavioural framing than do others". [Pollard, 1980, p. 48] This argument suggests that the nature of the subject (as interpreted by the teacher as an "object" of teaching) will have an influence on the interactions and activities of the classroom. In addition mathematics as a school subject *does* appear to have distinctive features It is generally perceived as abstract, symbolic and precise, demanding economy of notation and a high level of accuracy [see for example Williams, 1963]. Not only might such features affect classroom practice but also research suggests that they can be the root of much of the anxiety related to learning of the subject [see for example Nimier, 1976]

In terms of actual classroom interaction, the marked disparity between pupil/pupil and teacher/pupil discourse in different curriculum areas has often been noted [for example Barnes et al, 1971]. Brissenden [1980] describes one suggested manifestation of this difference: that the amount of information-giving at the start of what he defined as an episode (a descriptor taken from Nuthall and Church [1973] of a short sequence of events in the whole-class part of a lesson) is often distinctively small in a mathematics lesson; sometimes no longer than a lead-in sentence.

The fifth and final guiding principle of the Project is that the teachers in the Project should not merely be "objects" of research but active participating partners with a valuable contribution to make to the discussion. Not only should the aims and methodology of the research be made clear to the teachers but also they should be given the opportunity to give their view of findings and any interpretation of the findings

In summary, as can be seen from the above five principles, the Mathematics Teaching Project aims to "capture" the reality of the classroom of a good mathematics teacher from a range of perspectives, and to convey an awareness as to what it means to be a good mathematics teacher in a way that is meaningful and authentic to an audience of teachers. The Project therefore, it is hoped, is in tune with the Cockcroft Report which strongly emphasized the need for research to be relevant to the classroom and to affect classroom practice.

> "Even when teachers become aware of the existence of a research study on a topic, they very often find it difficult to appreciate its relevance to their own classroom". [Cockroft, 1982]

An enhanced understanding of a range of "particularities" built up from the point of view of the teacher herself, her pupils and from observation and analysis of actual classroom practice has been chosen as the most appropriate means of achieving this awareness.

The three perspectives

(a) *The teacher perspective*

There has been much discussion as to the nature and extent of the influence of teacher expectation on pupils' learning since the publication of the work of Rosenthal and Jacobson [1968] (see Joyles [1973] and Dusek [1975] for reviews of the discussion). Despite the controversy, it would appear that naturalistic studies using teachers' real expectations of their pupils have shown that high and low teacher expectations are related to differential teacher behaviour. Some researchers reported that teachers communicated more frequently and in longer exchanges with pupils deemed to be bright than pupils deemed to be less able [for example, Kester, 1969; Willis, 1970; Bryan, 1974; Lawlor and Lawlor, 1973]. In addition, differences in the quality of interaction have been reported. For example, communications with allegedly bright pupils have been described as friendly, encouraging and accepting [Kester, 1969], or demanding and reinforcing of quality performance and requiring explanation of work [Brophy and Good, 1970; Lawlor and Lawlor, 1973]. These are compared to interactions with allegedly less bright pupils which have been found to be often negative [Willis, 1970] critical [Cooper, 1977], control-oriented [Hargreaves, 1967; Rist, 1970; Cooper, 1977] or helpful and supportive without being challenging [Lawlor and Lawlor, 1973]. It has also been noted that teachers often ignore responses from pupils deemed to be dull [Willis, 1970; Hargreaves, 1967]. Cooper [1977] went further when he noted that teachers tended to perceive interactions with "low-expectation" pupils as time-consuming, and indeed they often used criticism in order to control the timing and content of interactions with these pupils All these studies suggest that teachers behaviour tends to reflect their expectations of their pupils. These expectations have also been found by Johnson et al [1963] to affect the personal qualities and source of motivation a teacher attributes to a pupil.

