

RECOUNTING CANTOR

DICK TAHTA

The life and work of Georg Cantor has been well documented by various scholars. In this article, I want to revisit some aspects that I believe could usefully be interpreted from a psychodynamic point of view. In recapitulating some historical details, I rely mainly on the scholarly accounts of Joseph Dauben and Ivor Grattan-Guinness. [1] I do not write as a historian or as a therapist, and here I can only sketch what others would be more qualified to flesh out in more detail. I would like to emphasise that anything I might imply about Cantor's inner life can only be conjectural.

Background

Cantor's father was the eldest of six children – the family had moved from Denmark to Russia; his wholesale trading firm went bankrupt in 1839 and he then became a stockbroker. In 1842, he married Maria Böhm, who came from a St Petersburg family of musicians. Some writers have claimed that Cantor's parents were of Jewish descent – the only evidence for this being a brief reference by Cantor in a letter to his father's Israelite ancestry. But in any case, his father was a devout Christian, who had been educated at an evangelical Lutheran mission, and his mother, who was born a Roman Catholic, adopted her husband's religion.

Georg Cantor was born in 1845 in St Petersburg. His brother Ludwig was born the following year, a sister, Sophie, two years later and another brother, Constantin, a year after that: so, Georg had three siblings by the time he was four years old. Their father retired in 1856 due to bad health and the family moved to a warmer climate in Germany, settling eventually in Frankfurt. Georg was sent to board at various schools.

On the occasion of his confirmation at the age of 15, Georg received a remarkable letter from his father which he is said to have kept with him always. This contained much worthy paternal advice about the “unforeseen and *unforeseeable* calamities and difficulties” he would have to fight, with warnings of the way even the most promising may be “defeated after a tenuous, weak resistance in their first serious struggle”, and exhortations to acquire “diverse technical knowledge and skills” in order to overcome the “jealousy and slander of open or secret enemies”.

I close with these words: Your father, or rather your parents and all other members of the family both in Germany and in Russia and in Denmark have their eyes on *you* as the eldest, and expect you to be nothing *less* than a Theodor Schaeffer [one of Georg's teachers] and, God willing, later perhaps a *shining star* on the horizon of science.

May God give you *strength*, persistence, health, sound character, and His best blessings! And therefore *you* should follow only his ways. Amen! [2]

In later life, Cantor was to be haunted by a sense of failure; and this may well have included a sense of having failed to meet the expectations heavily laid upon him in this letter.

Eric Bell, in his much criticised popularising account of Cantor [3], differs in his translation of the above extract by referring to “the engineering firmament” rather than “the horizon of science”. But Dauben has shown that this was to repeat a mistake made by a previous editor of the original letter [4]. This slightly modifies the force of Bell's suggestion that the father “recognising the boy's mathematical ability, obstinately tried to force him into engineering as a more promising bread-and-butter profession”. Nevertheless, he did spend two years at a *Gewerbeschule* or trade school, and Dauben notes that when he expressed a wish to study mathematics at university he “admitted the anxiety he had suffered in wanting to devote himself to science, while feeling a duty to honour his father's wish that he do something more practical”. When his father consented, Georg replied gratefully: “now I am happy when I see that it will no longer distress you if I follow my own feelings in this decision”. He goes on to hope that his father will still be proud of him one day and refers – significantly in hindsight – to “an unknown secret voice” (Bell phrases this as an “inner compulsion”) calling him to success.

In countering Bell's account of the father-son relationship, Dauben suggests that the father was “a sensitive and gifted man, who loved his children deeply and wanted them to live happy, successful, and rewarding lives”, and that the son “wanted nothing more than to fulfill the hopes of his parents”. But praiseworthy sentiments of this sort certainly do not preclude other more hostile – albeit sometimes unconscious – feelings on either side. It is, surely, a biographic over-simplification to ignore our present understanding of the possible ambivalence of family relationships. Fathers may well love their children deeply, but they may *also* be at some level deeply envious of them; sons may well want to be a credit to their father, but they may *also* want at some level want to destroy them. In throwing away Bell's bath-water, we risk throwing away the oedipal baby.