It has been found that pupils are aware of their teachers' expectations (by, for example, Barker Lunn [1970] and Hargreaves [1967] and this awareness could reasonably be expected to affect pupil response in the classroom. The situation is described by Amidon and Flanders in the following quotation:

> "In the process of this interaction, he (the teacher) influences the children, sometimes intentionally with planned behaviour, sometimes consciously without planning, but often without awareness of his behaviour and the effect of his behaviour on the learning process". [Amidon and Flanders, 1967, p. 1]

Different teacher behaviours have been found to affect pupils in different ways. For example, when the teacher was perceived as having a genuine concern and empathy for the needs of his pupils, the pupils were found to be both more productive and perform better (by, for example, Aspy [1965], Lawrence and Blagg [1974] and have a more positive attitude to work (by, for example, Kallingal [1972] and Bandura and Walters [1963]).

Of particular interest here is research which suggests that

the pupils can in fact pick up how they are seen as learners from their teachers. The early work of Brookover [1965] points to the importance of evaluation by others in the formation of self concept, in particular when the "other" is perceived by the individual concerned to have status and credibility (which presumably is the case when the "other" is a teacher). Brookover also found that self concept of ability was differentiated into specific self concepts corresponding to specific subject areas, a matter of interest in research focused on the mathematics classroom. More recently, Meyer et al [1979] found that praise for success at an easy task and the absence of criticism for failure at such tasks may lead the recipient of these feedbacks to infer that he or she is thought to be of lower ability. A similar point was made by Covington and Omelich [1979] who noted that teachers encourage achievement-through-effort, yet many pupils attempt to avoid the implication that they lack ability by not trying. Thus some teacher behaviours possibly undertaken in order to protect the self concepts of pupils perceived as being weak may actually convey an attributional message which undermines tis goal. Weiner et al [1981] also maintained that the impact of indirect ability cues on self-esteem increases with age. They conclude that, "Affective communications and inferred attributions could be among the emerging criteria that guide self-perception, including self ascriptions for failure." [Weiner et al, 1981, p. 285] Lorenz [1980] notes that the process model proposed by Heckausan [1974] to explain the teacher expectation effect enables several variables to be linked in a meaningful manner. These variables are: "student's self-concept with regard to his own performance, the explanation for this performance supplied by the teacher, the teacher's image of the student's ability, and, finally, the classroom interaction behaviour." [Lorenz, 1980, p. 14) As part of its brief the research proposed in the Mathematics Teaching Project is an attempt to produce empirical evidence related to these linkages.

Attribution theory as mentioned above in the context of the work of Weiner provides many thought-provoking insights of considerable potential for the mathematics teacher. A brief summary of some relevant findings will be given here. The guiding principle of attribution theory is that individuals search for understanding, seeking to discover why an event has occurred [Heider, 1958; Kelly, 1967; Weiner, 1980] Three dimensions of perceived causality have been identified [Weiner, 1979. 1980]: *locus*, that is whether the cause is perceived as residing within (internal to) or outside (external to) the individual concerned; *stability*, that is whether the cause is perceived as temporary or relatively enduring; and *controllability*, that is whether the cause is subject to volitional influence. The way teachers explain the achievement results of their pupils in terms of factors which are differentiated in terms of these dimensions has potential to provide insight into firstly the teachers general frame of reference and secondly how this varies in its application to individual pupils. (Some of the possibilities are given in Lorenz [1980] p. 15)

In the Mathematics Teaching Project, the importance of attribution theory is acknowledged and the way its implications are manifested in the classroom will be investigated. However, teachers' actions are also predicated on their conceptual systems, i.e. on their more general beliefs about the teaching of mathematics and on the general frame of reference used as a basis from which all pupils are perceived and classified. It is therefore intended to investigate a teacher's individual view of the pupils in his or her class, to consider how this might affect the way she/he tends to attribute success or failure amongst the pupils in the class, and finally to ascertain how this conceptual framework and habitual orientation to pupils is seen to influence practice.

In the exploratory work undertaken prior to this study as reported in Hoyles and Bishop [1982], the Rheinberg test was used to investigate differences in reference norm between teachers. As reported in that paper the basis of the test developed by Rheinberg [1977] is that teachers can assess a pupil's achievement as "good" or "bad" according to either the average level of the class (social reference norm NO) or the pupils prior performance (individual reference norm TO) or to a combination of the two. Thus whether a particular result is assessed as a success or as a failure will at least partly depend on the individual qualitative standard of comparison used by the teacher. Lorenz [1980] suggested that these differences in method of comparison would have important consequences for the teacher's interpretation of success or failure and the subsequent sanctions used by the teacher in the classroom situation. He reported after an empirical investigation [Lorenz, 1982] that differences in reference norm did seem to be related to different teacher strategies in the classroom, in particular to the distribution of actions concerned with "helping" the pupils. The exploratory work carried out by Hoyles and Bishop also suggested that data from this test (modified for use with English mathematics teachers) produced quite striking differences between teachers, but the nature of these differences, their meaning in terms of beliefs and expectations and, more importantly, their manifestation in terms of differences in practice, were not clear, though deemed worthy of further investigation.

As well as investigating a teacher's general frame of reference using the Rheinbeg test, the Mathematics Teaching Project intends to study the teacher's views about the pupils in their mathematics classes through the elicitation of personal constructs. Kelly's [1955] theory of Personal Constructs is based on the notion of "constructive alternativism", i.e. that the phenomena of experience may be construed in different ways, each equally valid. The appeal of the theory therefore for this research is that it begins with the uniqueness of an individual's interpretation of any experience and the importance of this interpretation for further action. Kelly proposes that an individual representation of any field of discourse takes the form of a limited number of bi-polar categories, called *constructs*, and developed his Repertory Grid Technique as a means of quantifying the relationship between constructs and of illustrating the internal structure of an individual's repertoire of constructs. In the Mathematics Teaching Project a triadic elicitation technique, by means of an interactive

computer programme PEGASUS [Shaw, 1980], is used. the elements of the grid are pupils taught by the teacher and chosen as "representative" of the Project classes The computer programme allows flexibility in procedure and has the advantage of providing ongoing feedback and analysis. It also enables an immediate cluster analysis of the repertory grid to be made as soon as the decision to end the elicitation procedure is taken. An important aspect of the elicitation in this research is that it is accompanied by indepth interview for reflection on the grid as a whole and for discussion of the meaning of pole names and the interpretation of clusters.

The use of personal constructs as a means of investigating a teacher's perceptions of pupils has been questioned by Hargreaves [1977] (in referring to the work of Nash [1973]). His main points are concerned with the need to establish a dynamic relationship between repertory grid typification and observational data; the need to recognise that the interpretation of the meaning of, for example, pole names is problematic; and the need to take into account the influence of familiarity and context on the teacher's constructs of pupils. These points have all been taken into account in the research design of this Project.

(a) *The pupil perspective*

Many research studies which have focused on the teacher have tended to neglect the views and perceptions of pupils and their interpretation of classroom events, yet these must be of significance in any attempt to build a "picture" of a teacher of mathematics. This perspective will of course be subject to individual pupil variation and, as such, will inform the analysis of classroom interaction (since the way a pupil responds to the teacher will in part be affected by his/her expectations of and judgements of that teacher). It is also believed however that, to a certain extent, a "normative" view of the teacher will emerge which encapsulates the dominant style and mode of teaching.

The study of the pupil perspective has been designed in two parts, each of which consists of two stages [Scott-Hodgetts, 1982]: the first part of the investigation is concerned with the elicitation, and relative standing, of characteristics considered by pupils to be of general importance in mathematics teachers, whilst the second part focuses on the dispositional judgements made by pupils of their current mathematics teacher (i.e those concerning permanent characteristics) and then explores the kinds of episodic judgements which have led to these more generalized attributions [Warr and Knapper, 1968]; in particular, the final stage of the study seeks to clarify the relationship between judgements of specific overt/covert characteristics and individual judgements of overt facts associated with them.