Cantor began his university studies in Zurich in 1862. Various letters from this time indicate that his father was still stressing the importance of not concentrating too exclusively on one subject – interests in art and music were enthusiastically approved. They also reveal that Georg was prone to anxieties about his work and about the costs of his education. Given the obvious importance of the father-son relationship – however it is interpreted – it is surprising that there has been little comment on the possible effects on the son of his father's death of tuberculosis in June, 1863. By the autumn, the family had moved to Berlin where Cantor was to continue his university studies, specialising in mathemat-

ics and philosophy. This is when he first came into contact with Kronecker, who was to become an ambivalent father-figure in his later professional life. Some recent writers have been as scathing about those earlier views that linked Cantor's later breakdowns with his conflicts with Kronecker as they have been dismissive of supposed links with unconscious conflicts with his real father. But in correcting the unsupported claims of previous historians, they seem to ignore any understanding of the way we internalise our parent figures and the emotions that are aroused when actual parents die.

Though sons may want to please their fathers, they also want to replace or surpass them. Freud wrote movingly about his ambivalent feelings when he stood for the first time on the Acropolis and recollected how much his father would have liked to have done that. He wrote of his sense of guilt about his enjoyment:

It was something to do with a child's criticism of his father, with the undervaluation which took the place of the overvaluation of earlier childhood. It seems as though the essence of success was to have got further than one's father, and as though to excel one's father was still something forbidden. [5]

The guilt that Freud refers to may be particularly manifest when a father dies. In Cantor's case, he had not only surpassed his father by getting to university, but also by being alive. It is a strange comment on our aspirations that the pleasure of achieving them can be soon tempered by guilt about our success. It would be contrary to everything we know about bereavement not to suppose that 1863 was indeed a crucial year for Georg Cantor.

Incidents

Another crucial year was undoubtedly 1874, when he married Vally Guttmann, who came from a Jewish family in Berlin, and when he wrote his first paper on infinite sets – with a proof that the reals were uncountable. The following decade was enormously productive for the young couple: she gave birth to four daughters and two sons, and he produced a stream of original articles and an important book. By his forties, Cantor was a respected professor at Halle University, with a large family, a large house and a growing reputation. But there had been an ominous warning of strain in 1884 – another crucial year – when he suffered the first of what was to become in his later years a series of mental breakdowns.

There is some mystery in the accounts given of this episode. It is known from letters to various correspondents that he was visiting Paris at the end of April of that year, calling upon French mathematicians and enjoying visits to theatres and galleries. Then, according to Dauben, he was “unexpectedly called back to Frankfurt to deal with some unknown family matters”. Whatever the business was, he seems to have been back home at Halle in May when, according to a later account by his eldest daughter, he began to behave in a strange and upsetting way. His mental depression seemed to have lasted about a month: by the end of June he is referring to his illness in letters and taking a holiday in a summer resort. In August he wrote to Kronecker about his

unhappiness over their rivalry; the reply was friendly – with warm references to Cantor's time as a student at Berlin and expressing some surprise that it had been supposed there was now any animosity between them.

This was also the time when Cantor became more and more anxious about proving the so-called continuum hypothesis, namely that the real numbers corresponded to his second transfinite number, *aleph one*. He worked on this again in the autumn of 1884, repeatedly announcing a proof in one letter and retracting it in the next. In the following year he had an article rejected by his hitherto sympathetic friend Gösta Mittag-Leffler and Cantor began to believe that he was being persecuted by Kronecker and the rest of the Berlin mathematical establishment. [6]

These experiences seem to have discouraged him so much that he became increasingly involved over the next decade in a number of other interests. He published various philosophical articles and, moreover, once applied unsuccessfully for his appointment to be changed to a chair in philosophy. He became preoccupied with Rosicrucianism, Freemasonry and Theosophy, as well as the theory that the Shakespearean plays were really written by Bacon. He did also, however, maintain some of his mathematical interests: he continued to try, fruitlessly, to prove the *continuum hypothesis*; he helped found a national organisation for mathematicians in 1890; he published an important exposition of his work in 1895, when he was 50 years old; in 1897, he attended the first *International Congress of Mathematicians*, which he had also actively promoted.