In order to assure that the characteristics selected for consideration be determined by pupils participating in the study, and to establish the classroom as the contextual centre of the investigation, the first stage asks these pupils to write a description of an "ideal" mathematics teacher; they are told to imagine that they personally have "ordered" this teacher, who has now arrived to teach their class. Using a method of content analysis, the most "popular" factors can be chosen for further consideration

Earlier studies have been undertaken specifically to elicit characteristics of "good" teacher [e.g. Taylor, 1962; Nash, 1974], and the broader studies of pupils' views of school have included sections on teachers [e.g. Woods, 1976; Furlong, 1976], but each of these studies came under criticism [Hargreaves, 1977]. Hargreaves sees as one main problem the static nature of the models being used, and points to the study by Gannaway, which proposes that teachers are progressively evaluated by means of subtle tests, as being the first in this area of research to recognise that typification is an on-going process [Gannaway, 1976]. Hargreaves maintains that there are at least three analytically distinct stages through which a pupil's typification of their teacher will progress; this has been taken into account in the planning of this research investigation, and it has been decided that all data from pupils should be collected during the summer term in order that a complex but stable teacher typification will have been acquired.

More recently, two studies involving pupils' perceptions of student teachers have yielded interesting results [Meigham 1978; Wilson 1981], but Kyriacou [1983] points out that since it is easier for pupils to comment on inadequacies rather than on positive aspects of a teacher's teaching, these studies tend only to provide feedback to the particular teachers concerned as to where they were going wrong. It therefore could be argued that this approach may only have limited value in provoking mathematics teachers in general to reflect on their practice. It is an awareness of this relative difficulty in finding exemplars of "good" practice which has led the research team in this Project to put a strong emphasis on positive factors.

Another comment made by Kyriacou which has been pre-empted by the design of this Project concerns the need for studies to take more account of the subject being taught, and of the age and ability of the pupils. In fact some pioneering work has been completed which relates specifically to mathematics teaching in, for example, Kiryluk [1980]; Hoyles [1982]; Dickins and Wood [1983], but no in-depth investigation of pupil typification has previously been undertaken involving such a well-defined sample.

A further question ignored by most earlier studies is that of the *relative* importance of those factors deemed to be desirable: is it valid to ask pupils to rank the factors, and if so how consistent will these rankings be? It is clear that simply asking the pupils to assign a rating to each factor would not yield convincing results, and a more sophisticated method is needed in order to overcome the problems of validity. The method of paired comparisons [Kendall, 1975] fulfils this need by providing a measure of agreement within a group of respondents, as well as an indication of the degree of consistency in the preferences of each individual respondent. A computer program has been developed [Scott-Hodgetts, 1983] which facilitates the analysis and representation of the responses of various subsets of the sample, in order to ascertain the extent to which the views of these groups differ.

Having determined, by means of a written questionnaire, which positive factors each pupil attributes to their teacher, a semi-structured interview will be used to investi-

gate the ways these characteristics are manifested (in the pupil's view) in the classroom; that is, actual episodes, recalled by the pupil to exemplify a certain "good" teacher characteristic, will be collected and examined. Thus an attempt is made to 'capture' the *pupil* perception of the teacher and to probe the *pupil* interpretation of the teacher factors which had been ranked in the written questionnaire. This approach, based on the descriptions of real situations rather than the collection of generalities, is felt to be more meaningful to the pupils and will allow an analysis of the teacher factors from the pupils' point of view. Although this strategy was arrived at independently, a rationale for the collection of accounts of actual events experienced by pupils can be found in the work of Bliss and Ogborn [1977] and Hoyles [1980].

A second point raised by Hargreaves is the possibility that it may not be valid to assume that data collected in a situation where the respondent (pupil) is required to make typifications of the other (teacher) outside their direct face-to-face relationship is representative of the typifications that take place within that relationship. It is argued that the contextualisation described above goes some way towards allaying those fears (and is indeed the sort of refinement suggested by Hargreaves), but of primary importance is the opportunity afforded by the Project design to relate the typification data to that obtained in the classroom.