He seems to have suffered from occasional depressions during this period. But in his fifties these became serious enough to involve hospitalisation for various periods. These breakdowns followed on a period of stress in which he complained of persecution by the University authorities. He was also disturbed by various deaths during this time: his mother died in 1896, and his youngest brother in 1899, another crucial year which ended even more tragically with the death of his youngest child, Rudolf, at the age of 12. In a moving letter to Felix Klein, written a week later, Cantor described the boy's musical talent and how this had reminded him of his own early interest in music: disillusioned with his mathematical career, he mourned for the alternative that he might have enjoyed vicariously through his son.

His son's death was a bitter blow from which Cantor never really recovered, though he continued to work in various ways: for example, he attended the third *International Congress* in 1904, where he was said to have behaved somewhat wildly after Jules König had presented a purported disproof of the *continuum hypothesis*. Accounts of this episode confirm other stories about his argumentative and dominating manner with colleagues; these compare curiously with what seems to have been a withdrawn role in his family. [7] He retired from his university post in 1905, at the age of 60 and subsequently spent increasingly longer periods in hospital up to his sad and lonely death in 1918.

Interpretations

Recent writers have been concerned to disprove earlier accounts of Cantor's breakdowns as being due to his reactions to external strains and stresses such as his despair over his

inability to prove the *continuum hypothesis*, or the antagonism towards his ideas from some of his contemporaries. Ivor Grattan-Guinness notes that the breakdowns were signalled by manic periods when Cantor would be admitted to hospital followed by a depressive phase in which he would be sent home to sit silently for hours. He observes that “the rudimentary nature of psychological treatment at that time precludes the possibility of a definitive professional evaluation of his mental illness”. But it is then surprising that he invokes what many would now feel is a very rudimentary notion, that of an *endogenous* depression. [8] Many psychiatrists used to be taught to distinguish between reactive and endogenous depressions. In the first case, some external factor is found to be the cause of the depression; but in the second case, the depression arises “from within” and this was usually taken to mean some innate genetic disposition. With this latter diagnosis, factors such as the problems incurred in his research or those encountered with colleagues become, for Grattan-Guinness, “little more than the clap that starts the avalanche”. He goes on to suggest that nowadays Cantor would have been treated “probably successfully” by drugs. We might add, he could have now also been subjected to ECT.

The issue about different diagnoses of depression became important because different treatments could be involved. But a clear-cut binary distinction must surely be too simple. According to Anthony Storr, the orthodox view he was given in his training as a psychiatrist merely cloaked medical ignorance.

In cases labelled reactive, the doctor would define an obvious cause for depression, like bereavement or bankruptcy, which he could see might cause depression in himself. Where no such obvious circumstance could be discerned, the doctor concealed his ignorance by calling the patient’s condition ‘endogenous’. [This word] ought to be forbidden in psychiatry [...]. Depression is not an illness which one catches like influenza, but a psychological reaction which can be provoked in any of us, given appropriate circumstances. [9]

Depression in this point of view is not a physical condition – though many people still prefer to see it as something like diabetes or epilepsy which can be controlled by drugs and endured without guilt. Nor is it always a perfectly natural reaction to particular events. Early biographers had been convinced that there were sufficiently depressing external circumstances that explained Cantor’s breakdowns. [10] But Storr calls attention to the nature of the psychological reaction rather than the specific triggers. It would not be unreasonable to speculate *why* such triggers had such a particularly devastating effect on Cantor. Not every loud clap starts an avalanche.

The metaphor of an avalanche is striking in this case, for it suggests a pile of stuff, an accumulation of feeling, held back – dammed up, repressed – until it erupts at the sound of a clap. A well-known therapist, Dorothy Rowe, has emphasised that a depressed person would not be too concerned to know who or what causes their condition: “All they know is that they feel very guilty, because they are depressed”. That’s bad enough, she writes, but the guilt is worse: “such guilt serves to strengthen the walls of the prison of depres-

sion”. [11] Moreover, I would suggest, this can also be reversed: depression may be a reaction to guilt. These seem useful ways of looking at the problem, for guilt is a universal and understandable feature of our lives and we do know something about the ways in which it can be triggered, denied or maintained.