Bill Brookes, in his introduction to Judy Morgan's report on the affective consequencs for the learning and teaching of mathematics of an individualised learning programme, made the following comments:

> "Investigations of the affective characteristics of the conditions of learning have been rare ... it is significant that when such investigations have been attempted, findings have been so circumspect that they have not found a way into the reality of the practice of teaching. One of the reasons for this is that the language of the researcher is not likely to be that of the teacher. Hitherto, this has resulted in pressure for teachers to learn about research so that they can appreciate research findings ... another possibility is that those with the facilities and time to enquire should learn about the ways in which a teacher actually views his work, how he asks questions and how he accommodates to problems and difficulties in his work so that 'real' issues are tackled." [Morgan, 1977]

Perhaps the most valid support for the study of this perspective has come from the teachers involved in the Project, who feel very strongly that there is a need for this feedback from the pupils, and that they can learn in a positive way from it; this view has also been expressed by other practising teachers who are not directly involved. It is hoped to present the findings in a way which takes account of the comments quoted above, as well as addressing the need identified by Hart [1983], to help teachers to distinguish between research and opinion.

(c) *The mathematics classroom*

As well as undertaking detailed investigations of the teacher and pupil framework as described above, the research project is studying the interactions and strategies of the teachers in actual classroom settings. This is a crucial aspect of the work since, as Hargreaves [1977] said, "Typification research may throw little light on how people interact, just as much attitude research shows a notoriously low correlation with people's conduct [Deutscher, 1965] and for the same reason, the testing situation is a special situation of its own which has too few equivalents in the everyday life of people. There is overlap between the two settings but there are additional factors in operation in direct interaction, which are not at work in third party talk, so the major research task before us now is to specify and analyse those additional features and their relation to the typification process. We must follow Schutz's lead and strive for a model of typification." [p. 282]

It thus seems inevitable that any attempt to characterise mathematics teachers must endeavour to get to grips with the "we-relation" between teachers and pupils in the classroom. [Hargreaves, 1977, p. 282] The gap between espoused theory and theory in action has been noted by many sociologists in a wide range of situations. For example, Elliott and Adelmann showed that some teachers "theories" logically entailed a gap between aspiration and practice. [Elliott, 1976 p. 15] Another perspective on this performance gap is also likely to occur in the classroom at the task level, that is between the tasks set for pupils by teachers and the task pupils actually come to work upon; and at the interaction level, that is between the message intended to be sent by the teacher, the words actually spoken and the message picked up by the pupil. The problems associated with the analysis of classroom interactions has been widely documented and indeed approaches to the question vary widely according to the theoretical position and the research aims of the study.

Studies in which the classroom is itself accepted as a legitimate area of study can be divided essentially into two major categories: studies using a pre-designed observation schedule of a limited number of variables, and ethnomethodological studies where a wide variety of sources of data (including observation, teacher interview, etc.) are used to probe the meaning behind classroom interactions. The latter paradigm, adopted here, was pioneered in 1972 by Parlett and Hamilton, and to a certain extent by Nash in *Classrooms observed* [1973], and has been taken up for extensive work in the "Teacher-Pupil Interaction and Quality of Learning Project" based at the Cambridge Institute of Education.

In reviewing research specifically in the area of mathematics education it is noticeable that, until quite recently, very little has been actually done, in the classroom. However, again a change in paradigm has occurred, stimulated by the influential work of Bauersfeld [1976] in Germany. He wrote:

> "The orientation to behaviouristic research paradigms borrowed from the natural sciences ... results in insufficient theoretical effort; that paradoxically, the intelligence is prevented from reflecting upon its own activity ... the *construction* of mathematical con-

cepts. Numerous studies on mathematics learning are mostly statistical in character, and accordingly focussed on average achievements and learning results, not learning processes, thus tending to conceal the decisive *recognition that mathematical insight ... may be achieved in individually highly different learning processes*" (Author's underlining)

It is the identification of these learning processes as they take place within the classroom context that is one of the aims of the Project.