The factors that might trigger feelings of guilt – and associated depression – are most likely to be found within relationships, especially those close relationships that mirror or reproduce in some way our earliest ones. This is why it might be important to consider the relationship between Cantor and his father, between Cantor and his brothers, between Cantor and his mother, and to think of the ways in which these might have been reflected in relationships with figures of authority, with his colleagues, with his wife. Bell may have been crude and inaccurate in his attempts to do this, but subsequent writers have sometimes smothered the issue in their urge to put him right. Inevitably, we are now somewhat trapped between the Scylla of the earlier commentators, who made rather crude psychoanalytical interpretations, and the Charybdis of the revisionists, who adopt a crude behaviourist approach.

What we can do is to look more closely at some of Cantor’s preoccupations in the later part of his life, including those from his manic phases. A recurrent theme that appears in different guises is that of a conspiracy. At one time, he supposed that his son had been killed by English agents, at another that there was a Freemasons’ plot, headed by Edward VII, to conquer the world. At various other times, he was gripped by paranoid suspicions – of the Berlin mathematicians, of the Halle University authorities, of the Ministry of Education, of those who were trying to disprove his *continuum hypothesis*. In his later years, he claimed descent variously from Nicholas II of Russia, from Henry VIII, from the Stuarts, from the Hapsburgs, and from a Spanish ancestor from Canterbury. He once claimed that Christ was only adopted by Mary, being the son of Mary Magdalene by Joseph of Arimathea. [12] These claims may be compared with his views on the authorship of Shakespeare’s plays: the common element is a denial of actual paternity.

Another, not unrelated, element is the concern to set things right, to establish the final truth against all temporary misguided error. “There is no honourable immortal place for those not worthy of it”, he wrote. And when he replaced Shakespeare by Bacon in the pantheon of immortality, we may speculate that he also meant that Kronecker should be replaced by himself. His own immortality was assured because he felt his work had been inspired – he had written to his father when he was 17 about the “unknown secret voice calling him to success”. His conviction of the truth of his theory of infinite sets, of the manifest existence of an actual, and conceivable, infinity stemmed from a passionate belief in direct inspiration from an actual, but incomprehensible, Absolute. Even before his first breakdown, he had told Mittag-Leffler that his theory had been given to him by “a more powerful energy”: Cantor felt that he had been specially directed to transmit God’s message to mathematicians. He was so vehement about this that it is tempting to suppose that some denial was involved. Dis-

avowing any failure would save him from feelings of guilt about betrayal – betrayal of the direct inspiration which he had been uniquely vouchsafed and also, perhaps, betrayal of the promises made to his earthly father. Depression can have positive uses.

Further implications

My main purpose in trying to reopen the case of Cantor is not, however, to make any further naive interpretations about Cantor's inner life. Applying psychoanalytic insights to figures from the past is a tricky enterprise because, of course, such insights arise as part of a dialogue, an exchange of conjectures by one party followed by refutations or verifications by the other. Cantor himself cannot now confirm or reject our interpretations. But in implying some possibilities myself, I have really wanted to draw attention to the role of unconscious processes in the creation of mathematics. I have tried to suggest that Cantor's life was "all of a piece" – not a matter of unconnected successive phases of creativity and despair. Many of the classical themes of psychoanalysis can be traced in Cantor's life without any denigration of his achievement as a mathematician or a man. His work illustrates the notion that mathematics stems from the unconscious as much as it derives from the external world. Such a completely "pure" topic as the theory of infinite sets could be said to have arisen only from some sort of self-observation [13]; moreover, its denial of tenets previously held to be self-evident – such as the assumption that the whole is greater than any part – confirms that what is being observed is not usual conscious thought.

That the theory of infinite sets might be an appropriate mathematisation of unconscious process has also been considered by the psychoanalyst Ignacio Matte Blanco, who has invoked Cantorian theory to explain a "logic of the unconscious". [14] In infinite set theory, a subset may be matched with the whole set: the part can be equal to the whole and all the subsets of this sort can be held in a sense to count the same. Matte Blanco suggests that it is a corresponding feature of the unconscious that the part may stand for the whole, that there is no order and so no difference. For example, in unconscious thought, Shakespeare is Kronecker and Bacon is Cantor, Father is Absolute and his voice is the inner compulsion – the secret voice within, Rudolf the musical son is the musical Georg that might have been, and so on. If this way of interpreting Cantor's inner life is accepted, then it casts another light on some of the issues raised by his biographers.