There has been some research work undertaken in mathematics classroom in U.K. which has influenced the Project design: for example, Joan Yates' study of "Four Mathematics Classrooms: A Study into Teaching Method" [1978] in which an open field work style of observation was used; and Nigel Langdon's work on "Teacher-Pupil Interaction in SMILE" [1981], a case study of one London teacher. Overall the approach adopted here is to recognise that classroom research does not merely seek to reproduce the original features of the situation under study but probes beneath the surface and makes connections between empirical data and theoretical frameworks. It must therefore be the researchers' task to make statements of interpretation of meaning. The stance of this investigation is to make public the rationale for the interpretations, and the choice of illustrative extracts, by first spelling out the parameters within which the observations are made and the transcripts considered. This approach would be in line with the notion of "Theoretical sampling" described by Todd [1981] whereby selections (in this case from transcripts) are made on criteria of theoretical purpose and relevance.

The investigation

Nine teachers in three mixed London comprehensive schools well known for their enthusiastic and competent mathematics departments have been chosen to participate in the Project. Two fourth years (14 year old) clases are to be studied for each teacher, one of high ability and one of middle/low ability. The following investigations are being undertaken:

The teacher perspective

- Elicitation of the teachers' personal constructs of their mathematics pupils.
- Cluster analysis of these constructs.
- Reflection and discussion on the nature of the constructs and clusters.
- Elicitation of the teachers' attributions of success or failure in a mathematical task for each individual pupil.
- Analysis of these attributions in terms of the three dimensions of perceived causality.
- Test of the reference norm and calculation of the teacher typification on the ideographic/normative continuum.

The pupil perspective

- Elicitation from all the pupils in the experimental classes of written descriptions of an "ideal" mathematics teacher.
- Extraction from these descriptions of up to ten of the most frequently mentioned factors
- Marking of the above factors by the use of a paired-comparison test completed by all the pupils
- Grading of the teachers by each of their pupils on a numeric scale for each of the factors.
- Individual interview of each pupil in order to obtain a description of classroom events chosen by the pupil as practical manifestations of one or more of the positive characteristics which they had attributed to their teacher.

The mathematics classroom

- Weekly observation and audio taping of the two teachers.
- "Reconstruction" of a selection of lessons in which observational notes of teacher comment (prior- and post- lesson), actions or non-verbal gestures are co-ordinated with sections of transcript in order to "bring the lesson to life".
- Analysis of the reconstructions in terms of the following eight categories: mode of teaching: instructional or interactional; type of questions; influences on content and pacing; assumptions made by the teacher; repetitions; vocabulary of the lesson; pupils' language; teacher's use of praise.
- Comparison of the two teachers in terms of the profiles drawn up on the basis of the above categorial system.
- Indentification from the transcripts and reconstructions of episodes of mathematical communication and development (using Grouws' work [1982] as a basis) and their analysis

Conclusion

So how far have we got along the road to a description of a good mathematics teacher? The answer is — not far yet, but the journey is fascinating and the outlook promising! What can be said, even at this stage in the work, is that the teacher characterisation, built up from the three perspectives, seems authentic and well-defined. There are surprises and inconsistencies which are interesting to identify and interpret but overall the pieces of the picture fit together neatly. A feeling of "knowing" the teacher is generated after reading all the data; one could almost feel confident enough to predict future responses and approaches! The cross-referencing between the three perspectives has been illuminating from each standpoint and has stimulated fresh ideas and interpretations. It has also provoked much discussion amongst the participants which is perhaps the most encouraging feature so far; since when the case studies are presented to teachers we would hope that there will be argument, more argument and then finally self-analysis and insight

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