Finally, I note that what I have been conjecturing has some implications for the various controversies that have surrounded Cantor's theory. If the theory is indeed a mathematisation of unconscious process, then a denial of its validity – for example, by finitists – involves a psychology that logicians have usually been at pains to discount. A psychoanalytic account offers a possible way of reconciling the sharply different ontological views about mathematical objects held by mathematicians and mathematics educators.

Notes

[1] Most of the historical details I recount (without giving any specific page reference in each case) are taken from Dauben, J. (1979) *Georg Cantor: his methods and philosophy of the infinite*, Harvard University Press and Grattan-Guinness, I. (1971) 'Towards a biography of Georg Cantor', *Annals of Science* 27(4), 345–392. Both authors provide detailed references to the various letters that are our main source for the important episodes in Cantor's life.

[2] The letter is given in full in Dauben, *op.cit.*, pp. 274–5.

[3] Bell, E. (1953) *Men of mathematics* 2, Pelican, chapter 29. Grattan-Guinness has called this "perhaps the most widely read modern book on the history of mathematics" and adds, "As it is also one of the worst, it can be said to have done considerable disservice to the profession" (*op.cit.*, p. 350, note 3). Without disputing the historical judgement, I can add that it certainly inspired me, and some of my sixth-form students, in the nineteen-fifties.

[4] Dauben translates from a typescript version of the original letter which refers to the horizon of *Wissenschaft* – this had been transcribed by an earlier biographer, Abraham Fraenkel, as *Ingenieure* (see Dauben, *op.cit.*, p. 356, note 4).

[5] Freud, S. (1936/1964) 'A disturbance of memory on the Acropolis', in Strachey, J. (ed.) *The standard edition of the complete psychological works of Sigmund Freud*, 22, Hogarth Press, p. 247.

[6] It is not surprising that Cantor often measured himself against Kronecker, for there are many significant similarities and differences in their lives. Both gained doctorates at the age of 22; both married in their late twenties and fathered six children while pursuing their mathematical researches. But whereas Kronecker became increasingly influential and enjoyed wealth, travel and fame, Cantor struggled for recognition, was beset by self-doubts and increasingly subject to mental breakdowns.

[7] According to Grattan-Guinness (p. 358), quoting a reminiscence of Cantor's eldest daughter, "he made no effort to dominate the household: indeed, at mealtimes he would sit silently and allow his children to lead the conversation, and then rise and thank his wife for the meal with: Are you content with me and do you then also love me?" This strange remark inevitably prompts speculations about the insecurity that would prompt such a regularly repeated question.

[8] In quoting this opinion, Dauben states that it was based on the advice of a psychologist who had examined Cantor's files in the hospital to which he was known to have been usually admitted. But this must have been a personal communication, for there is no reference to this in Grattan-Guinness's article.

[9] Quoted in Rowe, D. (1987) *Beyond fear*, Fontana, p. 285.

[10] This is also the view of the mathematician Laurence Young, whose parents (also mathematicians) were close friends of Cantor; he was scornful of the notion that the hostility of a number of fellow mathematicians would not dramatically affect a sensitive person like Cantor (see Young, L., 1981, *Mathematicians and their times*, North-Holland, p. 232).

[11] Rowe, D., *op.cit.*, p. 287.

[12] See Hermann, I. (1980) *Parallélismes*, Denoël, p. 237. This book is a translated collection of various articles – written in the nineteen-forties by a Hungarian psychoanalyst – on some nineteenth-century mathematicians and scientists.

[13] Cf. Spencer Brown, G. (1969) *Laws of form*, Allen and Unwin, p. xv:

In arriving at proofs, I have often been struck by the apparent alignment of mathematics with psycho-analytic theory. In each discipline we attempt to find out, by a mixture of contemplation, symbolic representation, communion, and communication, what it is we already know.

[14] Matte Blanco, I. (1975) *The unconscious as infinite sets*, Duckworth. An application of Matte Blanco's work in a discussion of mathematical textbooks may be found in Mordant, I. (1993) 'Psychodynamics of mathematics texts', *For the Learning of Mathematics* 13(1), 20–23.